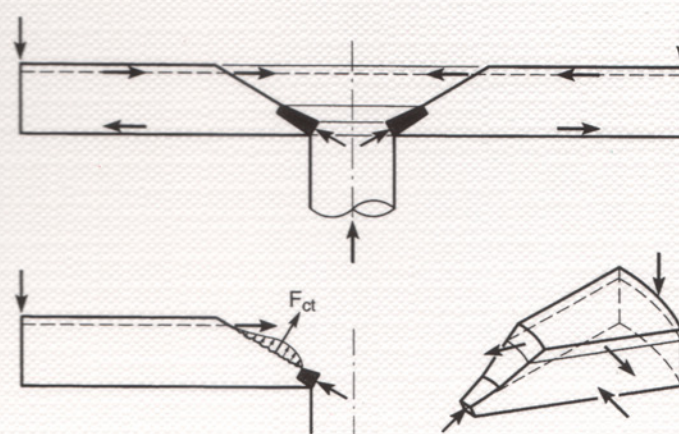




a) Model with concrete tension

b) Model by Kinnunen / Nylander

Punching of structural concrete slabs

Punching of structural concrete slabs

Technical report prepared by the CEB/*fib* Task Group

Utilisation of concrete tension in design

dedicated to

Prof. Sven Kinnunen

on the occasion of his 70th anniversary

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Technical Report	approved by a Task Group and the Chairpersons of the Commission
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Manual or Guide (to good practice)	approved by the Steering Committee of <i>fib</i> or its Publication Board
Recommendation	approved by the Council of <i>fib</i>
Model Code	approved by the General Assembly of <i>fib</i>

Any publication not having met the above requirements will be clearly identified as preliminary draft.

This Bulletin N° 12 has been suggested as a *fib* technical report in June 2000 by the Working Party on *Punching* of the Task Group *Utilisation of concrete tension in design* and has been recommended for publication by selected reviewers of *fib* Commission 4 *Modelling of structural behaviour and design*.

The Task Group on *Utilisation of concrete tension in design* started as a Task Group of former CEB Commission 2 *Material and Behaviour Modelling*. CEB and FIP merged in 1998 into *fib*. This report, therefore, is published in the new *fib* series of bulletins. Chapters authored by members of the Working Party are indicated below:

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Cover figure: Models for punching, see Fig. 4.10-1

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First published 2001 by the International Federation for Structural Concrete (*fib*)

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ISSN 1562-3610

ISBN 2-88394-052-5

Printed by Sprint-Druck Stuttgart

Foreword

The present report aims at supplementing the state-of-the-art report on punching shear in reinforced concrete published in CEB Bulletin 168 in 1985. Apart from new theoretical developments a comprehensive databank for comparisons with the experimental evidence is included. It is hoped that this report will encourage and facilitate further research and development in the field of punching and that it will contribute to an improved understanding and use of structural concrete.

The work on this report commenced in 1996 within CEB and was continued within the framework of *fib* Commission 4 in 1998. The 40th anniversary of the initial publication of the Kinnunen-Nylander model and the 70th birthday of Professor Kinnunen provide a fitting occasion for the publication of this report.

On behalf of *fib* Commission 4 I would like to sincerely thank Dr. Reineck and his group for their effort in putting together this valuable document.

Zurich, May 2000

Peter Marti
Chairman of *fib* Commission 4

Preface

Punching is one of the most difficult design problems in structural concrete so that relatively late in the history of concrete research attempts were made to develop mechanical models or theoretical analyses. Professor Sven Kinnunen started this development and contributed significantly to rationalize structural concrete design, and therefore this *fib* Bulletin "Punching of Structural Concrete Slabs" is dedicated to him.

This *fib* Bulletin reviews the development of design models and theoretical analyses for punching since the CEB Bulletin 168 was published in 1985. Thereby a special focus was placed on the role of the concrete tensile strength. In this respect this Bulletin is a consequent follow-up of the CEB Bulletin 237 published in 1997, where this Task Group "Utilization of Concrete Tension in Design" investigated the models for one-way members without transverse reinforcement.

A further focus was also on establishing a data bank on symmetrical punching tests, which form the basis for empirical formulae and certainly also for any theory or model. About 400 punching tests were critically reviewed and further evaluated in a consistent way. This is a first step to achieving a generally agreed selection of reliable tests. The high value of such a data bank is illustrated by the comparisons carried out between the data and some of the analytical proposals as well as with some well known empirical code formulae.

Finally, I want to thank all members of the Working Group on "Punching" of the *fib* Task Group 4.3 for their contributions, and especially for their enthusiasm, which made it possible that this Bulletin could be prepared and finalized in less than two years. I also want to thank Henrik Vocke for his services as co-editor of this Bulletin. This activity and the Bulletin itself demonstrates the high interest in this topic of analysing the structural behaviour of flat slabs and of modelling the punching failure. It clearly shows the need for further discussion between researchers and designers within *fib*.

Stuttgart, May 2000

Karl - Heinz Reineck
Convenor of *fib* TG 4.3

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1 Introduction

1.1 General remarks

Flat slabs without drop panels are nowadays widely used in many countries because of their economic and functional advantages, and Fig. 1-1 shows a recent example under construction. Despite its simple appearance, a flat slab has a complex load bearing behaviour in flexure and in shear. Therefore, much theoretical and experimental research has been conducted on the analysis of the bending moments and on the safe design for punching with and without moment transfer between slab and column. Punching is a shear failure within the discontinuity region (D-region) of the highly stressed slab at the column. An inclined crack forms around the column and finally the column with a punching cone separates from the slab. It is a brittle type of failure where the flexural reinforcement may not yield. Additionally to these structural aspects it must be stated, that the safety especially relies on good workmanship on the site especially concerning the concrete quality and the detailing of the reinforcement.

However, before concentrating solely on the punching topic one should not forget that from a structural point of view it is highly problematic to support a thin slab directly on columns; it really means supporting a thin plate on pin-supports. This statical discontinuity is one of the most critical D-regions occurring in concrete structures. In this D-region of the flat slab very high moments occur and the three-dimensional state of stress is extremely complicated. The problems are even aggravated if columns are placed directly at edges or corners or, if openings are placed near the columns. The absence of any transition between the column and the slab is extremely unfavourable and inappropriate for reinforced concrete, and the slab thickness is often determined by the punching problem. Flat slabs further unavoidably violate the principle, that the safety level of a structure should be equal throughout because the D-region with the punching zone, as against any other part of the slab, is not sufficiently ductile to gain capacity from the redistribution of forces. This is underlined by the fact that frequently steel devices or shear heads and shear studs are used and thereby demonstrate, that reinforced concrete is incapable of carrying the loads.

A brief historical reminder shows how the flow of forces may be directed, like the first mushroom slabs developed by Turner in the USA (Fig. 1-2 a) and the clear design by Maillart (Fig. 1-2 b), as well as the two structures by Nervi / Bartoli and by Maillart shown in Fig. 1-3. An insight into the flow of forces of Maillart's design is gained by the recent thorough review of Maillart's design approach, which was carried out by Füst and Marti (1996). This hopefully encourages more researchers to address historic topics for a better understanding of the development of concrete structures and design approaches.

These examples in Figs. 1-2 and 1-3 demonstrate that good solutions for reinforced concrete exist, which are satisfactory both from the engineering as well as from the architectural point of view. They also should remind researchers and code writers that clear models for the flow of forces in the whole structure as well as for the structural behaviour and failure is a prerequisite for good designs as well as for clear code rules. The user of a code must understand how the loads are transferred from the slab to the columns, and this basic equilibrium condition is not satisfactorily explained by checking nominal shear stresses along arbitrarily defined control perimeters, and neither are the failure mechanisms nor the failure causes explained.

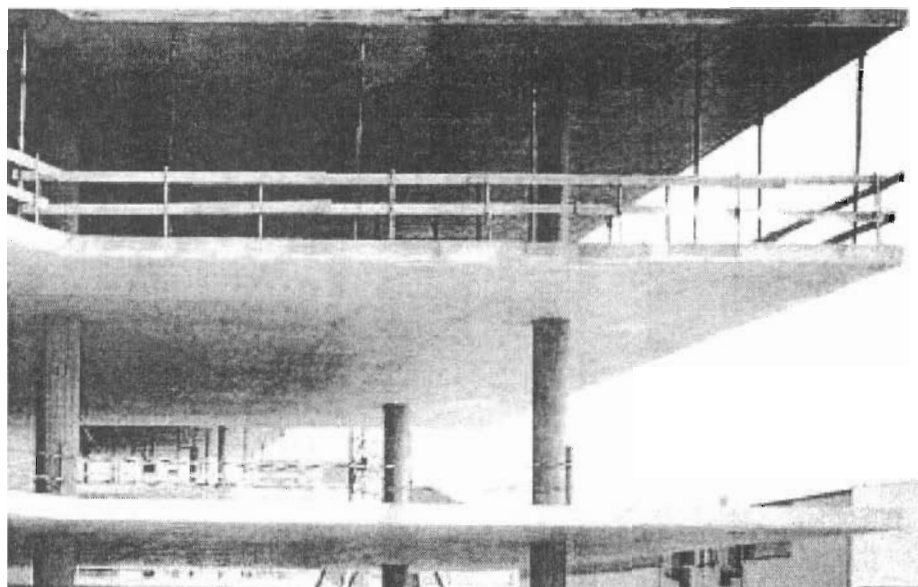
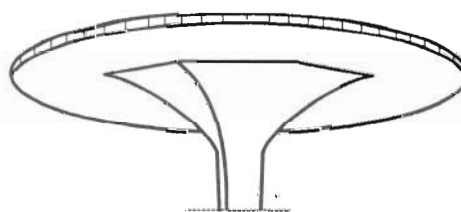
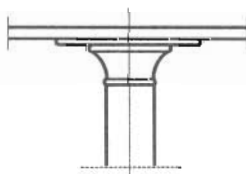
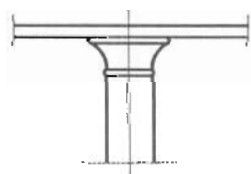


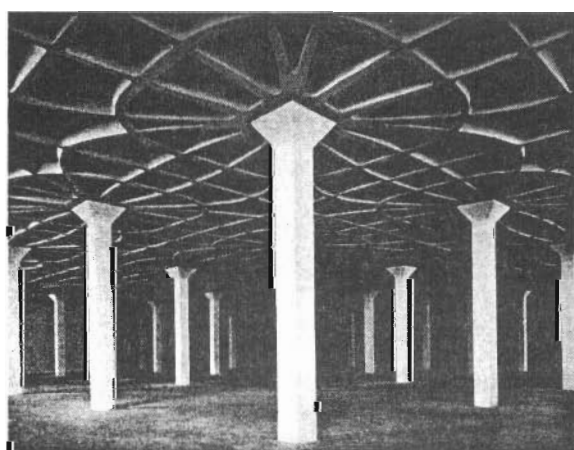
Fig. 1-1: View of a flat slab under construction.



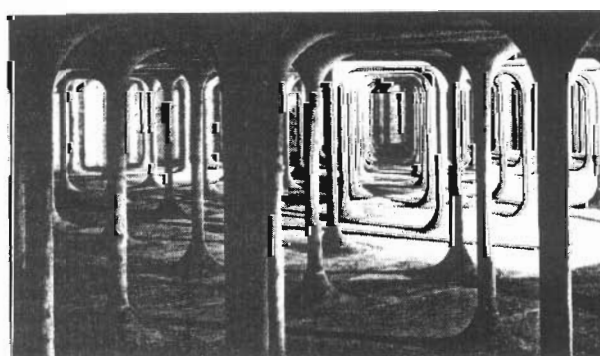
a) early solutions in the USA around 1900

b) Maillart 1910

Fig. 1-2: Historic solutions for flat slabs on columns



*a) wool manufacturer Gatti in Rome
by Nervi and Bartoli [Nervi (1957)]*



*b) jointless water reservoir of 65000 m³ in Munich
built by Dyckerhoff & Widmann [Dywidag (1973)]*

Fig. 1-3: Good structural solutions for flat slabs on columns

1.2 Scope and aim of the Bulletin

The main objective of this technical report by the *fib* TG 4.3 is to summarise the developments in the field of punching since the CEB Bulletin 168 on Punching was published in 1985. Thereby special consideration was given to the new developments for assessing the punching capacity at the ultimate limit state by means of mechanical models and to some extent also by nonlinear Finite Element Analyses.

A second focus was on the experimental evidence, the development of empirically derived code formulae and especially on establishing a data bank for punching tests, which allows an unbiased check of either models or empirical approaches. The phenomenon and definition of punching as well as the structural behaviour of the slab up to failure was described in CEB Bulletin 168 and is not repeated here, especially since some of this has also recently been done in the *fib* Bulletins 2 and 3 (1999), the Textbook to the CEB/FIP Model Code 1990.

The term “model” has been used in an ambiguous way in the past, and of course an empirically derived formula for the punching capacity may be seen as a mathematical model. However, it does not answer the question of how the load is transferred in the slab. This knowledge is only achieved by a mechanical model defining the forces in the steel and concrete and the failure mechanism. This may be done in a section or a failure surface, like the inclined punching crack surface, and such are the majority of models. However, a sectional approach is always in danger of not looking at the force transfer in the adjacent regions, as it is the case with truss models or strut-and-tie models. Such models are extremely useful for designers as well as researchers in order to understand the flow of forces and to identify critical issues. This especially refers to assessing to what extent the concrete tensile strength is utilised. An issue controversially discussed up till now and which is the main reason for the *fib* TG 4.3 “Utilization of Concrete Tension in Design” to pick up the topic mechanical models for punching.

Since the problem punching is so complex, no generally agreed upon design model is available for design practice. Codes still present empirically derived formulae for the shear force transferred at a specifically defined control surface and, therefore, Chapter 2 presents a review in this field. The comparison of some selected codes exhibits the variety of rules and definitions of the control perimeter as well as the wide scatter of the predicted punching capacities for a typical slab.

The Chapters 3 and 4 present and discuss the mechanical models for punching of slabs. Two chapters are used in order to avoid long heading numbers for subsections. First in Section 3.1 a brief review is made on the mechanical models and the shear transfer actions of one-way members, and then a classification of the different models for punching is given. In Section 3.2 some classical models are then described.

Chapter 4 aims at summarising the new developments for mechanical models since the state-of-the-art report in the CEB Bulletin Punching was published in 1985. It presents a survey and describes at least one of the different types of models proposed in the classification given before. Two of the models were more extensively described because they are typical for the opposite views on modelling the punching failure with respect to the role of the concrete tensile strength: whereas Menétrey in his model assumes that the load is transferred by tensile stresses in the concrete across the critical failure crack (section 4.7), Hallgren modifies the classical Kinnunen/Nylander model where the load is transferred by concrete members in compression, i.e. by struts (section 4.8). This aspect of whether concrete tension is activated in a model is especially pointed out in the discussion of the models in section 4.10 by analysing the different models with respect to the shear transfer actions utilised in the model.

Nonlinear Finite Element analyses have increasingly been used for modelling punching failures in the last 15 years since the CEB Bulletin 168 was published in 1985, and therefore in Chapter 5 the results of such numerical investigations are presented. The detailed treatment of the finite element method is avoided in view of the scope of another *fib* Task Group and the emphasis is put on the results of such analyses, which provide an insight into the behaviour in addition to experimental investigations. The sections 5.2 to 5.7 represent different approaches with respect to the finite element types and techniques as well as to the material models, and these are explained in some length in order to understand these approaches and also present the results gained.

A special feature of this Bulletin is the data bank on punching tests described in Chapter 6 and documented to some extent in the Appendices I and II. For all available tests the data were first collected as they were recorded by the authors, whereby the amount and quality of the information varied considerably. These data were critically reviewed and further evaluated and then presented in a second databank in a unified format. Special emphasis was put on further evaluating the values for the material strengths gained from different control specimens in order to derive unique values for the uniaxial strengths. Thus only these unique values for the uniaxial compressive strength f_{1c} and tensile strength f_{1ct} of concrete are handed over to the databanks for comparison with either empirical formulae or models.

Chapter 7 presents the results of the comparisons between the test results selected in the second databank for slabs without shear reinforcement with three classical models and four new models presented in this Bulletin. The ratios of tested to calculated punching loads are plotted versus the main parameters effective depth d , reinforcement ratio ρ_1 and uniaxial compressive strength f_{1c} and, along with the statistical data, provide an assessment of the quality of the different models.

Likewise, Chapter 8 reports the results of the comparisons between the tests without shear reinforcement selected in the second databank with six code equations from the German DIN 1045 (old and new), EC 2, MC 90, FIP Recommendations 1996, BS 8110 and ACI 318-95. Again, the statistical data and the above described plots are provided for discussing the results.

For slabs with shear reinforcement such comparisons were only made between codes and tests, and Chapter 9 presents a brief discussion on this topic.

Although the main emphasis of the Bulletin was on models and numerical analyses as well as the data bank, in Chapter 10 the Task Group also wanted to provide a brief review on interesting experimental investigations carried out in the last 15 years since the CEB Bulletin 168 was published.

Finally Chapter 11 gives the summary and conclusions along with some recommendations for further research in the field of punching of concrete flat slabs.

1.3 Topics out of the scope of the Bulletin

At edge and corner columns supporting a flat slab, even under gravity loads only, shear stresses in the slab are caused by both concentrated loads and moments that must be transferred between the slab and the column. Therefore, complicated non-symmetrical systems of stresses result in slabs around the edges and corners. This problem was studied by several researchers including Andersson (1966), Beresford (1967), Vanderbilt (1972), Hawkins and Corley (1974), Taylor and Clark (1976), Simmonds and Alexander (1987), Moehle (1988), Mortin and Ghali (1991), Lim et al. (1995) and Sherif (1996).

In design and construction of two-way reinforced concrete slab systems it is often necessary to construct openings near columns for sanitary, ventilation, heating, air-conditioning and electrical ducts. Due to practical as well as aesthetical considerations, the openings are mainly built next to the columns, often edge columns, which reduces the area of concrete that can resist transverse shear. This makes the slab-column connection weaker and more prone to the punching shear failure. This effect can be even more significant in the case of edge columns due to the fact that the transverse shear stresses are caused not only by concentrated loads but also by moments. The existing experimental data on slabs with openings includes tests on slabs with openings near centrally loaded interior columns done by Moe (1961), Hognestad, Elstner and Hanson (1964), Mowrer and Vanderbilt (1967) and Roll, Zaidi, Sabnis and Chung (1971). Tests on slabs with openings near the edge columns are presented by El-Salakawy et.al. (1998, 1999, 2000) and Polak and El-Salakawy (2000). These experimental investigations studied the influence of opening size and location, moment to concentrated load ratio and existence of shear reinforcement on the punching shear capacity of slabs.

Punching of prestressed slabs could not be dealt with in the given time schedule of this Bulletin. It was envisaged to carry out a review of recent tests and to collect all available information on punching tests in a data bank for prestressed slabs. This task remains to be dealt with by a future *fib* Task Group.

The punching capacity of slabs with shear heads and shear studs is dealt with in technical approval documents, so that this topic exceeds the scope of this Bulletin. There is a great activity within the ACI Building Code at this time. It is anticipated that formal design language for acceptance of shear studs will be in the next edition, ACI 318-2002. In addition, there are several studies underway on this topic, e.g. by McCabe at the University of Kansas and by Ghali at the University of Calgary.

Fibre reinforced concrete exhibits a considerably different structural behaviour than reinforced concrete and requires special attention which exceeds the scope of this document. Yet, it should be noted that generally the main aim of adding fibres is to increase the tensile capacity of concrete, and this indicates that a punching failure is seen to depend on the tensile strength of concrete. In normal weight concrete the residual post-peak-punching shear capacity is more increased by the amount of fibres than the peak resistance [Walraven, Pat and Markov (1987)]. For more information and recent experimental investigations see Azevedo and Hanai (1999), Shaaban and Gesund (1994), Kubat (1996) and Swamy and Ali (1982).

With respect to the fire design of flat slabs a brief reference can only be made here to an extensive review on the behaviour of flat slabs under fire by Kordina (1993, 1997). The investigations show the redistributions of bending moments and column loads of flat slabs, and the capacity in punching shear has been investigated in 10 fire tests on slabs.

The strengthening of slabs is a challenging topic and an area of new developments. The most common and easiest method to increase the punching load is to add new shear reinforcement. The use of vertical post-tensioned steel bolts has already been investigated in 1974 by Ghali et al. The influence of prestressing and injection has been experimentally studied by Menétrey and Brühwiler (1997). Recently Hassanzadeh and Sundquist (1998) compared different strengthening systems.

The topic strengthening reminds of the fact that reports on "real" punching failures of flat slabs in buildings are scarcely found and only rumours are heard. This is unfortunate and the topic failures should be addressed by a *fib* Task Group in the future.

Because of its relevance for designers, the calculation of the deflections of flat slabs is briefly mentioned here. Deflection calculations are based on the methods generally used for assessing the deflections of reinforced concrete flexural members, which was the focus of extensive research for many years and, as a result, various methods have been proposed for short and long term loading. Basically, there are two different approaches for calculating the deflections of a cracked reinforced concrete member. The first approach is the Effective Moment of Inertia (I_e) Approach where an effective moment of inertia I_e is calculated to be assumed constant over the full length of the member; the moment of inertia is interpolated between that of an uncracked section and the one of a fully-cracked section, like e.g. done by Branson (1977). This method is adapted by the ACI Building Code (1999). The second approach is the Mean Curvature (ψ_m) Approach like e.g. given by the CEB Manual for Deflections (1985) or by Ghali and Favre (1986). The deflection of a flat slab can be calculated by dividing the slab into strips treating each strip as a wide beam. For example the deflection of a midspan along the column axes is calculated as the deflection of the column strip, while the deflection of a midpanel is obtained by superimposing orthogonal column and middle strip deflections. The problems associated with determining the deflections of flat slabs in this way can be summarised as follows: First, restrained shrinkage stresses significantly influence the cracking of the slab so that a reduced modulus of rupture has to be used in calculating the cracking moments [Jokinen and Scanlon (1985)]. The second problem is to determine the factors for distributing the moments between the strips of the slab. Sherif (1998) proposed coefficients to distribute the bending moments between the slab strips to calculate the deflections of flat slabs as deflections of these strips. Third, the theoretical methods significantly underestimate the deflections of slabs subjected to low load levels just causing cracking to occur [Sherif (1998)].

2 Code Equations

Rüdiger Beutel

2.1 General

The purpose of this chapter is to compare the provisions for the punching shear resistance with and without shear reinforcement for Eurocode 2, Model Code 90, British Standard BS 8110-97, American Standard ACI 318-95 and the German Code DIN 1045.

Comparisons of codes provoke the following two questions “Which code gives the best approximation of the punching behaviour?” and “Which safety level is necessary for brittle punching?”

In Chapter 8 and 9 these codes are compared with test results and some answers are given to the first question. The second question can not be answered on the background of the test results as only the resisting shear capacity is judged. Different load values and partial safety factors for actions are not considered. Therefore in this chapter the variety of the punching shear capacities under consideration of partial safety factors at ultimate limit state for actions and for resistance is illustrated. Since live load values, partial safety factors for actions and for the resistance side are defined individually, the influences of the individual safety concepts are determined by a characteristic example (Table 2-1).

design code	dead load $g = 25 \text{ [kN/m}^3\text{]} \cdot 0,25 \text{ [m]} = 6,25 \text{ [kN/m}^2\text{]}$ life load $q = 5,00 \text{ [kN/m}^2\text{]}$		
	γ_g	γ_q	$\gamma_F = (\gamma_g \cdot g + \gamma_q \cdot q) / (g + q)$
DIN 1045 (07/88)	1,0	1,0	1,00
Eurocode 2 and Model Code 90	1,35	1,5	1,42
British Standard 8110-97	1,4	1,6	1,49
ACI 318-95	1,4	1,7	1,53

Table 2-1: Values of action partial safety factors γ_F for ultimate limit state of ordinary building

All considered codes define the nominal punching shear stress as a shear force $\gamma_F \cdot V_F$ divided by a control surface around the loaded area. The resistance partial safety factor to avoid punching is assessed by comparing the nominal shear stresses of punching tests with a strength parameter of the concrete, usually some measure of the tensile concrete strength.

An admissible punching shear strength on a serviceability load level could be determined if the punching shear capacity is divided by the partial safety factor for actions and for resistance. This admissible load level guarantees the comparability of different code formulae with individual safety concepts. Therefore most of the figures in this chapter show on the y-axis an admissible shear force V_F (eq. (2-1)).

$$V_F = \frac{V_R}{\gamma_F \cdot \gamma_R} \quad (2-1)$$

The calculation of the punching shear capacity can be standardised for all considered codes by the equations (2-2) to (2-5). The influence parameters were uniformly described to simplify the comparisons. Various definitions, e.g. between the concrete cylinder strength and the cube strength or of the flexural reinforcement ratio are arithmetically taken into account. For simplification and shortening of the code equations only the individual parameters are mentioned; all non-mentioned parameters of the equations (2-2) to (2-5) result in 1.

For the ultimate limit state the capacity must be greater than the demand:

$$\gamma_F \cdot V_F \leq V_{Rd} \quad (2-2)$$

The total punching shear resistance is:

$$V_{Rd} = \frac{V_c}{\gamma_c} + \frac{V_s}{\gamma_s} \leq \frac{V_{c,outside}}{\gamma_c} \leq \frac{V_{max}}{\gamma_c} \quad (2-3)$$

Consisting of two parts:

$$V_c = \tau_c \cdot k \cdot f(\rho_l) \cdot u \cdot d \quad [\text{MN}] \quad (2-4)$$

$$V_s = A_{sw} \cdot \kappa_s \cdot f_y \cdot \sin(\alpha) \quad [\text{MN}] \quad (2-5)$$

The parameters stand for:

V_F	characteristic value of the acting force
V_{Rd}	design value of the punching shear capacity
V_c	characteristic punching resistance without shear reinforcement or the concrete contribution of the punching resistance with shear reinforcement
V_s	characteristic shear capacity of the shear reinforcement
$V_{c,outside}$	characteristic shear capacity outside of the shear reinforced area
V_{max}	characteristic maximum shear capacity
γ_F	partial safety factor for the acting forces
γ_c	concrete partial safety factor
γ_s	partial safety factor of the shear reinforcement
τ_c	concrete shear capacity $[\text{MN}/\text{m}^2]$
k	size effect factor of the effective depth $[-]$
$f(\rho_l)$	function of the tension reinforcement $[-]$
ρ_l, μ_g	tension reinforcement ratio $[\%]$
u	control perimeter $[\text{m}]$
d	effective depth $[\text{m}]$
A_{sw}	cross section area of the shear reinforcement $[\text{m}^2]$
κ_s	efficiency of the shear reinforcement $[-]$
f_y	yield strength of the shear reinforcement $[\text{MN}/\text{m}^2]$
α	inclination of the shear reinforcement.

2.2 German design code DIN 1045 (88)

The design model for the punching resistance without shear reinforcement considers a shear capacity in dependence of the concrete cube strength and the flexural reinforcement ratio. The control diameter is given by the column diameter plus the effective depth (Fig.1). The admissible shear stresses τ_{011} and τ_{02} are determined under consideration of the uniaxial shear strength.

The design model for shear reinforced flat slabs and footings is based on tests and optical stress measurements by Franz (1963 and 1964) and Franz & Ritter (1964). They considered a punching cone with an inclination between 30° and 45° . Punching failures outside a shear reinforced area of $1,2 d$ were not investigated. The concrete contribution was calculated to be 25% of the ultimate load. Stirrups and bent-up bars reached the yield strength.

In contrast to the stirrups and bent-up bars, stud-rails and double headed studs are designed by a strut and tie model according to Andrä, Baur and Stiglat (1984). It has to be pointed out that the first two studs in the radial direction are provided as a suspension reinforcement (area c). The following studs are designed as a shear reinforcement with a concrete contribution.

The design rules of stirrups and bent-up bars are established in DIN 1045(88). The specifications for stud-rails and double headed studs are given in approvals of the German Institute for Civil Structures (Deutsches Institut für Bautechnik: DIBt) which are based on DIN 1045(88). The last revision of these technical rules in 1998 reduced the shear strength in the exterior control perimeter outside the studs, because a transition zone between the punching shear strength and the uniaxial shear strength of continuous supported slabs was investigated.

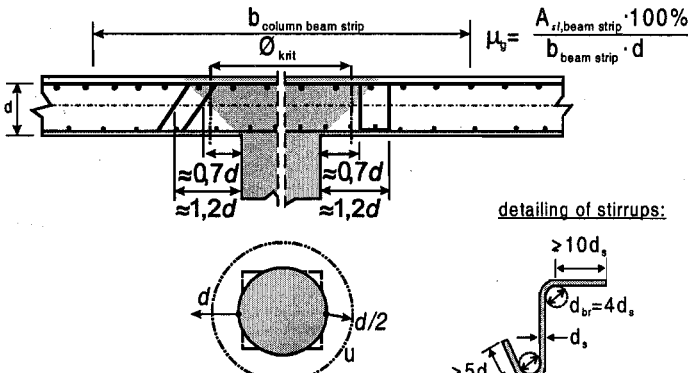
DIN 1045(88)	
Safety factors: $\gamma_c = 2,1; \gamma_s = 1,75$ Resistance without shear reinforcement: $\tau_c = \gamma_c \cdot \tau_{011}; \tau_{011} \cong 0,056 \cdot f_{c,cube 200}^{2/3};$ $f(\rho_l) = 1,3 \cdot \eta_s \cdot \sqrt{\mu_g}; \eta_s \cong 0,7 \cdot (1 + f_y / 500); 0,5\% \leq \mu_g \leq 25 \cdot f_{c,cube 200} / f_y \leq 1,5\%$ outside of the shear reinforcement: This failure type is not directly investigated maximum shear capacity: $V_{max} = \gamma_c \cdot \kappa_2 \cdot \tau_{02} \cdot u \cdot d; \tau_{02} \cong 0,21 \cdot f_{c,200}^{2/3}; \kappa_2 = \eta_s \cdot 0,45 \cdot \sqrt{\mu_g}$ within the shear reinforced area: $V_c = 0,25 \cdot V_F; V_s = A_{sw} \cdot f_y \cdot \sin(\alpha)$	
	additional rules: • minimum thickness of the plate: $h \geq 150 \text{ mm}$ • Stirrups have to enclose the tension and compression flexural reinforcement. • Rectangular columns can be transformed into concentric columns, if $a/b \leq 1,5$ and the cross section area is constant.

Fig. 2-1: Punching design rules of DIN 1045(88)

stud-rails and double headed studs		
maximum shear capacity: $V_{\max} = \gamma_c \cdot \kappa_2 \cdot \tau_{02} \cdot u \cdot d$		
stud-rails: $\kappa_2 = \eta_s \cdot 0,9 \cdot \sqrt{\mu_g} \leq 1$; double headed studs: $\kappa_2 = \eta_s \cdot 0,7 \cdot \sqrt{\mu_g} \leq 1$		
outside of the shear reinforcement:		
$V_{c, \text{exterior}} = \frac{\gamma_c}{1 + 0,25 \cdot l_s / d} \cdot \kappa_1 \cdot \tau_{011} \cdot u_1 \cdot d \geq \tau_{011} \cdot u_1 \cdot d$		
stud-rails / double headed studs:		
area c: $1,0 d \leq l_s$	$V_c = 0$; $V_s = \Sigma A_{sw} \cdot f_y$	
area d: $1,0 d < l_s \leq 2,0 d$	$V_c = 0,5 \cdot V_F$; $V_s = \Sigma A_{sw} \cdot f_y$	
$2,0 d < l_s \leq 4,0 d$	$V_c = 0,25 \cdot V_F$; $V_s = \Sigma A_{sw} \cdot f_y$	

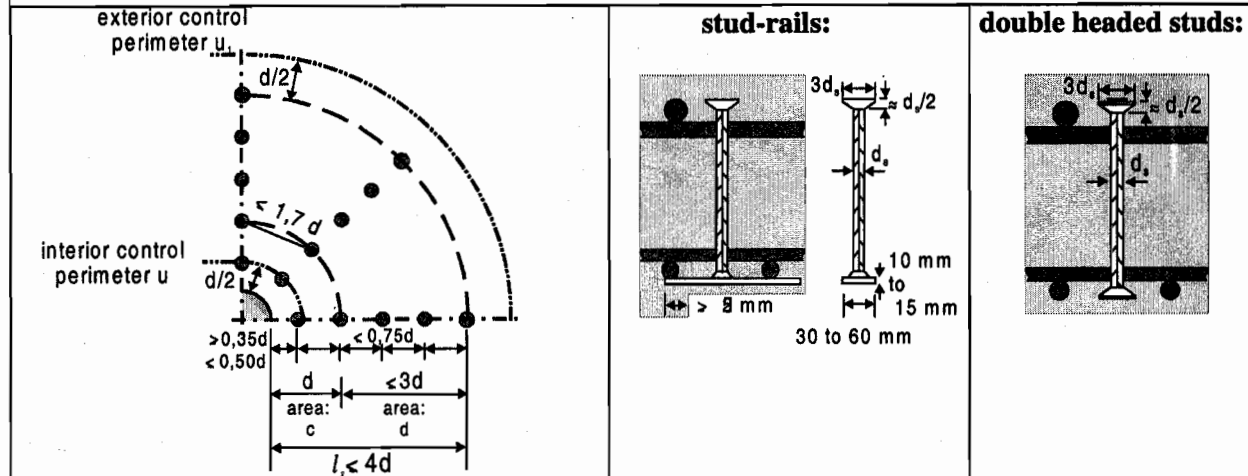


Fig. 2-2: Punching design rules of double headed studs and stud-rails according to DIN 1045 (88)

2.3 Eurocode 2, Model Code 90 and FIP-Recommendations 1996

Eurocode 2, Model Code 90 and the FIP-Recommendations 1996 "Practical Design of Structural Concrete" consider the following parameters:

- concrete cylinder strength: f_c
- flexural reinforcement ratio: ρ_l
- size effect of the effective depth: k
- and the shear capacity of the shear reinforcement: $f_{sw} \cdot A_{sw}$

The main differences between the Eurocode 2 and the Model Code 90 or the FIP-Recommendations are:

- (1) The diameter of the control perimeter: u
- (2) the value of the concrete shear capacity without shear reinforcement: V_c
- (3) the maximum shear capacity: $V_{\max}$
- (4) and the concrete contribution of the shear capacity within the shear reinforced area: V_c

The punching concrete contribution V_c and the stirrup contribution V_{sw} according to Eurocode 2 were criticised by Kordina (1994). His comparisons with tests have shown a better approximation of the test failure loads if the following items are considered:

- (a) The exponent of the function of the concrete strength changes from $2/3$ to $1/3$.
- (b) In comparison to the uniaxial shear strength the punching shear strength τ_c is increased by 20 % (see factor 1,2 in equation (2-5)).

$$\tau_c = \gamma_c \cdot 1,2 \cdot 0,09 \cdot f_c^{1/3} \quad (2-5)$$

- (c) The efficiency of stirrups and bent-up bars changes from 100% to 50% or 60% percent of the yield strength.

$$V_s = (0,5 \text{ or } 0,6) \cdot \sum A_{sw} \cdot f_y \cdot \sin(\alpha) \quad (2-6)$$

These improvements according to Eurocode 2 were taken into account in the German application document.

The motivation of the larger distance between the column face and the control perimeter of Model Code 90 according to Eurocode 2 can be explained by the evaluations of Kordina (1994). He pointed out, that the punching shear strength without shear reinforcement in the control perimeter according to Eurocode 2 is 20% higher than the uniaxial shear capacity. This discontinuity is not acceptable in a shear design concept. Therefore the following equivalent two ways are conceivable:

- (1) The punching shear strength is defined as the uniaxial shear strength, while the higher punching shear capacity is considered by a large control perimeter.
- (2) The punching shear strength and the uniaxial shear strength are different according to a fixed control perimeter but a transition zone is established.

Model Code 90 and the FIP Recommendations followed the first way. Assuming that the punching shear strength τ_c of Model Code 90 and Eurocode 2 are comparable, Kordina's evaluations showed that the punching control perimeter of Eurocode 2 can be increased. A large control perimeter has the secondary effect that local shear stress concentrations in the surrounding of the load area are fading away in the control surface. Therefore the influence of the geometry of the loaded area on the acting shear stress could be neglected. This leads to a robust failure criteria for low load levels (punching without shear reinforcement) but at high load levels local stress concentrations at the column face have to be limited (Model Code 90: formula of the maximum shear capacity).

The German technical design rules for double headed studs and stud-rails followed the latter way (Chapter 2.2). The control perimeter is defined close to the column face. The punching shear strength without shear reinforcement in this perimeter is higher than the uniaxial shear strength. Therefore a transition zone is defined. In this zone the punching shear strength is reduced in dependence of the radius of the shear reinforced area l_s . If the radius l_s reaches the value of $4d$, the uniaxial shear strength is reached.

Kordina (1994) recommended for Eurocode 2 a reduction of the admissible shear strength for stirrups and bent-up bars ($\sigma_{sw} = 0,5 \text{ or } 0,6 f_y$). If the yield strength f_y results to 500 MPa then this recommendation is almost equal to the upper limit of $\sigma_{sw} = 300 \text{ MPa}$ according to Model Code 90 and the FIP-Recommendations. This equivalence also applies to the required shear reinforcement quantity if the cross section area of each stirrup row is constant. For this case the factor $1.5 d/s_w$ of Model Code 90 shows the number of the shear reinforcement rows in the critical radius of Eurocode 2.

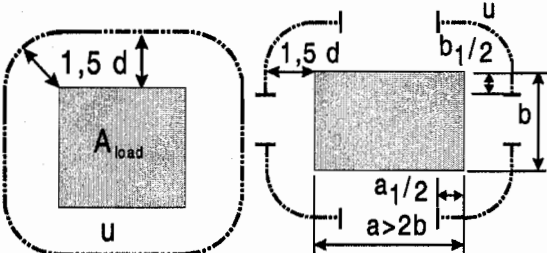
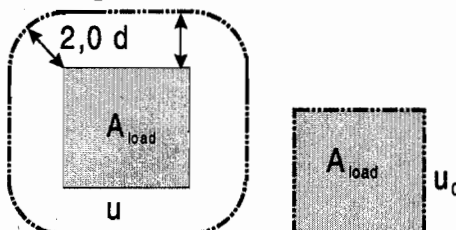
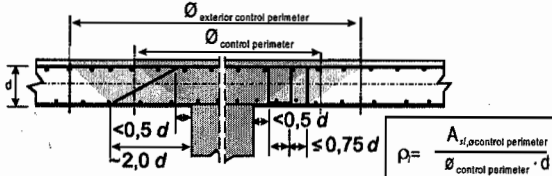
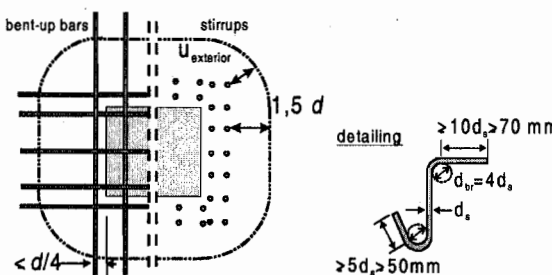
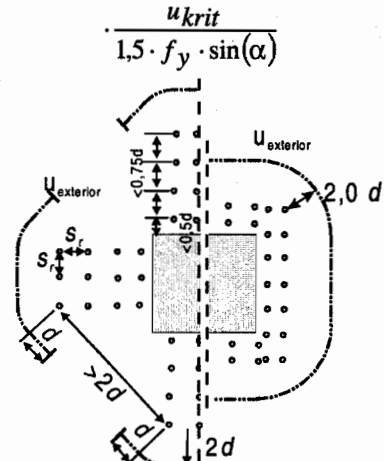
Eurocode 2, Part 1	Model Code 90 and FIP Recommendations 1996]
Partial safety factors :	
$\gamma_c = 1,5; \gamma_s = 1,15$	$\gamma_c = 1,5; \gamma_s = 1,15$
without shear reinforcement: V_c	
EC2, Part 1: $\tau_c = \gamma_c \cdot 0,035 \cdot f_c^{2/3};$ $k = 1,6 - d \geq 1;$ $f(\rho_l) = 1,2 + 40 \cdot \rho_l$ and $\rho_l \leq 1,5 \%$ control perimeter: 	MC 90: $\tau_c = \gamma_c \cdot 0,12 \cdot f_c^{1/3};$ $k = 1 + \sqrt{200/d}; \quad f(\rho_l) = \sqrt{100 \cdot \rho_l}$ FIP-Recommendations : $k = 1 + \sqrt{200/d} \leq 2,0$ control perimeter: 
outside of the shear reinforced area: $V_{c,outside}$	
EC2, Part 1: Similar to the formula of punching without shear reinforcement under consideration of an exterior control perimeter.	MC90/ FIP-Recommendations: Similar to the formula of punching without shear reinforcement under consideration of an exterior control perimeter..
maximum shear capacity: V_{max}	
$V_{max} = 1,6 \cdot V_c$	MC90: $V_{max} = 0,5 \cdot 0,6 \cdot (1 - f_c / 250) \cdot f_c \cdot u_0 \cdot d$ FIP-Recommendations: $V_{max} = 0,5 \cdot 0,85 \cdot f_c \cdot u_0 \cdot d$
within the shear reinforced area: $V_c + V_s$	
EC 2, Part 1: V_c: punching capacity without shear reinforcement; $V_s = \sum A_{sw} \cdot f_y \cdot \sin(\alpha); h \geq 200 \text{ mm}$ $\min A_{sw} = 0,6 \cdot (0,11 \text{ resp. } 0,13 \%) \cdot (A_{krit} - A_{load}) / \sin(\alpha)$  	MC90 / FIP-Recommendations: $V_c = 0,75 \cdot V_{c, \text{without shear reinforcement}}$ $V_s = 1,5 \cdot d / s_r \cdot A_{sw} \cdot f_y \cdot \sin(\alpha)$ $f_y \leq 300 \text{ MPa}$ $\min A_{sw} / s_r = 0,03 \cdot (100 \cdot \rho_l \cdot f_c)^{1/3}$ 

Fig. 2-3: Punching design rules of Eurocode 2, Model Code 90 and FIP Recommendations

2.4 British Standard 8110, Part 1, 1997

British Standard 8110-97 considers the same parameters as Eurocode 2 and Model Code 90. The last review of the British Standard in 1997 takes into account the beneficially acting membrane forces of cracked shear reinforced flat slabs. This approach is based on tests by Chana and Desai (1992) and is empirically established in the design rules of the shear reinforcement. Another significant difference concerns the efficiency of the shear reinforcement. The British Standard from 1985 assumed a shear crack inclination of 45° in the shear reinforced punching zone. The new formula considers a shear crack inclination of 33° and, therefore, the quantity of the shear reinforcement can be reduced up to 50%.

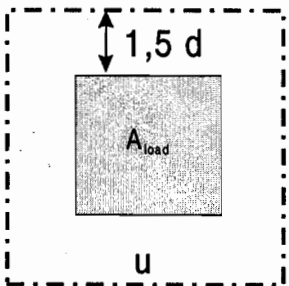
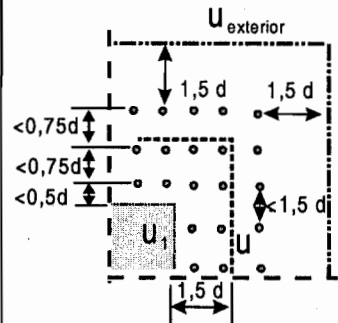
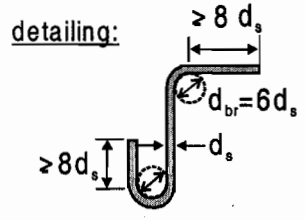
British Standard 8110. Part 1, 1997		
partial safety factors		
$\gamma_c = 1,25; \gamma_s = 1,15$		
without shear reinforcement: V_{ct}		
$\tau_c = \frac{1,5 \cdot d}{a_v} \cdot 0,27 \cdot f_{c,cube}^{1/3}; k = \sqrt[4]{400 / d}; f(\rho_l) = (100 \cdot \rho_l)^{1/3} \text{ und } \rho_l \leq 3,0 \%$		
a_v : distance between the control perimeter and the column face ; d in mm		
outside of the shear reinforced area: $V_{c,outside}$		
Similar to the punching formula without shear reinforcement.		
maximum shear capacity: V_{max}		
$V_{max} = 2,0 \cdot V_c \leq u_0 \cdot d \cdot \min \left(\frac{5}{0,8 \cdot \sqrt{f_{c,cube}}} \right)$		
within the shear reinforced area: $V_c + V_s$		
<u>$V_F \leq 1,6 \cdot V_c :$</u>	V_c see punching without shear reinforcement; $V_s = 0,95 \cdot \sum A_{sw} \cdot f_y \cdot \sin(\alpha); f_y \leq 460 \text{ MPa}$	
<u>$1,6 \cdot V_c < V_F \leq 2,0 \cdot V_c :$</u>	$V_c = 1,42 \cdot V_{c, \text{without shear reinforcement}} ;$ $V_s = 0,27 \cdot \sum A_{sw} \cdot f_y \cdot \sin(\alpha); f_y \leq 460 \text{ MPa};$	
minimum shear reinforcement:	$\min \sum A_{sw} \sin(\alpha) \geq 0,4 \cdot u \cdot d / (0,95 \cdot f_y); h \geq 200 \text{ mm}$	
		

Fig. 2-4: Punching design rules of British Standard 8110, Part 1, 1997

2.5 ACI 318-95

The punching resistance of ACI 318-95 depends on the concrete strength, the column geometry and the length of the control perimeter. The design rules of ACI 318-95 are identical to that of ACI 318-89. The recommendations of Dilger and Ghali (1989) on the design strength outside the shear reinforcement are not considered. Additionally the well known dependency of the punching shear strength on the flexural reinforcement ratio is not considered.

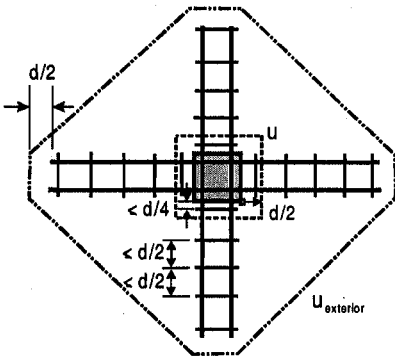
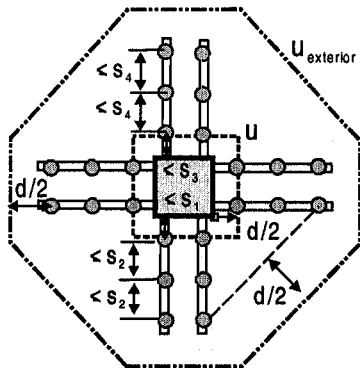
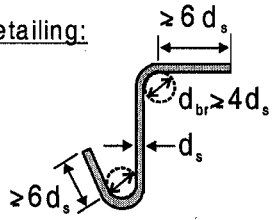
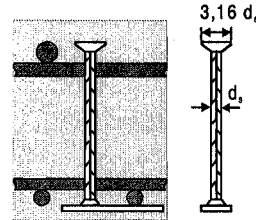
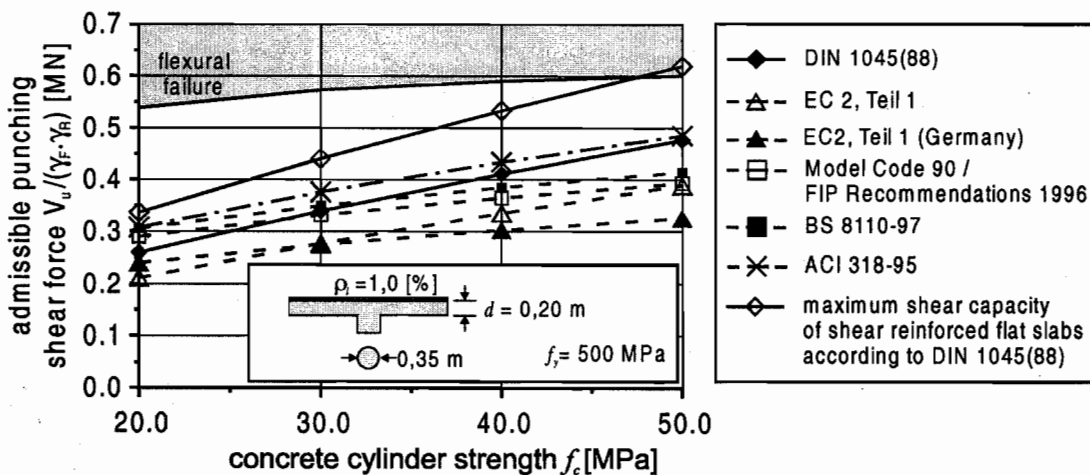
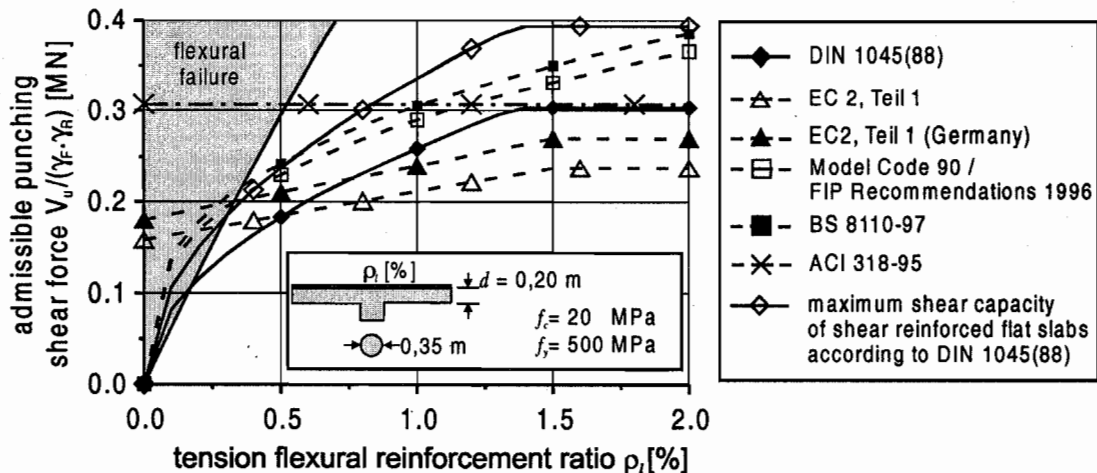
ACI 318-95	
Partial safety factors:	
$\gamma_c = \gamma_s = 1,176$	
without shear reinforcement and outside of the shear reinforced area: V_c	
$\tau_c = \min \left\{ 0,33 \cdot \sqrt{f_c}; \quad 0,083 \cdot \sqrt{f_c} \cdot \left(2 + \frac{4}{\beta_c} \right); \quad 0,083 \cdot \sqrt{f_c} \cdot \left(2 + \frac{\alpha_0 \cdot d}{u \text{ resp. } u_{\text{exterior}}} \right) \right\}$	
$\alpha_0 = 40$ at interior columns; β_c = ratio of the column side length ≥ 1	
Recommendations of Dilger and Ghali (1989):	
Dilger: $\tau_c = \min \left\{ 0,33 \cdot \sqrt{f_c}; \quad 0,083 \cdot \sqrt{f_c} \cdot \left(2 + \frac{4}{\beta_c} \right); \quad 0,083 \cdot \sqrt{f_c} \cdot \left(2 \cdot \frac{\alpha_0 \cdot d}{u \text{ resp. } u_{\text{exterior}}} \right) \right\};$	
$\alpha_0 = 40$ at interior columns	
Ghali: $\tau_c = \min \left\{ 0,33 \cdot \sqrt{f_c}; \quad 0,083 \cdot \sqrt{f_c} \cdot \left(2 + \frac{4}{\beta_c} \right); \quad 0,083 \cdot \sqrt{f_c} \cdot \left(1,6 + \alpha_0 \cdot \left[\frac{d}{u \text{ resp. } u_{\text{exterior}}} \right]^2 \right) \right\}$	
$\alpha_0 = 1000$ at interior columns	
maximum shear capacity of stirrups: $V_{\max}$	maximum shear capacity of stud-rails: $V_{\max}$
$V_{\max} = 0,5 \sqrt{f_c} \cdot u \cdot d$	$V_{\max} = 0,67 \sqrt{f_c} \cdot u \cdot d$
within the shear reinforced area: $V_c + V_s$	
$V_c = 0,167 \cdot \sqrt{f_c} \cdot u \cdot d$; $V_s = A_{sw} \cdot f_y$; $f_y \leq 414 \text{ MPa}$	
	
	
stirrups have to enclose the bottom and top layer of the flexural reinforcement	
detailing:	$\frac{V}{\gamma \cdot u \cdot d} \leq 6 \cdot \sqrt{f_c} :$ $s_1 \leq 0,5d ; s_2 \leq 0,75d$ $\frac{V}{\gamma \cdot u \cdot d} > 6 \cdot \sqrt{f_c} :$ $s_3 \leq 0,35d ; s_4 \leq 0,5d$
	

Fig. 2-5: Punching design rules of ACI 318-95

2.6 Comparative examination of selected codes

2.6.1 Punching without shear reinforcement

The punching shear capacity without shear reinforcement of DIN 1045(88), Eurocode 2, Model Code 90, British Standard and ACI show significant differences for the admissible load level. For illustration, the admissible punching shear capacity is printed versus the flexural reinforcement ratio and the concrete cylinder strength in Fig. 2-6 and 2-7. A scale for the scatter is given by the maximum punching shear capacity of shear reinforced flat slabs according to DIN 1045(88).



The following points can be observed:

- (1) Eurocode 2 estimates conservative punching shear capacities in dependence of the flexural reinforcement ratio ρ_l and the concrete cylinder strength f_c .
- (2) BS 8110-97, Model Code 90 and FIP-Recommendations evaluate the highest punching shear capacities for a low concrete cylinder strength ($f_c = 20$ MPa) and high flexural reinforcement ratios ($\rho_l > 1\%$). For a reinforcement ratio $\rho_l = 2\%$ even the punching capacity

of shear reinforced flats according to DIN 1045(88) is reached. If the concrete strength is increased the punching capacity tends to the conservative values of Eurocode 2.

- (3) For a relatively high concrete strength greater than 30 MPa, DIN 1045(88) and ACI 318-95 give the upper value of the punching shear.

2.6.2 Punching outside of the shear reinforcement

In Fig. 2-7 the maximum shear stresses according to DIN 1045(88) for stirrups in various exterior control perimeters are shown. In comparison to this acting shear stress line the shear strength lines according to a failure outside of the shear reinforced area of ACI 318-95, Eurocode 2 and for the recommendations of Dilger and Ghali (1989) are printed. The exterior control perimeters depend on the radius l_s of the shear reinforced area. Therefore Fig. 2-8 shows the ratio l_s/d on the x-axis. The intersection points of the acting shear stresses line and the resisting shear strength lines scatter in a wide range. In the case of ACI 318-95 and the recommendations of Dilger and Ghali (1989) the radius of the shear reinforced area l_s results in $0,6 d$. In contrast to this value the shear strength line of Eurocode 2 requires a radius of $1.9d$. This radius is 3,1 times larger than the required radius of ACI 318-95 which points out that a failure outside the shear reinforced area is assessed very variously. The reasons are:

- (1) The value of the shear strength in the exterior control perimeter differs in a wide range.
- (2) The intersection between the acting shear stress line and the resisting shear strength lines can show a low angle.

This example (Fig. 2-7) shows that the required shear reinforced area differs in a wide range and therefore the required quantity of shear reinforcement differs, too. Due to this behaviour a comparison of the shear reinforcement quantity was irrelevant.

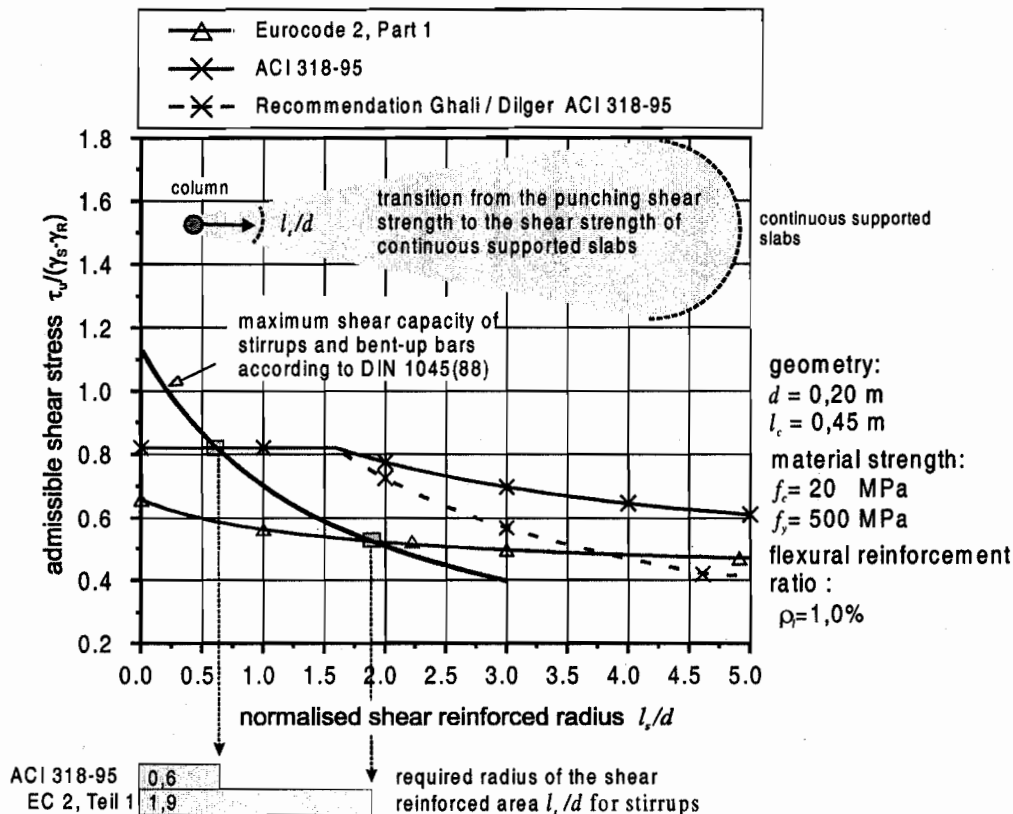


Fig. 2-8: Required radius of shear reinforced areas for ACI 318-95 and Eurocode 2

2.6.3 Maximum punching shear capacity of shear reinforced flat slabs

The assessments of the maximum punching shear capacity between DIN 1045(88), BS 8110-97, Eurocode 2 on the one hand and ACI-318-97, Model Code 90 and the FIP Recommendations on the other hand are very different. From the examples in Fig. 2-9 and Fig. 2-10 it can be observed that:

- (1) Eurocode 2, DIN 1045(88) and BS 8110-97 indicate a similar maximum load level. Significant differences depend on the flexural reinforcement ratio.
- (2) For a concrete strength of 20 MPa, ACI 318-95 and Model Code 90 indicate a similar load level. This shear capacity for stirrups is approximately equal to the shear capacity of stud-rails or double headed studs according to DIN 1045(88), although the anchorage of the studs is much better than the anchorage of the stirrups.
- (3) The FIP-Recommendations calculate the highest punching resistance.

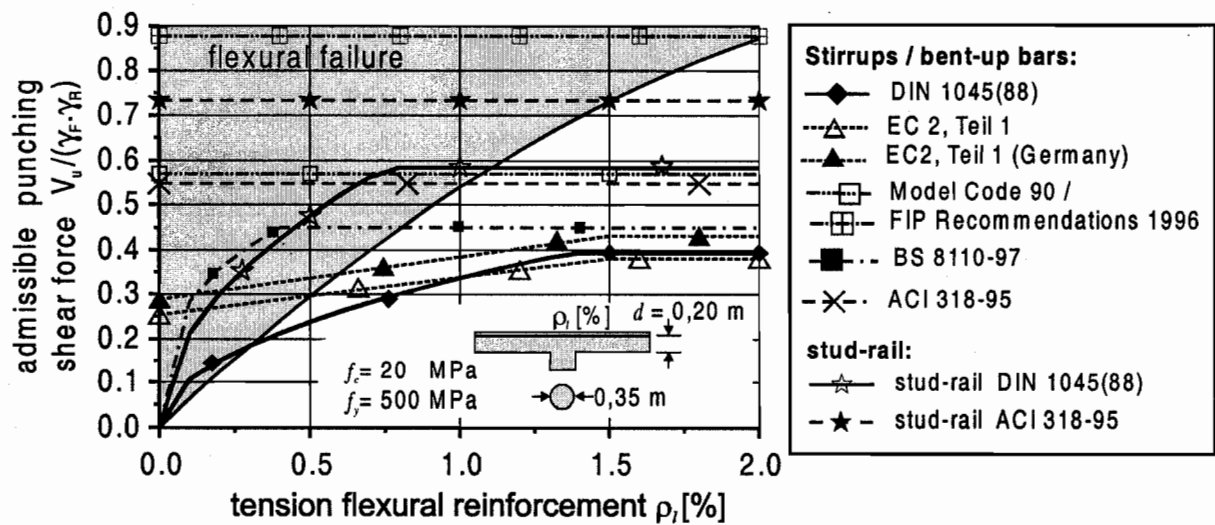


Fig. 2-9: Maximum punching shear capacity with shear reinforcement versus tension flexural reinforcement ratio for seven design codes

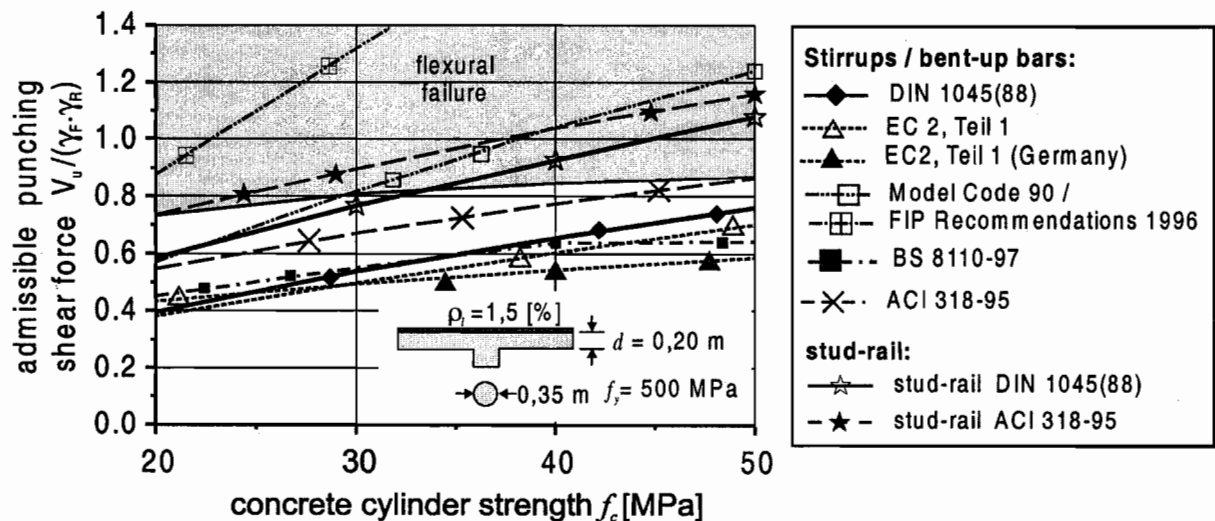


Fig. 2-10: Maximum punching shear capacity with shear reinforcement versus concrete cylinder strength for seven design codes

2.7 Conclusions

The comparisons of the punching resistance according to DIN 1045(88), Model Code 90, Eurocode 2, ACI 318-95 and BS 8110-97 on an admissible load level show a wide scatter of the results for all failure modes. The reasons are the different safety levels for actions and for resistance and different assessments of the punching shear capacities within these codes.

Essential differences of the punching shear capacities without shear reinforcement are caused by the parameters of the flexural reinforcement ratio and the concrete compressive strength. Different approaches for the size effect of the effective depth have to be taken into account for thick plates, either. An example of an ordinary interior flat slab-column connection shows that Eurocode 2 determines a conservative punching shear capacity. All other codes show higher capacities but the ranking depends on concrete compressive strength and the flexural reinforcement ratio.

The required radius of the shear reinforced area to avoid a punching failure outside of the shear reinforcement shows such significant discrepancies, that a comparison of the required shear reinforcement amount was irrelevant.

Eurocode 2 also shows a conservative result for the maximum shear capacity. The FIP-Recommendations 1996 "Practical Design of Structural Concrete" yield the highest values for the shear capacity. This shear strength for stirrups exceeds the maximum shear capacity for studs, although the anchorage behaviour of studs is much better than that of stirrups. The failure loads according this capacity can be divided into two different cases. On the one hand all codes from Europe show approximately the same maximum shear capacity. Significant differences are caused by the influence of the flexural reinforcement ratio. On the other hand Model Code 90 and ACI 318-95 result in higher maximum shear capacities.

The arrangement of the shear reinforcement provided in the European codes and ACI 318-95 is non-uniform. In Europe the shear reinforcement is uniformly distributed within the required shear reinforced area. ACI 318-95 prefers a concentrated distribution in the direction of the column-axis. Model Code 90 considers both arrangements but in the case of the concentrated stirrup distribution the length of the exterior control perimeter is reduced. Therefore the punching shear capacity outside of the shear reinforcement is also reduced.

Summarising this state of the punching code rules is unsatisfactory from the mechanical point of view. A uniform and neutral base would be desirable to judge the approaches. For punching resistance, this task can be solved in a first step on the basis of the presented data-bank of punching tests with and without shear reinforcement (Chapter 8 and 9).

3 Mechanical models for punching

3.1 Mechanical models and shear transfer actions for structural concrete

Karl - Heinz Reineck

3.1.1 Design models for uniaxial members without transverse reinforcement

A prerequisite for understanding the effect of punching of slabs is of course the understanding of the structural behaviour of the simpler members beams or one-way slabs which normally do not contain transverse or shear reinforcement, i.e. no stirrups or bent-up bars. Such members are slabs, foundations, footings, joists and members of minor importance like lintels in buildings or are even shells in silos or concrete offshore platforms, which with respect to shear are presently designed in codes by means of empirical formulae.

Comparatively little attention has been paid to the structural behaviour of these members as compared to carrying out tests and deriving or modifying empirical formulae for the ultimate capacity to be implemented in codes. Few researchers have worked at developing design models for members without transverse reinforcement stressed in uniaxial flexure and shear, but this topic has received greater attention recently. Some of the design models were reviewed in the recent state-of-the-art report by the ASCE-ACI Committee 445 (1998), and the different views on the failure of members without transverse reinforcement and how to model it also were presented in the CEB Bulletin 237 (1997). Referring to these recent reports only a brief account on modelling members without transverse reinforcement is given in the following.

The design of these members requires special consideration, because a brittle failure may occur prior to the yielding of the longitudinal steel. In the beam regions subjected to flexure and shear initially slightly inclined cracks at about 60° ("diagonal" cracks) and slightly curved form, which may suddenly propagate with a flat inclination further into the compression zone and lead to collapse. In some cases it also was observed that the failure crack formed as a new crack near the end support and developed over the initial cracks.

A simple strut-and-tie model with a straight strut from the load to the support cannot cover this failure mode and is unsafe for slender members although the model complies with a lower bound of the theory of plasticity: the inclined strut supposed to carry the whole load crosses the inclined cracks having formed at earlier load stages, and it also crosses the failure crack which widely opens at failure. In the beginning of reinforced concrete this model was the first one to be proposed for such members, but then plain bars with poor bond were used inducing only few cracks.

The ACI - ASCE Committee 426 (1973) presented a thorough state-of-the-art report on shear and also gave a detailed explanation of the structural behaviour and failure of members without transverse reinforcement. The different failure modes and shear transfer mechanisms were described (see section 3.1.2). In the report of Committee 426 the main parameters influencing the shear failure also were discussed and numerous examples were given for empirical formulae.

As described above, the first models were the "arch" for a member with distributed loads or the model with an inclined strut in case of a member with point loads. In both cases the load is directly transferred to the end supports which are tied together by the bottom longitudinal reinforcement. This model does not allow the staggering of the longitudinal reinforcement which however is commonly done in practice.

A further development of these simple models are truss models or strut-and-tie models with concrete ties, i.e. with members in the model utilising the concrete tensile strength to some extent. Such models were e.g. proposed by Al-Nahlawi and Wight (1992) and Reineck (1982, 1990, 1991), and they are valid for slender members. This truss model of course explains the decrease of the forces in the chords towards the supports and so the reinforcement may be staggered in the B-region with shear force.

A completely different approach for modelling the failure of members without transverse reinforcement than those described above is the tooth model, where the crack pattern is modelled and the different shear transfer actions (mechanisms) at the cracks are applied, which act on the concrete "teeth" between the cracks. The teeth, therefore, represent elements in the B-region of a member which connect the top and bottom chord, and this leads to a decrease of the chord forces towards the support like in the truss model.

General it may be said that the tooth model is a discrete model, where the cracks are individually defined with their shape and crack spacing. The opposite approach are truss models and strut-and-tie models, which are smeared models describing the state of stress and flow of forces in the whole member independently from any cracks. With respect to the failure criteria the fact must be considered that failure is always a local phenomenon occurring at a single crack or in a limited volume of concrete, and this especially applies to non-ductile failures like that of members without stirrups. When using smeared models therefore, such local effects have to be expressed in terms of stresses and limiting strengths for the members used in the model, and this is often done empirically. An example for this empirical procedure is the v -factor for the strengths of struts in the variable strut angle method for the shear design or for struts of strut-and-tie models or the effectiveness factor sometimes used in the theory of plasticity. Similarly when using concrete ties, it cannot be expected that attaining the uniaxial tensile strength in the ties defines the failure of the member, but failure occurs at far lower tensile stresses caused by the loss of strength of one or several of the shear transfer actions. Therefore, Reineck (1990, 1991) derived the truss model from the state of stresses in the member defined by the shear transfer actions in the discrete tooth model so that both models comply to each other.

It must be pointed out that truss models only apply for members with a distinct B-region, i.e. for members with a slenderness of e.g. $a > 3d$ for point loads in a distance a from the support axis or a slenderness of $l/d > 12$ for members with distributed loading. For less slender members the failure of the shear transfer actions may not define the failure and a further load increase is possible by means of the load directly transferred to the supports by an inclined strut or an arch forming in the remaining uncracked part over the cracks, i.e. in the compression chord. The whole member then practically forms a D-region. In such a case a change of the load transfer takes place during load increase after the and this requires also a change of the model or statical system for calculating the ultimate load of the member. The role of the slenderness is, therefore, very pronounced in the structural behaviour and shear failure of uniaxially spanned members and so the relevant consequences have to be drawn for modelling. The failure load of a non-slender member cannot be assessed by a magnifying factor applied to the maximum load of the truss model for slender members, as it is often done when using empirical formulae.

3.1.2 Shear transfer actions

A brief survey on the shear transfer actions is given in the following for uniaxially spanned members, i.e. beams, without transverse reinforcement, whereby reference is made to the profound state-of-the-art report by the ACI - ASCE Committee 426 (1973) and to the recent descriptions given in the ASCE-ACI 445 (1998) and in the *fib* Textbook (1999), sect. 3.3.6 and 4.4.1 (5). The term shear transfer "action" is preferred in this context instead of the often used term "mechanism", which indicates more the deformational response at failure. The description of these shear transfer actions is a valuable tool, because these are directly related to the possible failure criteria and failure causes, and also the role of the concrete tensile strength f_{1ct} can clearly be described and defined.

In the cracked tension zone of a member the following shear transfer actions can be discerned:

- cantilever action of concrete teeth,
- friction stresses along cracks,
- dowel action of tension chord,
- residual tensile stresses across cracks.

Most of these transfer actions in the tension zone are directly related to the concrete tensile strength, i.e. the uniaxial strength f_{1ct} , like the cantilever action, the dowel action and the residual tensile stresses across cracks. The transfer by friction on one side relies locally on the transfer of high compressive forces at particles of the crack surface, but it induces a tension field in the adjacent concrete, so that altogether friction strengths have mostly been related to the concrete tensile strength.

The following two shear transfer actions are in the compression zone:

- shear stresses in compression zone,
- inclined compression chord.

The latter action does not imply the concrete tensile strength, unless at the end anchorage by means of bond. The shear stresses in the compression zone of course lead to principal tensile stresses.

It is noteworthy now, that for symmetrical punching no additional shear transfer actions occur, because the load is transferred in the radial sectors representing the webs. In the ring direction no load or shear force is transferred in the symmetrical case. Of course, in the ring direction there are high compressive stresses in the concrete and tensile stresses in the ring reinforcement and these have to be accounted for in any model. However, the triaxial state of stress only influences the failure criteria and perhaps also the constitutive laws for the different shear transfer actions.

All the mentioned actions occur in the loading history and have to be considered as possible failure criterion or failure cause, unless proof is given that one or the other diminish prior to failure and a change of load transfer occurs.

3.1.3 Survey on models for punching

In the following different types of models for punching are discerned and a certain - classification of models is proposed. Since this is mainly based on the classification of theoretical analyses given in CEB Bull. 168, only those items are further explained where either some modifications are made or where new evidence was developed. Such a classification is meant as a help to characterise theories or models and to understand and compare them, but there are often models which do not exactly fit into a classification. Therefore, the ambition to classify models should not go to far; it is more important to clearly understand the flow of forces and the failure causes, and for this the investigation is more relevant, which shear transfer actions were utilised in the different models.

Flexural capacity approach

Since in many punching tests the ultimate loads did not differ significantly from the flexural capacities in some early models punching strength of reinforced concrete slabs was either derived from the flexural capacity or somehow combined with it. Such models were e.g. developed by Yitzhaki (1966), Long and Rankin (1987), Long (1975), Moe (1961) and Gesund et al (1970), as describe in CEB Bull. 168 (1985), and also Reimann (1963) and Nölting (1984) may be grouped among these.

Plasticity Approach

The theory of plasticity has mainly been used to determine upper bound values for the punching load, like e.g. by Braestrup, Nielsen et al. (1976). A failure mechanism is assumed in which the deformation is concentrated in a rotationally symmetric surface, and the concrete punch is assumed to be perpendicular to the slab. By equating the rate of external work done by the applied load with the rate of internal work dissipated by the failure, an analytical expression is derived. The obtained punching load is an upper bound value for which it is necessary to reduce the compressive and the tensile strengths of concrete to reconcile with test results. The flexural reinforcing bars are not taken into account in this formulation. Another simplified upper bound value for the punching load was derived by Marti and Thürlimann (1977).

Lower bound solutions were discussed by Braestrup in CEB Bulletin 168 (1985), and among them the lower bound approach by Pralong (1982), which involves the tensile strength of concrete.

The Kinnunen/Nylander approach

The approach by Kinnunen and Nylander (1960) has been extensively presented in the CEB Bulletin 168 (1985) and only its main features are given here. The model consists of rigid sectors outside the punching cone supported by a triaxially compressed truncated conical shell extending from the column to the root of the flat punching crack. The failure criteria were expressed in terms of a limitation of the inclined radial compressive stress and a limitation of the tangential compressive strain at the shear crack.

This model was the first mechanical model for punching which not only yielded good agreement with test results but also visualized the flow of forces. It was likewise attractive to engineers and researchers and this explains why it has influenced many other researchers throughout the last four decades until now, like e.g. Broms (1990), Shehata (1990) and recently Hallgren (1996), see sections 4.1, 4.4. and 4.8.

More generally the Kinnunen/Nylander approach may be characterised as a failure mechanism approach, where rigid bodies separate at defined failure surfaces, here in case of punching the radial cracks and the inclined shear crack surface. The location and geometry of the cracks either have to be pre-defined or are computed based on material properties and slab geometry, and they play an important role in the theory. This especially refers to the inclined shear crack surface, the location and inclination of which is mostly based on test observations and sometimes also on results from non-linear finite element analyses.

Failure mechanism approaches with concrete tensile stresses in failure surface

The same approach as used by Kinnunen/Nylander but assuming other shear transfer actions was applied by other researchers. The extensive and increasing research on fracture mechanics encouraged researchers to utilize the transfer of tensile stresses across the crack surface at the boundary of the punching cone, like Georgopoulos (1988,1989) or Menétrey (1994), see sections 4.3 and 4.7.

The theories based on fracture mechanics have somehow to be grouped among this approach, like that by Bazant and Cao (1987), see section 4.2, but the explanation in terms of a mechanical model with residual tensile stresses crossing the crack surface is not clearly addressed.

Truss models or strut-and-tie models

Truss models or strut-and-tie models with concrete ties describe the load transfer in the slab and thus may be classified as smeared models. The cracks are not explicitly addressed but may only be considered in one or the other failure criterion, like a reduced value for the concrete compressive strength. Truss models with concrete ties have been proposed by Reineck for one-way shear (see CEB Bulletin 237 [2]) and for punching by Pralong (1982) as well as by Andrä (1982), and also the beam model by Alexander and Simmonds (1990, 1992) may be ranked among these type of models.

Fracture mechanics

Since 1985 models based on fracture mechanics have increasingly been proposed, and the same applies to numerical analyses, which also often consider fracture mechanics. These topics are presented separately in chapter 5. In some models also the failure criterion is based on fracture mechanics, like that by Hallgren.

3.1.4 Design parameters

The models and theories have to be reviewed under especially all the aspects, which are relevant for design, and most of them are well known from the empirical formulae:

- the role of main parameters like the concrete compressive strength, the reinforcement ratio ρ_1 , the effective depth d (size effect) and the amount of transverse reinforcement.
- the role of the concrete tensile strength in models for punching.

3.2 Review on classical models

Markus Staller

3.2.1 Introduction

In the following some selected classical models are presented, which are published until 1986. An extensive list of other models can be taken from CEB-Bulletin 168 (1985). All existing mechanical punching models can be divided into different groups as shown in section 3.1.3. In this section the most important models are shortly introduced. These are the models of Kinnunen/Nylander, Moe and Bræstrup/Nielsen *et al.*

3.2.2 The Model of Kinnunen/Nylander

The derivation of the *Kinnunen/Nylander* model is based on the 61 tests in 1960 on circular slabs with circular column stubs. The observations of these tests, especially the formation of shear cracks, the deformation of the sector elements and the expansion of the concrete and steel are important fundamentals of their theory.

The basic idea is to create an equilibrium of forces acting on the sector element (see Fig. 3.2-1). The stiff sector element is limited at the sides by the radial cracks and in the front by the tangential shear crack. Under load action it turns around a centre of rotation located in the root of the shear crack. The failure criterion is defined by the ultimate shear expansion of the concrete at the bottom surface of the slab ($\epsilon_c = -1,96 \text{ ‰}$). The original theory is derived for slabs with ring reinforcement and expanded to slabs with two-way reinforcement by introducing a correction factor ($\kappa = 1,1$) to compensate the dowel action. The circumference of a square column must be transposed into the corresponding circumference of a circular column.

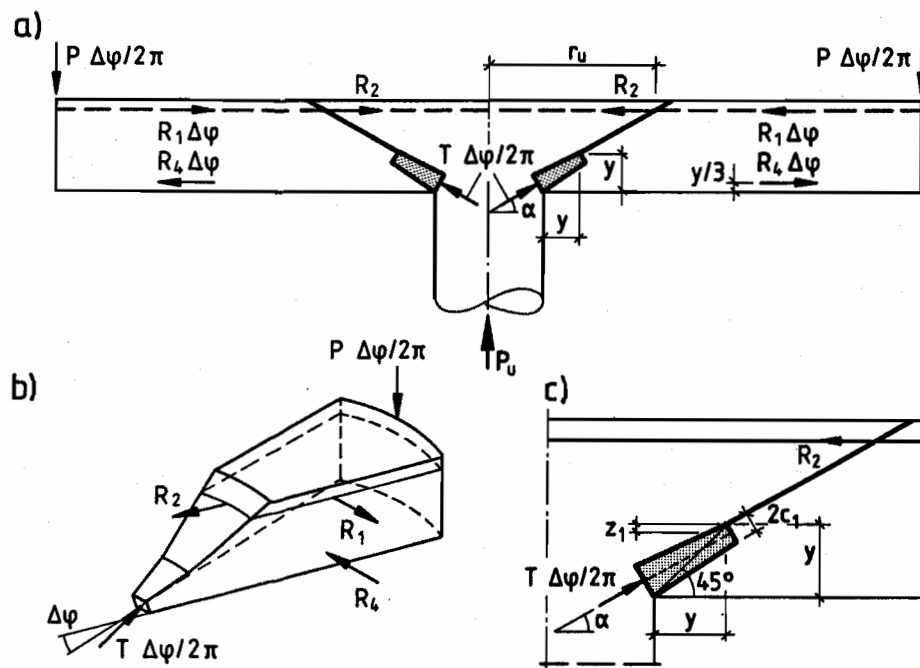


Fig. 3.2-1: Mechanical Model of Kinnunen/Nylander

To get the predicted ultimate load of non shear reinforced slabs the two following eq. (3.2-1) and eq. (3.2-2) have to be set equal by an iteration on the ratio of the concrete compression zone k_x . The first eq. (3.2-1) for $V_{u,c}$ which depends on the ultimate concrete stress σ_{cu} is derived by setting $\Sigma V = 0$.

$$V_{u,c} = \kappa \pi \eta d^2 k_x \frac{1 + \frac{2k_x}{\eta}}{1 + \frac{k_x}{\eta}} \sigma_{cu} f(\alpha) \quad (3.2-1)$$

$$\text{where: } \eta = \frac{c_2}{d} \quad \text{and} \quad f(\alpha) = \frac{\tan \alpha (1 - \tan \alpha)}{1 + \tan^2 \alpha}$$

The angle α describes the inclination of the conical shell. Further abbreviations and symbols are explained in chapter 6.

The second eq. (3.2-2) $V_{u,s}$ depends on the yield stress of steel f_y , the ratio of reinforcement ρ and the shape. It can be derived by setting the sum of flexural moments to zero $\Sigma M = 0$. Three different cases have to be differed in dependence on the location where the yield stress of the steel is reached. Here only the equations for the case that yield stress is reached inside the area of the conical shell, with the radius r_u , is given. The radius of this circular area inside the shell is called r_f .

$$V_{u,s} = \kappa 4 \pi \rho f_y d r_f \left[1 + \ln \left(\frac{\delta d}{2 r_u} \right) \right] \frac{1 - \frac{k_x}{3}}{\delta - \eta} \quad \text{where: } \delta = \frac{l_{slab}}{d} \quad (3.2-2)$$

3.2.3 The Model of Moe

The results of *Moe's* research are basic for the US-code ACI-Standard 318 of the year 1963. To derive his model *Moe* considered only his tests on square slabs. Two different limit states are used to describe the punching failure. First the ultimate force V_{flex} carried by the column is calculated considering only flexural stress. Second the shear carrying capacity V_{shear} of the column is derived. eq. (3.2-3) describes basically the postulated relation between the flexural and the shear capacity, the resulting ultimate shear carrying capacity is given by V_u .

$$\frac{V_u}{V_{shear}} + A \frac{V_u}{V_{flex}} = 1 \quad (3.2-3)$$

The Factor A is derived and calibrated by test results. Finally eq. (3.2-4) results expressing the ultimate shear carrying capacity.

$$V_u = \frac{1,246 (1 - 0,059 \eta) u_{col} d \sqrt{f_c}}{1 + \frac{0,436}{V_{flex}} u_{col} d \sqrt{f_c}} \quad (3.2-4)$$

$$V_{flex} = C \omega (1 - 0,59 \omega) d^2 f_c \quad (3.2-5)$$

where: $\omega = \rho \frac{f_y}{f_c}$ and $\eta = \frac{c_2}{d}$

The factor C gives the ratio between the ultimate flexural moment of the slab and the corresponding force inside the column V_{flex} . The magnitude of C depends on the method of determining the static forces and is not explicitly given by Moe . It essentially affects the size of V_u . The ratio of reinforcement and the yield stress of steel are indirectly considered in eq. (3.2-5). It has to be said, that major variables of eq. (3.2-4) are derived on empirical basis. This hole theory is not primary based on a geometric and material laws, a physically failure criterion is missing too.

3.2.4 The Model of Bræstrup/Nielsen et al.

The model of *Bræstrup/Nielsen et al.* published in 1976 is based on the theory of plasticity. These basic assumptions are supposed:

1. Concrete is a perfectly plastic material.
2. The failure criterion is based on *Coulomb's* law: $\tau + \sigma \cdot \tan \Phi - c = 0$
3. The concrete tensile strength is set very low. Optimal results can be achieved by calculating with $f_{ct} = f_c / 400$.

The ultimate cracking force P_u is calculated by comparing the fracture energy of the conical shell with the work performed by the applied loads. The shape of the conical shell is added by a straight line and a catenary curve.

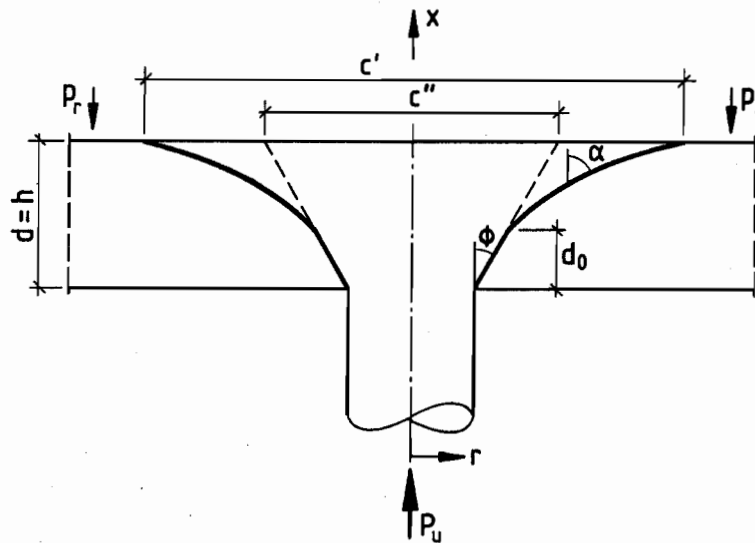


Fig. 3.2-2: Form of the punching cone according to Bræstrup, Nielsen et al.

It is ascertained by an iterative variation calculation which gives the factors A , B and C do obtain the minimum punching load. V_{u1} describes the cracking force of the conical shell, V_{u2} gives the cracking force of the catenary shell.

$$V_{u1} = 0,5 \cdot \pi \cdot f_c \cdot d_0 \cdot (c_2 + d_0 \cdot \tan \Phi) \cdot \frac{1 - \sin \Phi}{\cos \Phi} \quad (3.2-6)$$

$$V_{u2} = 0,5 \cdot \pi \cdot f_c \cdot \left[k_2 \cdot C \cdot (d - d_0) + k_2 \cdot \left(\frac{c'}{2} \cdot \sqrt{\frac{c'^2}{4} - C^2} - A \cdot B \right) - k_3 \cdot \left(\frac{c'^2}{4} - A^2 \right) \right] \quad (3.2-7)$$

where: k_2, k_3 ... material constants

$$P_u = V_{u1} + V_{u2} \quad (3.2-8)$$

The geometrical parameters used in eq. (3.2-6) and (3.2-7) are shown in Fig. (3.2-2).

This theory is suitable to give an upper-bound solutions for the ultimate forces. Especially the first assumption, that concrete is a perfectly plastic material, means a drastically simplification of the concrete behavior. The ratio of reinforcement is not considered in this model.

4 New developments for mechanical models

In the present chapter some important models for the punching failure which were developed since the Bulletin 168 in 1985 will be presented in chronological order. A classification and discussion of the models will be given in section 4.10.

4.1 Modified model by Shehata

Shehata (1985) proposed a detailed mechanical model for the punching failure of flat slabs. However, in the following section the simplified model by Shehata (1990) will be described, because it has the same general aspects like the more complete one.

Based on observed crack patterns and linear deflection profiles of test specimens, Shehata considers that the punching region of a slab is divided into rigid radial segments that rotate around a centre of rotation located at the column face and at the level of the neutral axis (Fig 4.1-1). Considering a radial segment of the slab of sectorial angle $\Delta\phi$ bounded by two radial crack planes and the inclined crack surface, Shehata identifies the following external and internal forces acting on it in a radial plane (Fig. 4.1-1):

- a) the external applied load $P(\Delta\phi/2\pi)$, at a radius $r = r_p$;
- b) the radial component of the resultant ring tension forces $F_{st} \cdot \Delta\phi$ due to slab deformation;
- c) the radial component of the resultant ring compression forces $F_{ct} \cdot \Delta\phi$ due to slab deformation;
- d) the inclined bearing force dF_{cr} at the column face acting on the axis of symmetry of the prismatoid frontal part of the segment;
- e) the radial net force dF_{sr} ;
- f) dowel forces dD on the steel cutting across the inclined crack. The dowel forces will be ignored here because yielding of the reinforcement is assumed.

Shehata defines three critical states at which the frontal part of the radial segment fails to support the force at the column face:

1. If the angle α of the compressive force reaches 20° , there are principal tensile stresses in the compressed front part and failure occurs by splitting of the concrete.
2. If the average radial strain on the compressed face reaches a value of 0.0035 in the plastic length starting from the column face, there is a radial crushing of the concrete.
3. If the tangential strain of the compressed face reaches 0.0035 at a distance x from the column face, there is a tangential crushing of the concrete.

To define the ultimate punching resistance, it is required to find the rotation ψ_u at which any of the three aforementioned critical states is first achieved. Subsequently, for a given value of ψ , the punching capacity P_u can be found with the conditions of equilibrium on a radial segment:

$$\begin{array}{ll} \Sigma H^{\text{ring}}: & F_{st} \cdot \Delta\phi = F_{ct} \cdot \Delta\phi \\ \Sigma H^{\text{radial}}: & dF_{sr} = dF_{cr} \cdot \cos 10^\circ \end{array} \qquad \begin{array}{ll} \Sigma V: & P \cdot (\Delta\phi/2\pi) = dF_{cr} \cdot \sin 10^\circ \\ \Sigma M^0: & m_r = F_{st} \cdot \Delta\phi \cdot z = F_{ct} \cdot \Delta\phi \cdot z \end{array}$$

As a result, the punching capacity P_u can be calculated according to equation (4.1-1):

$$P_u = 2\pi \cdot r_0 \cdot x \cdot n_c \cdot f_c \cdot \tan 10^\circ \quad (4.1-1)$$

where: r_0 = diameter of column
 x = height of compression zone at flexure in tangential direction when punching occurs
 n_c = stress concentration factor to account for the concrete strength under a multiaxial state of stress
 f_c = concrete cylinder strength

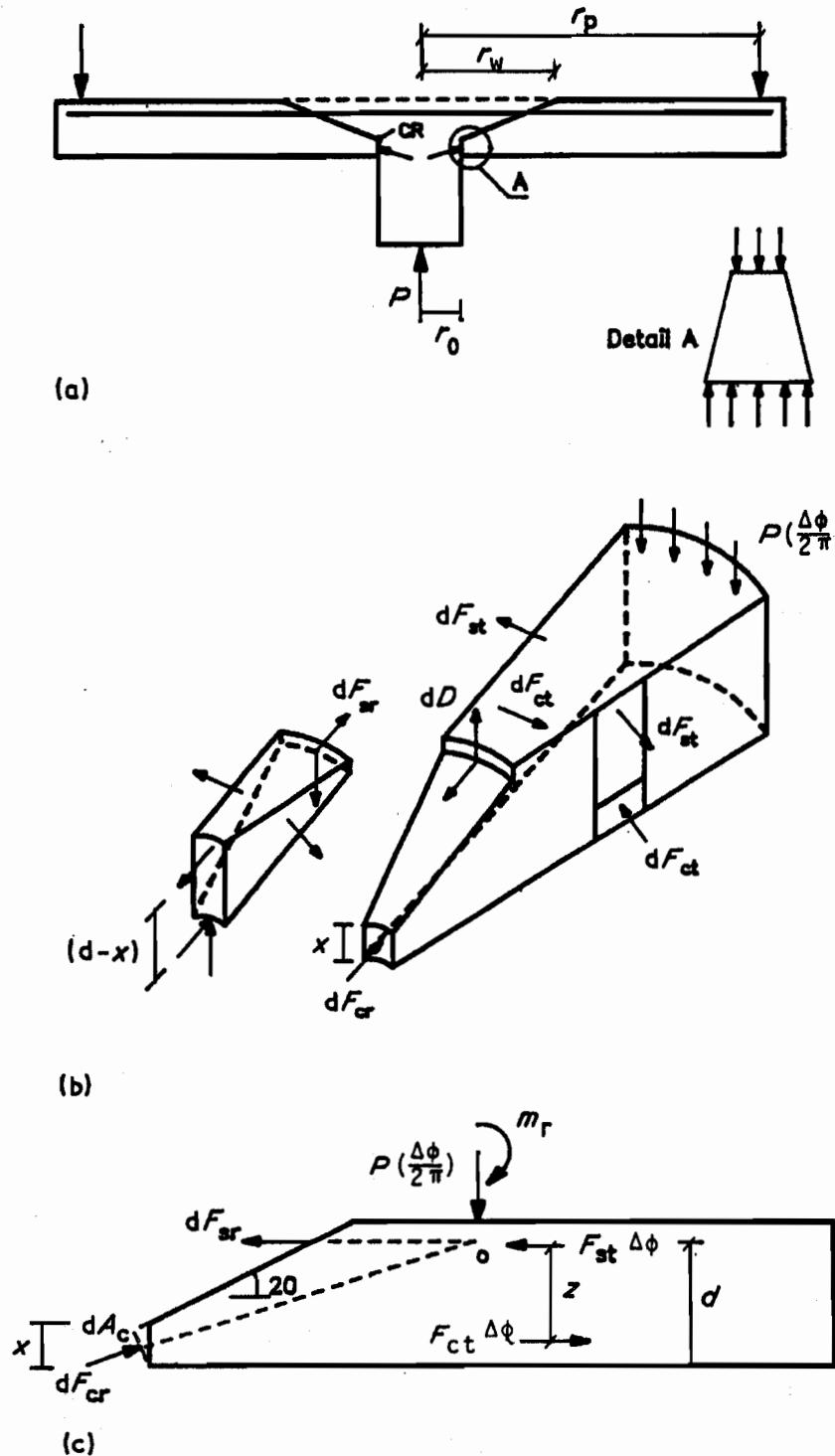


Fig. 4.1-1: Punching failure model: a) stress concentration region, b) forces acting on a segment, c) forces acting on a segment in a radial plane

4.2 Fracture mechanics model by Bazant and Cao

The basic concept of Bazant and Cao (1987) is that the punching failure does not occur simultaneously along the ultimate failure surface (except for very small structures) but that the failure zone propagates across the structure, with the energy dissipation localized into the cracking front. Therefore, the punching load should be predicted by fracture mechanics instead of plastic limit analysis, i. e. based on energy and stability criteria instead of strength criteria.

The most important difference between these methods is the size effect. Fig. 4.2-1 demonstrates that the nominal shear stress at failure $\sigma_N = P_U/bd$ (d = slab thickness, d = punch diameter) of geometrically similar structures for plastic limit analysis is size-independent, whereas for fracture mechanics it decreases as the structure size increases. However, linear fracture mechanics generally overestimate the size effect of reinforced concrete structures. Therefore, nonlinear fracture mechanics have to be applied to represent a gradual transition from the failure criterion of limit analysis (for very small structures) to the linear elastic fracture mechanics (for very large structures).

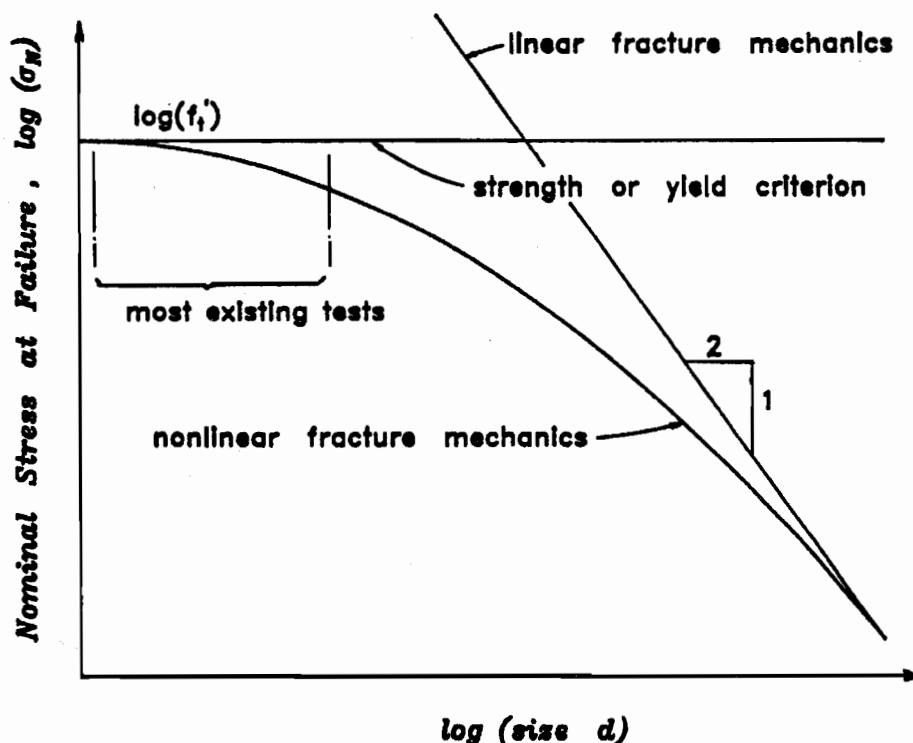


Fig. 4.2-1: Size effect laws according to plastic analysis, linear and nonlinear fracture mechanics

Punching tests

In order to quantify the size effect on the punching phenomenon, Bazant and Cao carried out nine punching tests on geometrically similar micro-concrete specimens in three different thicknesses. The circular slabs contained an orthogonal reinforcing mesh at the bottom and were 25.4 mm, 50.8 mm and 101.6 mm thick with a column diameter equal to the slab thickness and a span of four times the slab thickness.

Fig 4.2-2 shows the typical load-displacement diagrams which exhibit a sharp peak followed by gradual softening, i. e. the failure was caused by brittle cracking and not by plasticity of the concrete. It is interesting that the larger the specimen the steeper is the decline of load after the peak point.

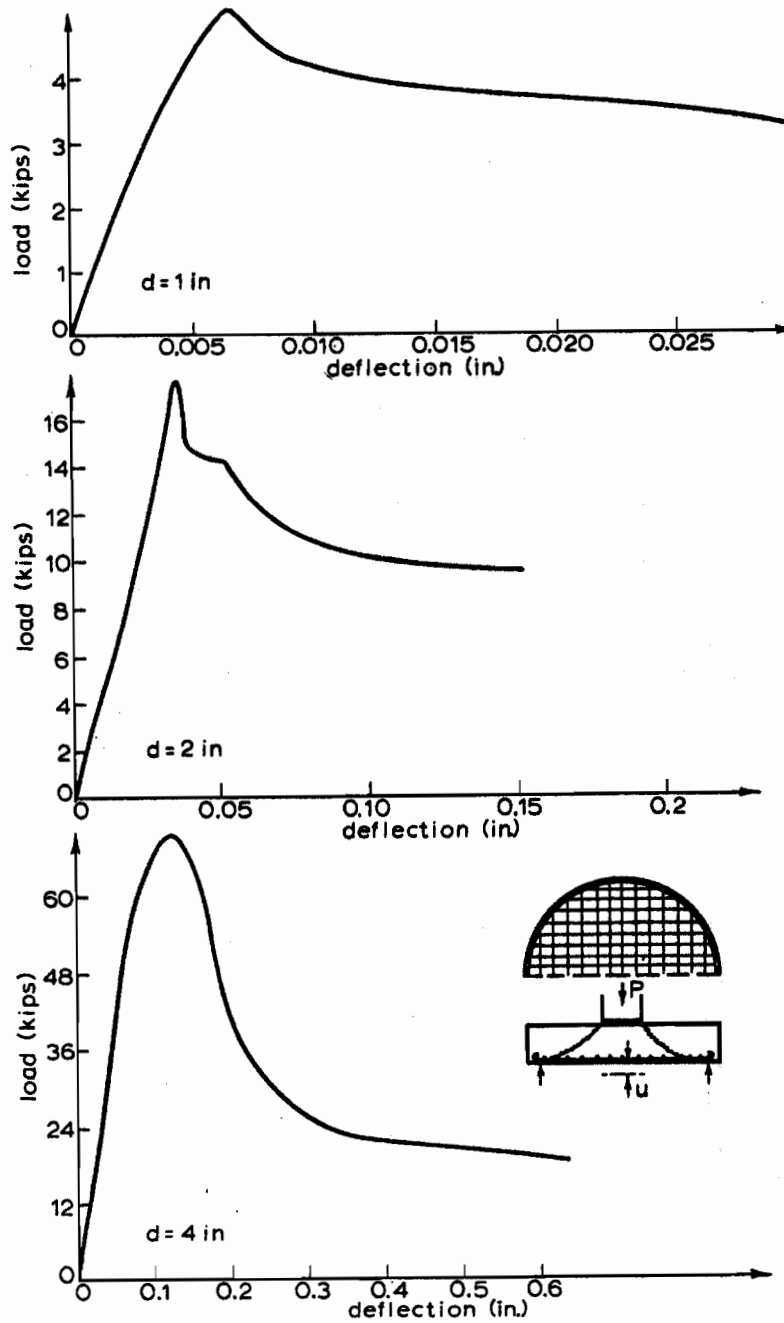


Fig.4.2-2: Typical measured load-deflection diagrams

Bazant and Cao concluded that the response of a large specimen is less ductile than that of a smaller specimen and regard this behavior as an indirect confirmation of the size-effect law. Moreover, the test results confirm a significant decrease of the nominal shear stress at failure $\sigma_N = P_U/bd$ with an increase of the slab thickness.

Proposed formula

From the results of the punching tests Bazant and Cao derived equation (4.2-1) for the calculation of the punching load:

$$v_u = C \cdot \left(1 + \frac{d}{\lambda_0 \cdot d_a} \right)^{-1/2} \quad (4.2-1)$$

where: v_u = nominal shear stress
 C = value of the nominal shear stress according to plastic limit analysis
 $= k_1 \cdot f_c \cdot \left(1 + k_2 \cdot \frac{d}{b} \right)$
 d = slab thickness
 b = diameter of the punch cone
 k_1, k_2 = empirical constants
 λ_0 = empirical parameter characterizing the fracture energy of the material and the shape of the structure
 d_a = maximum aggregate size

Finally, Bazant and Cao compare the proposed equation (4.2-1) to previous test data from the literature. However, the test data neither validate nor contradict the proposed formula due to their enormous scatter.

4.3 Model by Georgopoulos based on tensile strength

Georgopoulos (1988, 1989) developed a procedure for the calculation of the punching load and the punching angle of a flat slab without shear reinforcement. In his approach the concrete tensile strength $\bar{\sigma}_1$ and the mechanical reinforcement ratio ω are the main parameters.

Fig. 4.3-1 shows the load-bearing model for the punching failure. Georgopoulos supposes that the punching load is carried by the principle tensile stresses $\bar{\sigma}_1$ in the expected punching crack and by the inclined compressive forces in the compression shell around the column. Assuming that approximately 75% of the punching load is carried by the principle tensile stresses $\bar{\sigma}_1$ in the expected punching crack, the punching load can be calculated from the vertical component of the resulting tensile force Z_B in the assumed punching crack according to equation (4.3-1).

$$P_u = \frac{Z_B \cdot \cos \alpha}{0.75} \quad (4.3-1)$$

where: Z_B = resulting tensile force Z_B in the assumed punching crack
 α = inclination of the punching crack

Determination of the tensile stresses in the punching crack

Georgopoulos assumes a distribution of the tensile stresses in the surface of the expected punching crack according to Fig. 4.3-2 as a polynom of third order. Moreover, he estimates the height of the compression zone to $x = 0.2 \cdot h_m$. The integration of the tensile stresses along the surface of the expected punching crack as well as equilibrium in the vertical direction according to equation (4.3-1) lead to equation (4.3-2) for the calculation of the punching load:

$$P_u = 4.13 \cdot \bar{\sigma}_1 \cdot h_m^2 \cdot \cot \alpha \cdot \left(\frac{\lambda}{2} + 0.20 + 0.35 \cdot \cot \alpha \right) \quad (4.3-2)$$

where: $\bar{\sigma}_1$ = maximum principle tensile stress in the expected punching crack
 $\approx 0.17 \cdot (f_{ck, cube})^{2/3}$
 h_m = effective depth
 α = inclination of the punching crack according to equation (5)
 λ = d_{st}/h_m
 d_{st} = diameter of the column

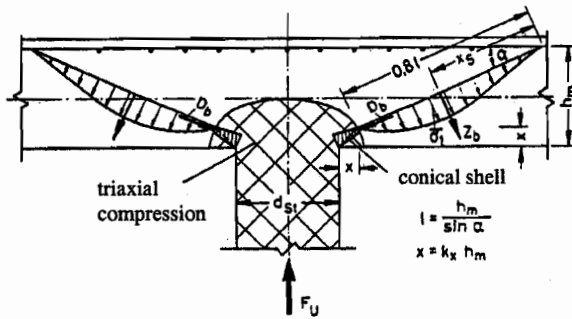


Fig. 4.3-1: Load-bearing model

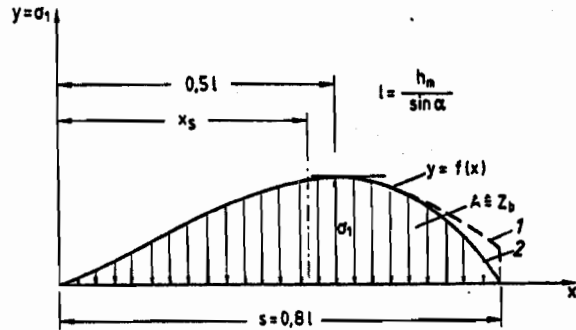


Fig. 4.3-2: Distribution of the concrete tensile stresses

Determination of the inclination α of the punching crack

Georgopoulos assumes that the inclination α of the punching crack is a function of the mechanical reinforcement ratio ω according to equation (4.3-3):

$$\tan \alpha = \frac{0.056}{\omega} + 0.3 \quad (4.3-3)$$

where: $\omega =$ mechanical reinforcement ratio
 $= \rho_l \cdot f_{yk} / f_{ck, \text{cube}}$

Fig. 4.3-3 illustrates the good agreement of equation (4.3-3) with the inclination angles α observed in punching tests.

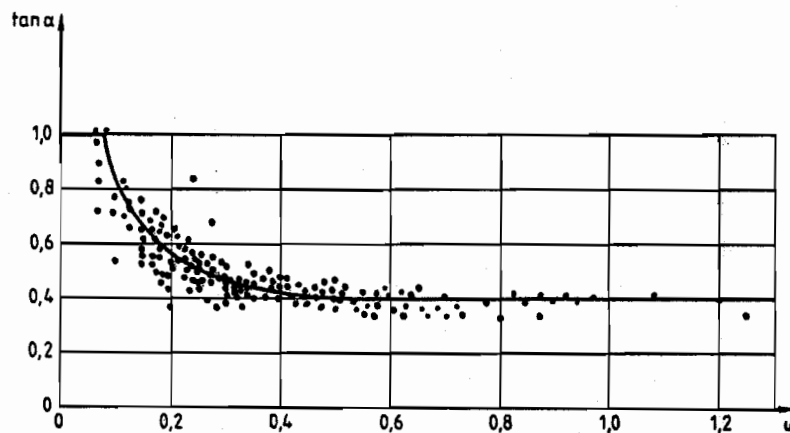


Fig. 4.3-3: Inclination α of the punching crack as a function of the mechanical reinforcement ratio ω

4.4 Modified model by Broms

In analogy to the theory of Kinnunen/Nylander (1960), the hypothesis of Broms (1990) is that punching occurs when the concrete in compression near the column is distressed either by a high tangential strain or a high radial stress. However, in his theory Broms introduces two principal modifications of the Kinnunen/Nylander approach. First, Broms utilizes generally recognized values for concrete properties instead of calibrating his model against specific test results. Second, he calculates different heights of the compression zone in the radial and tangential direction instead of Kinnunen/Nylander's iteration procedure. Furthermore, the design method is expanded to include unsymmetrical punching and size effect.

High tangential compression strain mechanism

Broms assumes that if the tangential concrete strain in the compression zone around the column exceeds a critical value ϵ_{cpu} macrocracks will start to form parallel to the compression direction. These macrocracks allow the inclined shear crack to propagate to the column face and punching to occur. Based on the behaviour of uniaxially compressed cylinder specimen Broms defines the critical tangential concrete strain ϵ_{cpu} as a function of the concrete strength and the thickness of the compression zone according to equation (4.4-1):

$$\epsilon_{cpu} = 0.0008 \cdot \left(\frac{150}{\alpha x_{pu}} \cdot \frac{25}{f'_c} \right)^{0.333} \quad (4.4-1)$$

where: 150 = diameter of a standard test cylinder specimen
 x_{pu} = height of the compression zone at flexure in tangential direction when punching occurs
 αx_{pu} = height of the equivalent rectangular stress block with the stress f'_c
 f'_c = concrete cylinder strength

Knowing the critical tangential concrete strain ϵ_{cpu} the punching load V_e can be calculated in the framework of the classical bending theory. I. e., based on linear-elastic bending moments and bilinear section properties the punching load V_e is determined using conditions of equilibrium and of compatibility (Bernoulli) as well as stress-strain curves for steel and concrete. Moreover, the calculation is carried out as a function of the reinforcement ratio in order to take into account possible yielding or failure of the reinforcement.

High radial compression stress failure mechanism

Broms assumes that punching occurs when the radial concrete compression stress in an imaginary conical shell with a constant thickness and an inclination of 15° (fig. 4.4-1) reaches the critical value $1.1 \cdot f'_c$ at the bottom of the shear crack. The factor 1.1 is derived from the strength increase of the concrete due to the biaxial state of stress around the column.

As a result, the punching load V_σ can be determined by the condition of equilibrium in the vertical direction according to equation (4.4-2). In equation (4.4-2), the factor $(150/0.5y)^{0.333}$ is introduced to account for size effects and the height of the compression zone in the radial direction y is determined from the position of the neutral axis in a reinforced concrete flexural member under elastic conditions.

$$\begin{aligned} \Sigma V: V_\sigma &= \left[\pi \cdot \left(B + 2y \frac{1}{\tan 30^\circ} \right) \cdot \frac{y \cdot \sin 15^\circ}{\sin 30^\circ} \cdot 1.1 \cdot f'_c \cdot \left(\frac{150}{0.5y} \right)^{0.333} \right] \cdot \sin 15^\circ \\ &\approx 0.46 \cdot (b + 3.5y) \cdot y \cdot f'_c \cdot \left(\frac{300}{y} \right)^{0.333} \end{aligned} \quad (4.4-2)$$

where: 150 = diameter of a standard test cylinder specimen
 y = approximate thickness of conical shell
 f'_c = cylinder strength
 b = diameter of column

Finally, the effective punching load is determined as the lesser value of V_e and V_σ . Moreover, Broms extends his theory to account for possible moment transfer between the column and the slab.

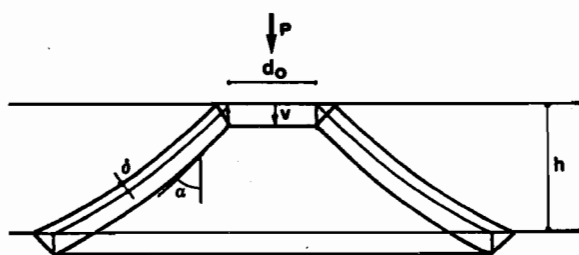


Fig. 4.5-1: Failure mechanism

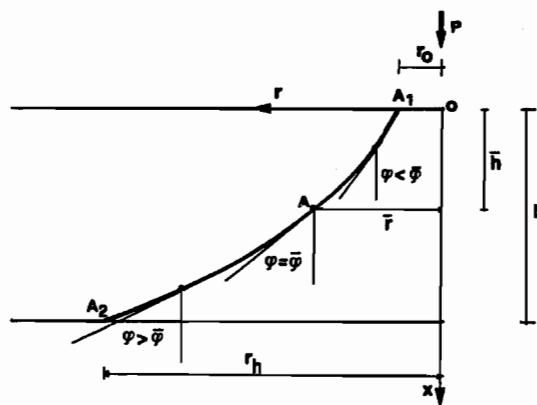


Fig. 4.5-2: Failure profile

In the next step, integration over the failure surface and application of the work equation lead to an upper bound solution for the punching load. To find the lowest upper bound solution, Bortolotti carries out a variational calculus using the Euler equation. Finally, some corrective mechanism factors are derived from a comparison of the theoretical failure load with the results of punching tests.

4.6 Bond model by Alexander and Simmonds

The so-called bond model by Alexander and Simmonds (1992) is the result of a modification of the truss model proposed earlier by Alexander and Simmonds (1987). As in the truss model, a slab-column connection is described as an assembly of steel tension ties and concrete compression struts.

While in the truss model the compression struts are straight and inclined at a constant angle α , in the bond model curved compression struts are assumed (Fig. 4.6-1) which were derived from strain measurements in punching tests. The curved arches are located in four radial strips extending from the column parallel to the reinforcement as shown in Fig. 4.6-2. The geometry of the curved arch is governed by the interaction between the arch and the adjacent quadrants of the two-way plate. The radial strips separate the column from the slab so that all load reaching the column must be carried by the radial strips. Within each strip, shear is carried in the radial direction by the vertical component of a curved compression strut or arch. Each strip is loaded on its side faces by the maximum, one-way shear that can be carried by the adjacent quadrants of plate. To deliver an internal shear to the strips, the plate must maintain moment gradients perpendicular to the radial strips. These moment gradients require force gradients in the reinforcement.

The force gradient in a reinforcement bar may be limited by bond failure or it may be limited by the spread of yielding along the length of the bar. Whatever the reason, the loss of reinforcing bar gradient in the quadrants of a plate reduces the ability of the slab to carry shear in the vicinity of the column. Therefore, all punching failures, whether or not they are preceded by extensive yielding of the slab reinforcement, may attribute to the loss of moment gradient and, hence, shear capacity in the vicinity of the column.

Based on this theory, the authors derive the maximum loading of the radial strips and thus the punching capacity from the bond strength of the reinforcement perpendicular to the radial strips using the bond model of Morita and Fujii (1982). However, the maximum loading of the radial strips can also be calculated using code predictions (e. g. ACI, BS) for a limiting nominal one-way shear stress that can be transferred from the quadrants of the plate to the radial strips.

It can be summarized, that the model of Alexander and Simmonds combines radial arching

action with the concept of a critical shear stress on a critical section (beam action shear). For brittle punching failure, bond strength of the reinforcement is seen as the significant factor limiting the beam action shear.

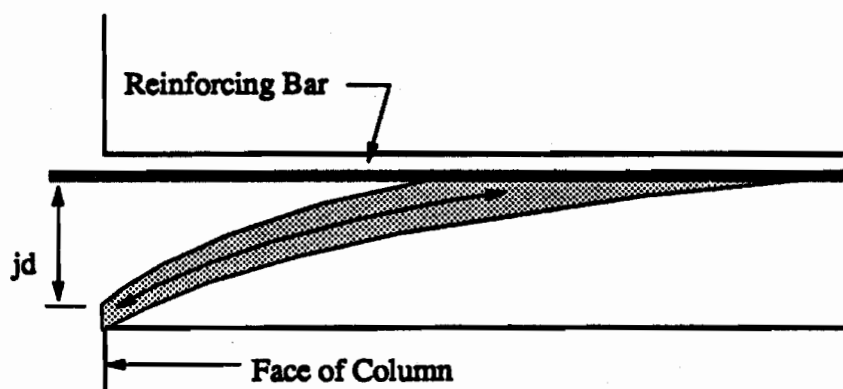


Fig. 4.6-1: Curved compression strut

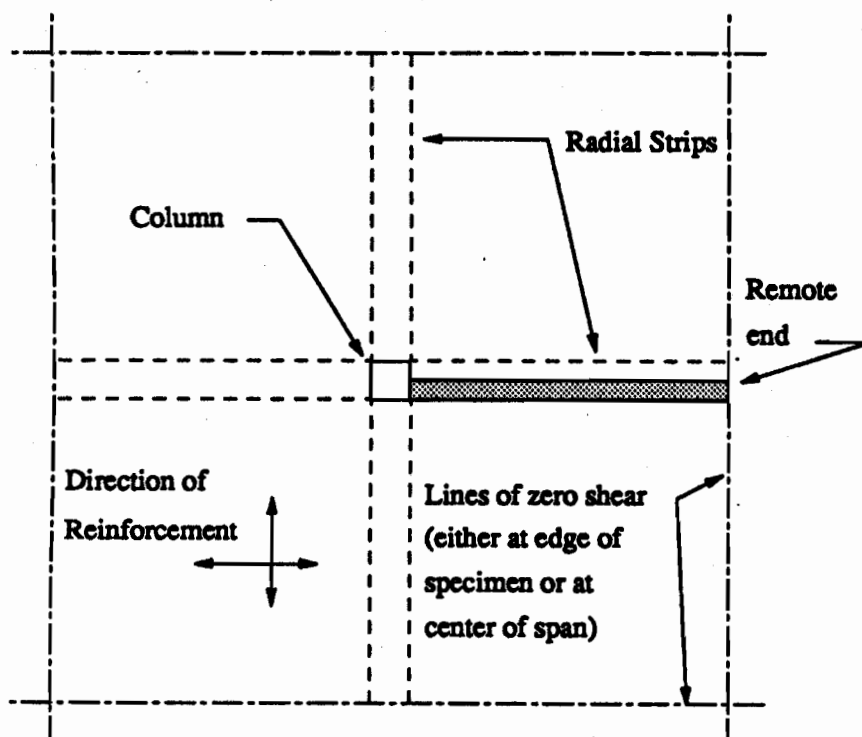


Fig. 4.6-2: Layout of radial strips

4.7 Analytical model by Menétrey

Ph. Menétrey

4.7.1 Preliminaries

The analytical expression proposed by Menétrey (1996) to compute the punching load was developed based on results obtained with the numerical simulation of the failure mechanism (Menétrey et al. (1994) and (1997) as described in sec. 5.4). It is enhanced with the treatment of the contribution of the shear reinforcements along the line of the work presented by Menétrey (1999). Finally, the influence of the flexural capacity is linked to the punching capacity with the punching crack inclination as proposed by Menétrey (1998) as presented in sec. 10.4.

The complete load transfer from the point of load-application up to the support assumed by the proposed model is illustrated in fig. 4.10.3a) with the strut-and-tie analogy. As opposed to most existing model, the concrete ties extend up to the D-region. This strut-and-tie model is detailed in the vicinity of the column in fig. 4.7-1 so that the concrete tie which crosses the punching crack can be visualized.

The basic idea of the model is to assume that punching failure corresponds to the failure of the concrete tie, so that *the tie strength is equivalent to the punching strength*. The resulting punching load is computed by integrating the vertical components of the concrete tensile stresses around the punching crack. This is performed according to the concept of non-linear fracture mechanics, for which tensile stresses can still be transmitted across a crack as presented in sec. 5.4.2 (exponential decrease of the residual tensile strength). The model is generalized to take into account the reinforcements by simply adding to the integral of the concrete tensile stresses, the contribution of each reinforcements which are crossing the punching crack. The resulting punching load is expressed as:

$$F_{pun} = F_{ct} + F_{dow} + F_{sw} + F_p, \quad (4.7-1)$$

where F_{ct} is the vertical component of the concrete tensile force, F_{dow} is the dowel-contribution of the flexural reinforcement, F_{sw} is the vertical component of the force in the studs, stirrups or bent-up bars which are well-anchored and F_p is the vertical component of the force in the tendons. The punching load of a slab is consequently computed in a unified manner by summation of all the vertical tensile forces crossing the punching crack as illustrated in fig. 4.7-1.

The punching failure is a sudden failure characterized by a rapid decrease of the load carrying capacity as shown in sec. 10.4.3 (specially on fig. 10.4-3 and 10.4-4). However, the punching crack causing the punching failure is not formed suddenly, but is preceded by the formation of internal micro-cracks. This micro-crack formation is reported experimentally by Moe (1961) and Regan (1983). Along the same line, Menétrey et al. (1997) showed that punching failure results from a coalescence of micro-cracks (progressive phase) followed by a crack propagation (sudden phase) as shown in fig. 5.4-4. Because the micro-crack formation is progressive, steel resistance is gradually activated. Therefore, the force in the reinforcement can be added to the concrete tensile force as proposed by the general model in eq. 4.7-1 (as long as the slab is under reinforced as given in eq. 4.7-11).

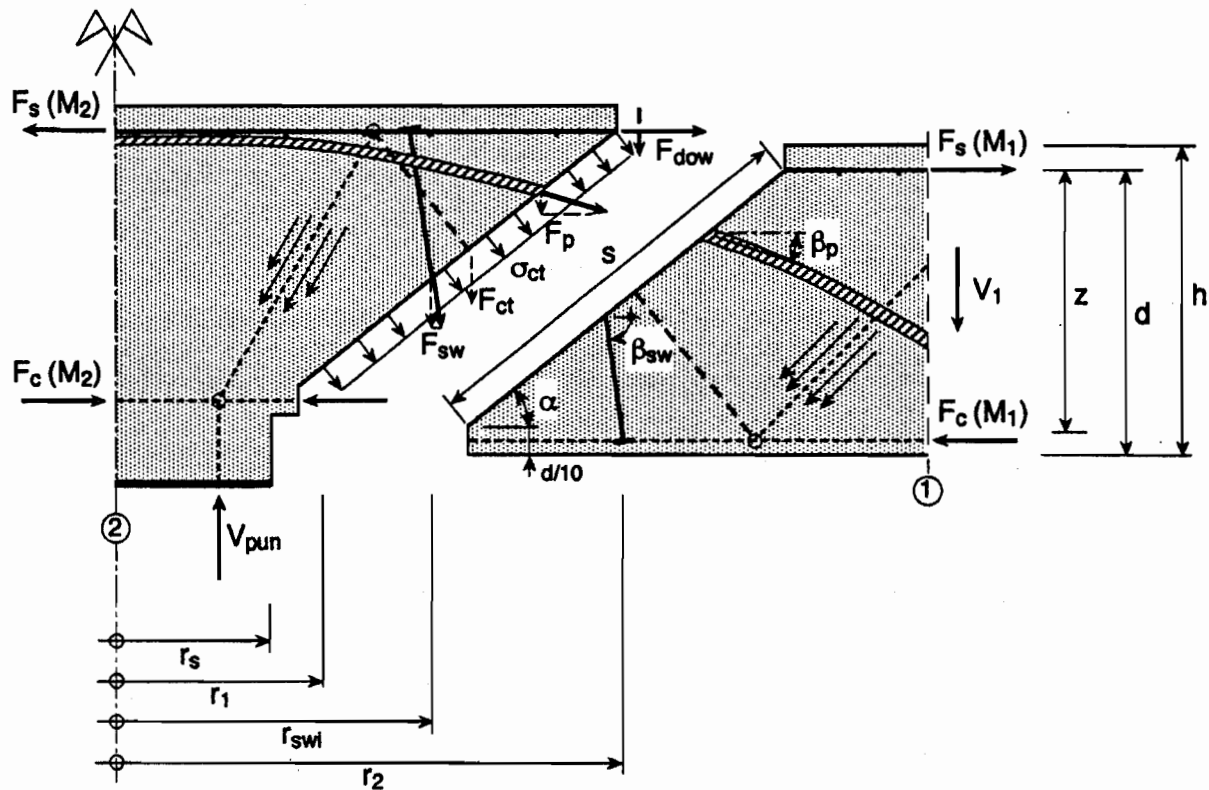


Figure 4.7-1: Representation of the punching shear capacity of a general reinforced slab

4.7.2 Concrete tensile force

The concrete tensile force is determined by integrating the vertical tensile stresses σ_v around the punching crack. It should be noted that no shear transfer in concrete is assumed around the punching crack (no aggregate interlock, no friction stress) as summarized in tab. 4-10.1. The punching crack is assumed to be a truncated cone in shape comprised between two radii r_1 and r_2 as shown in fig. 4.7-1. The radius r_1 is assumed to correspond to 1/10 of the effective depth of the slab (analogy with flexural behavior) so that:

$$r_1 = r_s + \frac{1}{10} \frac{d}{\tan \alpha}, \quad (4.7-2)$$

where r_s is the radius of the column and α the punching crack inclination. The radius r_2 extends up to the flexural reinforcing bars so that:

$$r_2 = r_s + \frac{d}{\tan \alpha}. \quad (4.7-3)$$

The surface between r_1 and r_2 is: $\pi(r_1 + r_2)s$ where s is the inclined length:

$$s = \sqrt{(r_2 - r_1)^2 + (0.9d)^2}. \quad (4.7-4)$$

Around the punching crack, a constant tensile stress distribution is assumed for simplicity. The analytical expression to compute the concrete tensile force is consequently expressed as follows:

$$F_{ct} = \pi(r_1 + r_2) s \sigma_v = \pi(r_1 + r_2) s f_{ct}^{2/3} \xi \eta \mu. \quad (4.7-5)$$

The concrete tensile force is influenced by the tensile strength of concrete f_{ct} (shown in fig. 5.4-5a)) and the analytical relation proposed by Menétrey et al. (1997) given

by: $F_{ct} \propto f_{ct}^{2/3}$ is considered. Three parameters ξ , μ , and η reproduce respectively the influence of the percentage of flexural reinforcement, the slab thickness and the radius of punching crack initiation.

The influence of the percentage of reinforcement ρ on the tensile stresses was determined with the numerical simulation of punching failure in slabs with a percentage of reinforcement which varies from $0.2\% \leq \rho \leq 2\%$ (presented in fig. 5.4-6). It was observed that the intensity of tensile stresses and their inclinations increase with increasing percentage of reinforcement. The analytical expression developed to compute the concrete tensile force reproduces the influence of the percentage of reinforcement with the parameter ξ (unit of the parameter ξ is $\text{MPa}^{1/3}$). The numerical results are matched with a parabolic expression given by:

$$\xi = \begin{cases} -0.1\rho^2 + 0.46\rho + 0.35 & 0 < \rho < 2\% \\ 0.87 & \rho \geq 2\%, \end{cases} \quad (4.7-6)$$

where ρ is in percent.

The size-effect laws obtained numerically (presented in fig. 5.4-7 and given in eq. 5.4-5 and 5.4-6) are combined and included in the developed analytical model with the parameter μ expressed as:

$$\mu = 1.6(1 + d/d_a)^{-1/2} \quad (4.7-7)$$

where d_a is the maximum aggregate size.

This size-effect law was derived for a constant ratio $h/r_s = 2$ where h is the slab thickness and r_s is the radius of the column. However, as reported experimentally, the radius of the column or similarly the radius where the punching crack is initiated influences the size-effect law. Corley and Hawkins (1968) tested several slabs containing shearhead reinforcement and they observed that the nominal stress at failure—computed at a control section at the extremity of the shearhead—is lower than the value obtained without shear reinforcement. Along the same line, Van der Voet et al. (1982) tested slabs with shear reinforcement and established that the nominal shear stress—computed at a distance $d/2$ away from the last row of shear elements—decreases with the distance from the face of the column. Therefore, increasing the radius where the punching crack is initiated, results in a decrease of the nominal stress. This experimental observation is also reproduced numerically as shown in table 5.4-1. Based on this table, an analytical relation is proposed to reproduce the influence of the radius of the column, or more generally the radius of the punching crack initiation:

$$\eta = \begin{cases} 0.1(r_s/h)^2 - 0.5(r_s/h) + 1.25 & 0 < r_s/h < 2.5 \\ 0.625 & r_s/h \geq 2.5. \end{cases} \quad (4.7-8)$$

This parameter is determinant to predict the punching load of slab with shearhead or shear reinforcement.

4.7.3 Dowel-effect contribution

The dowel-effect is a shear mechanism transferred by flexural reinforcements crossing the punching crack. This mechanism increases the punching load significantly for slabs with orthogonal reinforcement. Regan and Braestrup (1985) reported that up to 34% of the total punching load of slabs with orthogonal reinforcement can be attributable to dowel-effect. Consequently, the prediction of the experimental punching load for slabs with orthogonal reinforcement should consider the dowel-effect.

The shear force which can be transferred by reinforcing bars crossing the punching crack is computed by adapting the approach proposed by the CEB-FIP model code (1990) to the punching failure mechanism so that:

$$F_{dow} = \frac{1}{2} \sum^{bars} \phi_s^2 \sqrt{f_c f_t (1 - \zeta^2)} \sin \alpha, \quad (4.7-9)$$

where the summation is performed for all bars crossing the punching crack and ϕ_s is the diameter of the corresponding bars. The design values of the materials are replaced by their characteristic values (f_c is the concrete compressive strength and f_t is the steel yield strength). A parabolic interaction is assumed between the axial force and the dowel force in the reinforcing bar. This parabolic interaction is expressed with the term: $(1 - \zeta^2)$ where $\zeta = \sigma_s / f_t$ and σ_s is the axial tensile stress in the reinforcing bar (given in eq. 4.7-10). The tensile stress σ_s is obtained by projection of the force in a compressive strut parallel to the punching crack: $F_{pun} / \sin \alpha$ in the horizontal reinforcement which gives $F_{pun} / \tan \alpha$ and by dividing by the sum of the area of the reinforcing bars crossing the punching crack so that:

$$\sigma_s = \frac{F_{pun} / \tan \alpha}{\sum^{bars} A_s}. \quad (4.7-10)$$

The dowel-contribution given in eq. 4.7-9 is reduced with the term: $\sin \alpha$, as suggested by Pruijssers (1988) to take into account the angle between the flexural reinforcing bar and the punching crack (in a vertical plane). In eq. 4.7-9, the factor 1/2 gives the best approximation of the dowel-effect contribution because the reinforcing bars are not crossing the punching crack at a right angle (in an horizontal plane) as orthogonal and not radial reinforcement is considered.

4.7.4 Shear reinforcement contribution

The computation of the failure load in slabs with shear reinforcements such as studs, stirrups, bent-bars or bolts should differentiate the following failure mechanisms (illustrated in fig. 4.7-2 for a slab with studs).

- 1) A failure mechanism for which the punching crack is located in between the column face and the first row of shear reinforcement. The computation of the corresponding failure load must consider the interaction between the punching F_{pun} and the flexural capacity F_{flex} in terms of the punching crack inclination α as described in sec. 10.4.5 and 4.7.6 and expressed in eq. 4.7-16. The punching crack inclination is: $\alpha_a = \arctan((r_{swi} - r_s)/h)$.
- 2) A failure mechanism for which the punching crack is initiated outside the last row of shear reinforcement. The punching shear capacity is computed similarly to the one of a normal reinforced concrete slab except that instead of the radius of the column r_s , the radius of punching crack initiation r_{sc} should be considered. The influence of this radius on the size-effect is treated with the parameter η given in eq. 4.7-8.
- 3) A failure mechanism for which the punching crack crosses the shear reinforcement.

The computation of the punching load for the third failure mechanism is further addressed here. In order to treat the contribution of the shear reinforcements crossing the punching crack, their bond properties are differentiated: (1) shear reinforcements made

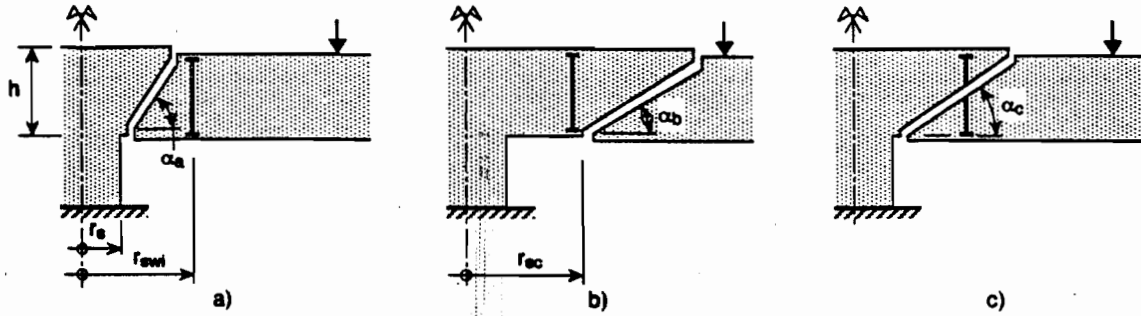


Figure 4.7-2: Three punching failure mechanisms : (a) punching crack located in-between the column face and the first row of studs, (b) punching crack outside the last row of studs, and (c) punching crack crossing the studs

with plain bars and anchorage denoted as *studs*, (2) shear reinforcements made with high-bond bars denoted as *stirrups*. The contribution of injected strengthening bolts (set after perforating the slab) is determined similarly to the contribution of studs or stirrups according to their bond properties. As a special case, for non-injected strengthening bolts, the concrete and the shear reinforcements contributions cannot be added because they do not interact as established by Menétrey and Brühwiler (1997).

In what follows, only slabs with shear reinforcements which are *under-reinforced against punching failure* are treated, that is slabs for which the tensile strength of concrete is smaller than the yield strength of the shear reinforcement (both obtained by integrating the force around the punching crack). This condition can be written as:

$$\text{Condition for under-reinforced slab: } F_{sw} < F_{ct} \quad (4.7-11)$$

The reason for this restriction is that for higher percentage of shear reinforcement, the tensile force in the shear reinforcement cannot reach the yield strength at the same time as the concrete tensile force reaches its maximum value.

The contribution to the punching load of studs is derived for the axisymmetric reinforced concrete slab illustrated in fig. 4.7-3. This slab is reinforced with vertical studs well anchored at their extremities and made with plain bars characterized by a cross-section A_{sw} and the yield strength f_{sw} . By considering the definition of the fracture energy in sec. 5.4.2, eq. 5.4-1, the crack rupture opening can be approached by:

$$w_r \approx \frac{5G_f}{f_{ct}}. \quad (4.7-12)$$

The failure mechanism for which the punching crack crosses the studs is initiated by micro-cracks. Due to micro-cracking the slab thickness increases, resulting in the loading of the studs. Consequently, the studs sustain a so-called displacement control loading for which the displacement corresponds to the opening of one crack w_r between both studs extremities l . The studs deformation is $\epsilon_s = w_r / (l \cos \alpha)$ and the maximum force is expressed as:

$$F_{sw} = \sum A_{sw} E_{sw} \frac{5G_f}{f_{ct} l \cos \alpha} \sin(\beta_{sw}) < \sum A_{sw} f_{sw} \sin(\beta_{sw}) \quad \text{for studs} \quad (4.7-13)$$

where the summation is performed over each studs crossing the punching crack. This force is limited by the yield strength of the studs. For high studs length, the contribution

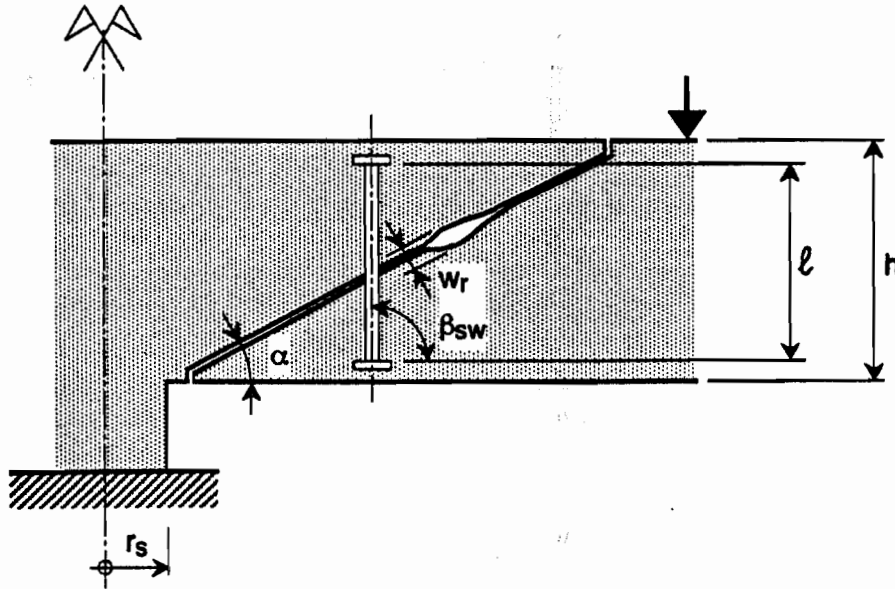


Figure 4.7-3: Punching crack formation in slab reinforced with studs

to the punching load is reduced, reflecting a size-effect inversely proportional to the studs length.

The tensile force in the stirrups is transmitted by bond stresses to concrete over the transmission length (defined as the length over which slip between steel and concrete occurs CEB-FIP Model Code (1990)). If this length is available, the yield strength of the stirrups can be reached so that :

$$F_{sw} = \sum A_{sw} f_{sw} \sin(\beta_{sw}) \quad \text{for stirrups} \quad (4.7-14)$$

where the summation is performed over each stirrups crossing the punching crack. If this length is not available, the carrying force in the stirrup is function of the anchorage of the stirrup's extremity. Consequently, the anchorage of the stirrups has to be carefully set, specially in the top face of slabs (positive bending) and for shorts stirrups.

As a conclusion based on these developments, it can be inferred that: *studs are more adapted for thin slabs and stirrups are more adapted for thick slabs.*

4.7.5 Prestressing tendons contribution

The punching load of a slab reinforced with inclined prestressing tendons is enhanced by adding the vertical projection of the tendons force. This vertical projection of the tendons force F_p is expressed as:

$$F_p = \sum^{tendons} A_p \sigma_p \sin(\beta_p), \quad (4.7-15)$$

where the summation is performed for the prestressing tendons crossing the punching crack, A_p is the area of the prestressing tendons, σ_p is the tensile stress of the prestressing steel, and β_p is the inclination of the tendons at the intersection of the punching crack with the plane of the slab as shown in fig. 4.7-1. The prestressing stress σ_p is considered and not the yield strength f_p of the tendons, because a shear failure does not allow large deformation, so that the yield strength cannot be reached. The influence of unbonded tendons is considered only with the vertical projection of the tendons force of eq. 4.7-15. For bonded tendons, in addition to this contribution, the percentage of flexural

reinforcement ρ_s used to compute the parameter ξ in eq. 4.7-6 is modified with the percentage of prestressing tendons ρ_p in the neighborhood of the punching cone so that: $\rho = \rho_s + \rho_p$.

4.7.6 Link between punching and flexural capacity

The geometry of the failure surface plays an important role, especially when the failure surface is forced into a shape different from that giving the normal punching resistance as suggested by Regan (1984). The influence of the punching crack inclination is presented in sec. 10.4.5 based on experimental results. It is shown that *the punching crack inclination* α allows to link both flexural and punching failure loads and the analytical expression (given in eq. 10.4-5). By assuming a standard punching crack inclination of $\alpha_0 = 30^\circ$ the general failure load of a slab can be expressed as:

$$F_{fail} = F_{pun} + (F_{flex} - F_{pun}) \left\{ \sin\left(\frac{3}{2}\alpha - 45^\circ\right) \right\}^{1/2} \text{ with } 30^\circ \leq \alpha \leq 90^\circ. \quad (4.7-16)$$

It can be noted that for a punching crack inclination $\alpha = 30^\circ$ then $F_{fail} = F_{pun}$ and for $\alpha = 90^\circ$ then $F_{fail} = F_{flex}$. For design application, the flexural failure load F_{flex} can be determined with the yield-line theory as presented by Gesund and Kaushik (1970).

4.7.7 Design application

The analytical method presented here can be adapted for design purposes so that:

- Safety factors are reducing the values of the tensile strength of concrete, the tensile strength of shear reinforcement and stress in the prestressing steel.
- Dowel-contribution should be neglected for two reasons: (1) it can only be activated for large deformation, so that the load is close to the punching load; (1) the determination of the dowel contribution is iterative and time demanding as the stress in the reinforcement σ_s is required.
- The mechanism a) given in fig. 4.7-2 should be prevented by limiting the radius r_{swi} and the flexural strength V_{flex} .

The resulting design model for punching load prediction can be classified as a strut-and-tie model and the failure of the tie close to the column corresponds to the punching failure. The model is based on solid theoretical background and can be easily visualized and understood. Furthermore, the proposed model is one of the few one which treats all the reinforcements (flexural, shear and prestress reinforcement) so that it range of application covers most of the slabs. Finally, with the above mentioned simplifications, the model is easy to apply.

4.8 Modified Mechanical Model by Hallgren

Mikael Hallgren

4.8.1 Introduction

The mechanical model proposed by Kinnunen & Nylander (1960) gives a physically realistic description of the mechanisms prior to the failure. However, the failure criterion is given as a set of semi-empirical expressions, which are based on strains measured in punching shear tests. Furthermore, the model does not consider the size effect on the punching shear strength.

In Hallgren (1996), it was shown that a significant increase of the punching shear strength can be gained by using high strength concrete. However, the observed punching shear strength was not proportional to the compressive strength, nor to the tensile strength of concrete. It appeared that the brittleness of concrete with higher strength reduces the rate of increase of punching shear strength.

The present chapter presents a modification of the mechanical model by Kinnunen and Nylander (1960) based on previous test observations and previous numerical simulations, (Hallgren 1996, 1997). The failure criterion is derived from a simple fracture mechanical model, which reflects both the brittleness of concrete and the size effect.

4.8.2 The Model

As in the original model by Kinnunen & Nylander, the proposed modified model considers a circular polar-symmetrical slab supported on a centric column and externally loaded with a uniformly distributed load at the circumference of the slab, see Fig. 4.8-1. The slab portions outside the shear crack, which are bounded by the shear crack and by radial cracks, are assumed to rotate as rigid bodies. The external load P is computed from the equilibrium of forces acting on a segmental slab portion, with the centre angle $\Delta\phi$, outside the shear crack.

The load is carried from the slab portion outside the shear crack to the column through a truncated wedge. The inclined compression force in the truncated wedge is denoted T . The length of the truncated wedge along the bottom surface of the slab in radial direction is given by $y = x(1 + \tan\alpha)$, where α is the inclination of the compression force T and x is the depth of the tangential compression zone.

The distance c_0 from the column centre to the shear crack at the level of the flexural reinforcement is taken as

$$c_0 = \frac{B}{2} + x + \frac{d - x}{\tan(1.5\alpha)} \quad (4.8-1)$$

where B is the diameter of the column, d is the average effective depth of the slab, and 1.5α is the angle of the shear crack. The value 1.5 is based on test observations and results from finite element analyses (Hallgren 1996, 1997).

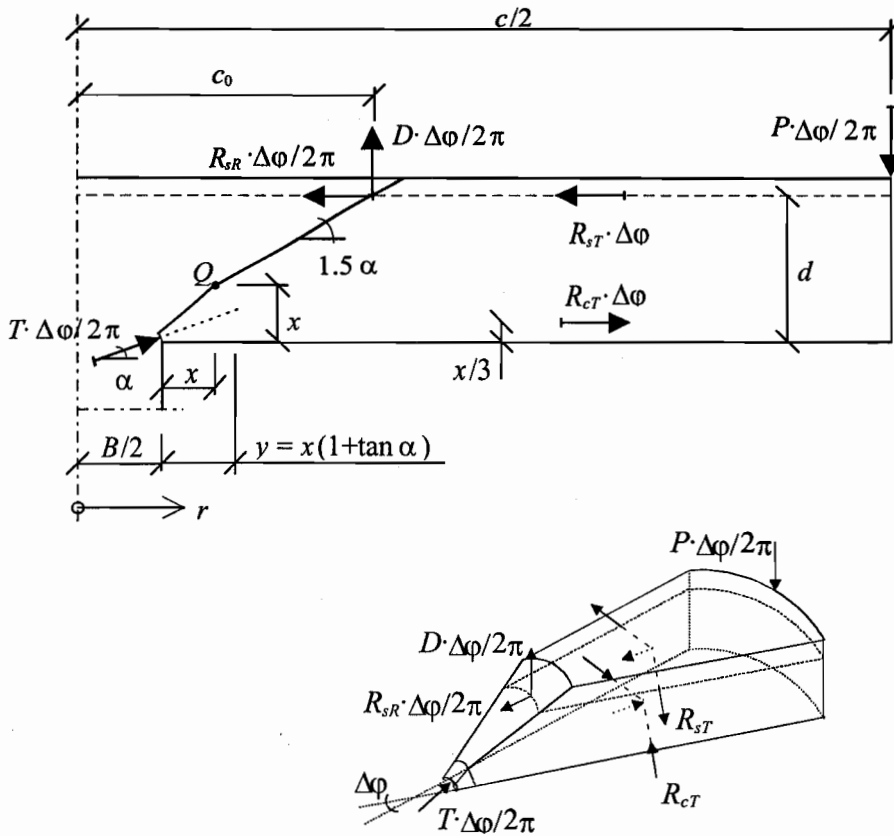


Fig. 4.8-1: The model

4.8.2.1 States of stress and strain

Based on test observations and on results from the finite element analyses, the tangential strains in the concrete on the compression side of the slab and in the reinforcing steel on the tension side of the slab are assumed to be inversely proportional to the radius r from the slab centre, see Fig. 4.8-2.

The constitutive laws of concrete in compression and steel in tension are both simplified by bi-linear curves. The depth x of the tangential compression zone is determined from the tangential concrete and steel strains, and the stress-strain states, i.e. if the materials are in elastic or plastic states. The distribution of strains across the tangential cross section is assumed to be linear.

If steel and concrete both are in the elastic state, x is given by

$$x = \rho \frac{E_s}{k_E \cdot E_c} \left(\sqrt{1 + \frac{2 \cdot k_E \cdot E_c}{\rho \cdot E_s}} - 1 \right) \cdot d \quad (4.8-2)$$

where ρ is the ratio of reinforcement, E_s and E_c are the moduli of elasticity of steel and concrete, respectively, and k_E is a factor which considers the increase of stiffness in bi-axial compression. The factor is given by

$$k_E = \left(1 - \frac{\nu + 2\nu^2}{2 + \nu} \right)^{-1} \quad (4.8-3)$$

where ν is Poisson's ratio.

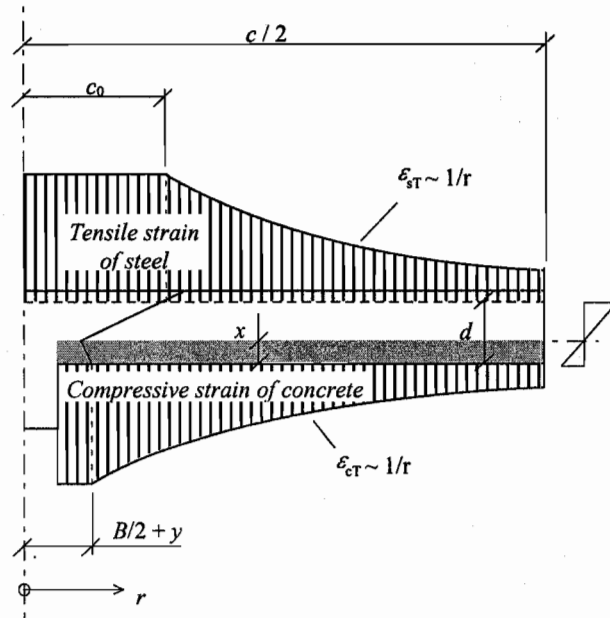


Fig. 4.8-2: Distribution of tangential steel and concrete strains along the radius r

If concrete has yielded at $r = c_0$ and steel is in the elastic state, then x is computed from

$$x = \frac{\rho \cdot E_s \cdot \varepsilon_{cT0}}{2 \cdot \alpha_{c0} \cdot f_{cc}} \left(\sqrt{1 + \frac{4 \cdot \alpha_{c0} \cdot f_{cc}}{\rho \cdot E_s \cdot \varepsilon_{cT0}}} - 1 \right) \cdot d \quad (4.8-4)$$

where f_{cc} is the compressive strength of concrete, ε_{cT0} is the tangential concrete strain at $r = c_0$, and

$$\alpha_{c0} = 1 - \frac{\varepsilon_{cy}}{2 \cdot \varepsilon_{cT0}}, \text{ where } \varepsilon_{cy} = \frac{f_{cc}}{E_c}.$$

At ultimate load, the tangential concrete strain at $r = c_0$ is given by

$$\varepsilon_{cT0} = \varepsilon_{cTu} \frac{(B/2 + y)}{c_0} \quad (4.8-5)$$

where ε_{cTu} is the ultimate tangential concrete strain, i.e. the failure criterion.

If steel has yielded at $r = c_0$ and concrete is in the elastic state, then x is computed from

$$x = \frac{2 \cdot \rho \cdot d \cdot f_{sy}}{\varepsilon_{cT0} \cdot k_E \cdot E_c} \quad (4.8-6)$$

where f_{sy} is the yield stress of the reinforcing steel.

If both steel and concrete have yielded at $r = c_0$, then x is expressed as

$$x = \frac{\rho \cdot d \cdot f_{sy}}{\alpha_{c0} \cdot f_{cc}} \quad (4.8-7)$$

4.8.2.2 Forces acting on the segmental slab portion

The resulting forces of concrete in compression and steel in tension, i.e. R_{cT} , R_{sT} , and R_{sR} , see Fig. 4.8-1, are computed by integration of stresses over the area they act on. The stresses are determined from the concrete and steel strains, and the stress-strain states.

If all concrete in tangential direction is in the elastic state, then the resulting tangential concrete force at ultimate load can be expressed as

$$R_{cT} = \frac{1}{2} E_c \cdot x \cdot \epsilon_{cTu} \left(\frac{B}{2} + y \right) \cdot \ln \left(\frac{c}{B + 2y} \right) \quad (4.8-8)$$

If the tangential concrete strain reaches the yield strain ϵ_{cy} at $r > B/2 + y$, then this yield radius can be determined from

$$r_c = \frac{\epsilon_{cTu}}{\epsilon_{cy}} \left(\frac{B}{2} + y \right) \quad (4.8-9)$$

The distribution of the tangential concrete stresses in the compression zone along the radius of the slab for this case is shown in Fig. 4.8-3.

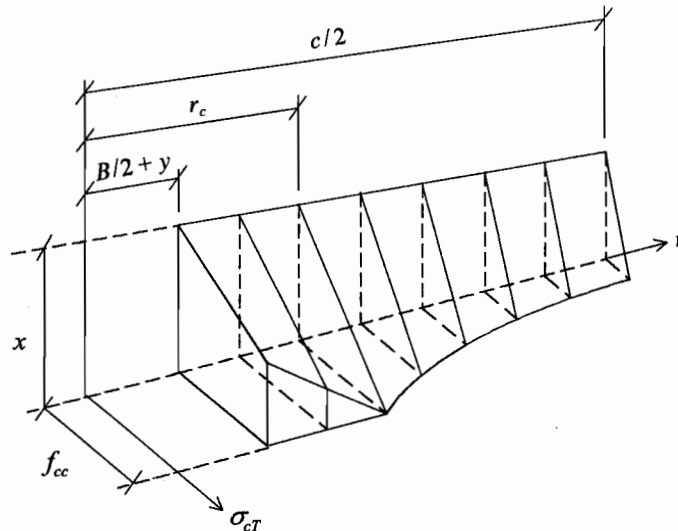


Fig. 4.8-3: Distribution of tangential concrete stresses σ_{cT} in the compression zone along the radius r of the slab.

Hence, if the concrete has yielded in tangential direction, Equation (4.8-8) has to be rewritten as

$$R_{cT} = f_{cc} \cdot x \cdot \left[r_c \left(\frac{3}{4} + \frac{1}{2} \cdot \ln \left(\frac{c}{2r_c} \right) \right) - \left(\frac{B}{2} + y \right) + \frac{1}{4 \cdot r_c} \left(\frac{B}{2} + y \right)^2 \right] \quad (4.8-10)$$

If all steel in tangential direction is in the elastic state, then the resulting tangential steel force can be expressed as

$$R_{sT} = \rho \cdot d \cdot E_s \cdot \epsilon_{cTu} \left(\frac{d}{x} - 1 \right) \left(\frac{B}{2} + y \right) \cdot \ln \left(\frac{c}{2c_0} \right) \quad (4.8-11)$$

If the tangential steel strain reaches the yield strain ϵ_{sy} at $r > c_0$, then this yield radius can be determined from

$$r_s = \frac{\epsilon_{cTu}}{\epsilon_{sy}} \left(\frac{d}{x} - 1 \right) \left(\frac{B}{2} + y \right) \quad (4.8-12)$$

The distribution of tangential steel stresses of the flexural reinforcement along the radius of the slab for this case is shown in Fig. 4.8-4.

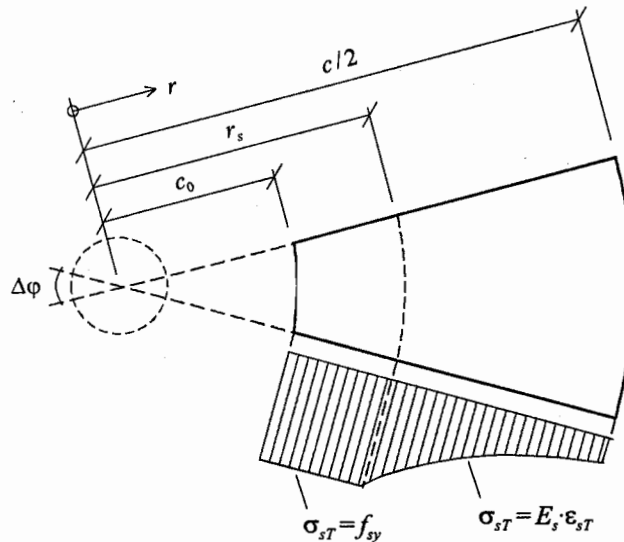


Fig. 4.8-4: Tangential steel stresses σ_{sT} of the flexural reinforcement along the radius r of the slab.

Hence, if the reinforcing steel has yielded in tangential direction, Equation (4.8-11) has to be rewritten as

$$R_{sT} = \rho \cdot d \cdot f_{sy} \left[r_s \left(1 + \ln \left(\frac{c}{2r_s} \right) \right) - c_0 \right] \quad (4.8-13)$$

The radial steel strain of the reinforcement intersecting the shear crack at $r = c_0$ is assumed to be equal to the tangential steel strain at $r = c_0$. Thus, the resulting radial force R_{sR} of the reinforcement at $r = c_0$ is determined from the following set of equations:

If $r_s < c_0$, then

$$R_{sR} = \rho \cdot d \cdot 2\pi \cdot E_s \cdot \epsilon_{cTu} \left(\frac{d}{x} - 1 \right) \left(\frac{B}{2} + y \right) \quad (4.8-14)$$

and if $r_s \geq c_0$, then

$$R_{sR} = \rho \cdot d \cdot 2\pi \cdot c_0 \cdot f_{sy} \quad (4.8-15)$$

The dowel force D is determined from a semi-empirical expression derived from previous beam tests by Hamadi & Regan (1980):

$$D = 27.9 \cdot \emptyset^{2/3} \cdot c_0 \left(1 - 1.6 \frac{\rho \cdot d}{\emptyset} \right) f_{cc}^{1/3} \quad [N] \quad (4.8-16)$$

where $\emptyset$ is the diameter [mm] of the reinforcement bars. If the reinforcement is in the state of yielding, the dowel force would be essentially reduced. For the sake of simplicity, D is here set equal to zero if the reinforcement at $r = c_0$ yields.

4.8.2.3 Equations of equilibrium

From the equilibrium of forces and moments, the external load P is given by the following two equations:

$$P = [R_{sR} + 2\pi(R_{sT} - R_{cT})] \tan \alpha + D \quad (4.8-17)$$

and

$$P = \frac{(2\pi R_{sT} + R_{sR})(d - x) + D \left(c_0 - \frac{B}{2} - x \right) + \frac{[R_{sR} + 2\pi(R_{sT} - R_{cT})]x}{2\cos^2 \alpha} + 2\pi R_{cT} \frac{2x}{3}}{0.5(c - B) - x} \quad (4.8-18)$$

The load P can be determined by satisfying Eqs. (4.8-17) and (4.8-18) by iteration, using the angle α as the iteration variable. The ultimate load P_u is found when the tangential concrete strain in the extreme compression fibre, and at the distance y from the column face, reaches a certain ultimate value ε_{ctu} which states the failure criterion.

4.8.2.4 Failure criterion

In previous finite element analyses (Hallgren 1996, 1997), the concrete between the tip of the shear crack and the slab-column root was found to be in a three-axial state of compressive stress, see Fig. 4.8-5. At the distance of between one to two times the depth of the tangential compression zone, the concrete in the tangential compression zone is in a state of bi-axial compression in the horizontal plane and tensile strain in the vertical direction. Hence, in this area, the concrete is close to horizontal cracking. This is believed to initiate the punching shear failure. When this crack appears, the confinement giving the three-axial state of compressive stress at the slab-column root is lost and the shear crack can break through the radial compression zone, causing a sudden loss of the load capacity. This theory is the basis for the failure criterion used in the modified mechanical model.

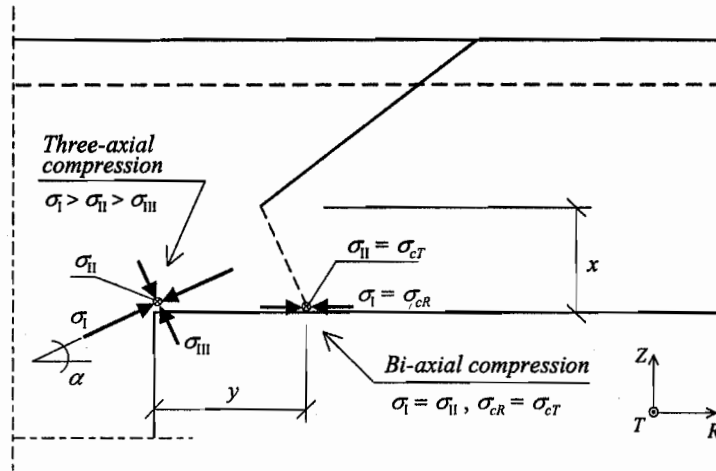


Fig. 4.8-5: States of stress

Based on the previous finite element analyses, the concrete in the compression zone at $r = B/2 + y$ is assumed to be in a bi-axial state of compression, where the tangential strain ϵ_{cT} is equal to the radial strain ϵ_{cR} . Kupfer et al. (1969) tested plain concrete specimens in bi-axial compression. They noted that the volume of the specimen started to increase at a stress level close to the ultimate stress. It is fair to conclude that macro-cracks started to develop at this level. Furthermore, in the case of equally large stresses in both directions, the compressive strains at this level were approximately equal to the transverse tensile strain.

For the proposed punching model, this implies that immediately before failure, the strain ϵ_{cTu} is equal to the vertical tensile strain. Based on this, and by adopting the Fictitious Crack Model by Hillerborg et al. (1976), the ultimate tangential strain is set equal to the average strain across the compression zone when the critical crack width w_c is reached:

$$\epsilon_{cTu} = \frac{w_c}{x} \quad (4.8-19)$$

Hence, the failure criterion depends on the depth of the tangential compression zone and on the brittleness of the concrete. The critical crack width is determined from the bi-linear tension-softening curve proposed by Petersson (1981), see Fig. 4.8-6, which gives:

$$w_c = 3.6 \frac{G_F}{f_{ct}} \quad (4.8-20)$$

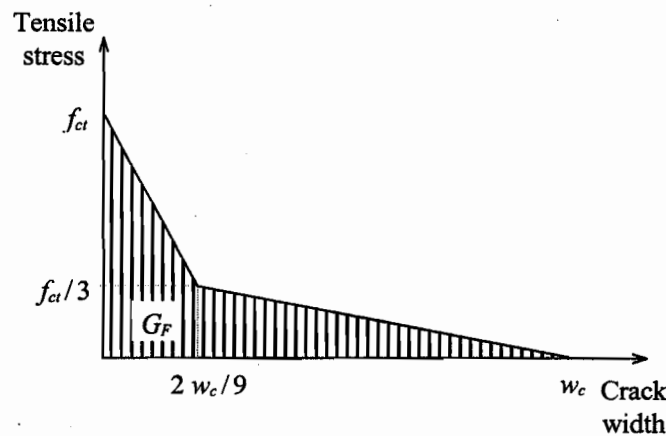


Fig. 4.8-6: Bi-linear tension-softening curve proposed by Petersson (1981)

Several previous tests and theories indicate that the fracture energy G_F determined from tests is size dependent, and increases with the structural size up to a level where it is constant for very large sizes. This is conveniently described with the Multifractal Scaling Law (MFSL) by Carpinteri and Chiaia (1995), see Fig. 4.8-7. By inserting Eq. (4.8-20) and the MFSL into Eq. (4.8-19), the failure criterion is expressed as:

$$\varepsilon_{cTu} = \frac{3.6 \cdot G_F^\infty}{x \cdot f_{ct}} \left(1 + \frac{13 \cdot d_a}{x} \right)^{-1/2} \quad (4.8-21)$$

where d_a is the maximum aggregate size and G_F^∞ is given by:

$$G_F^\infty = G_F^R \left(1 + \frac{13 \cdot d_a}{d^R} \right)^{1/2} \quad (4.8-22)$$

where G_F^R is the fracture energy determined from test, or estimated from empirical expressions, and d^R is the size of the test specimens on which the value of G_F^R is based.

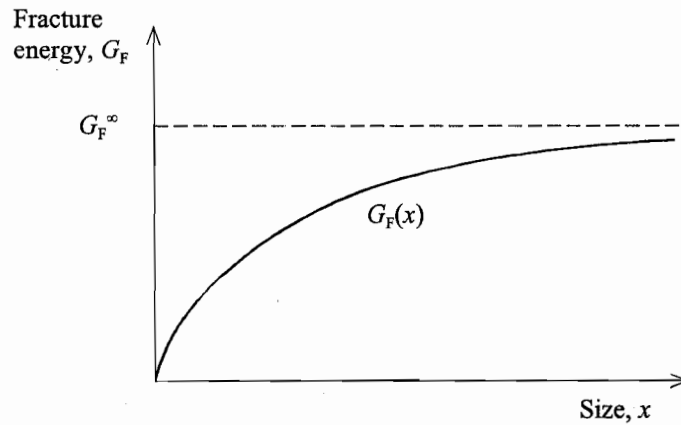


Fig. 4.8-7: Multifractal Scaling Law for fracture energy, according to Carpinteri and Chiaia (1995)

For a typical normal strength concrete (NSC), and for a typical high strength concrete (HSC), Fig. 4.8-8 shows how the ultimate tangential strain ε_{cTu} decreases with increasing depth of the tangential compression zone. Due to the higher brittleness of the HSC, the ultimate tangential strain ε_{cTu} at a given value of the depth x is smaller for HSC than for NSC. Hence, the proposed model of the failure criterion not only takes into account the size effect, but also reflects the brittleness of the considered concrete.

4.8.2.5 The algorithm

Equations (4.8-1) to (4.8-22) form an algorithm which is an iterative process. A flow-chart of the algorithm is shown in Fig. 4.8-9. The angle α is determined by iteration in an outer loop and the depth of the tangential compression zone x is determined by iteration in an inner loop.

The inner loop converges to a solution when the depth x is equal to x^k given from the previous iteration step and, thus, complies with the stress-strain states (elastic or plastic) of concrete and steel, and with the failure criterion ε_{cTu} of the previous iteration step.

The outer loop converges to a solution when the load P determined from the equation of vertical projection (Eq. 4.8-17) is equal to the load P determined from the equation of moment (Eq. 4.8-18). At convergence, this load is the ultimate load P_u according to the proposed modified mechanical model.

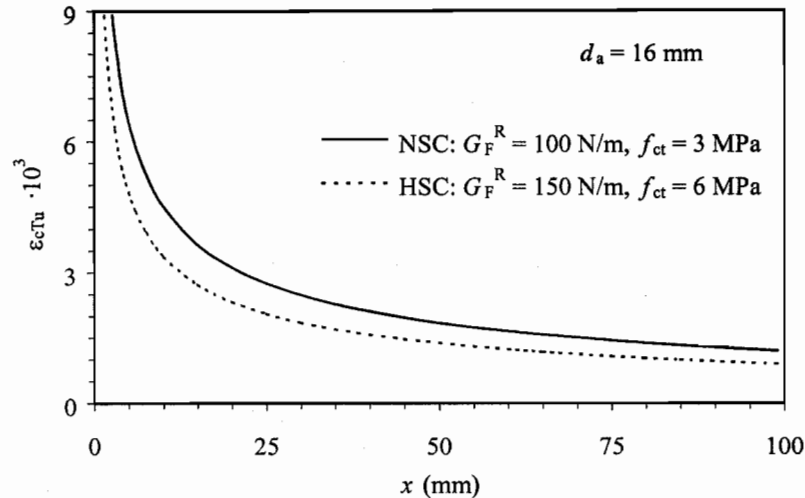


Fig. 4.8-8: Ultimate tangential concrete strain ϵ_{cTu} as function of the depth x of the tangential compression zone. Curves for a typical high strength concrete (HSC) and for a typical normal strength concrete (NSC).

4.8.3 Conclusions

Based on previous finite element analyses and on test observations, a modified mechanical model of punching of reinforced concrete slabs without shear reinforcement is proposed. The failure criterion, which is taken as the ultimate tangential concrete strain, is derived from a physical model based on non-linear fracture mechanics. The ultimate tangential concrete strain increases with increasing ductility of concrete and decreases with increasing depth of the tangential compression zone. Thus, the failure criterion reflects both the brittleness of concrete and the size effect. The upper load limit given by the flexural capacity of the slab is automatically included in the model as it is based in the equilibrium of internal forces from concrete in compression, reinforcing steel in tension (including yielding) and external forces from loads and support reactions.

The proposed model is in its present form limited to the analysis of symmetric punching of reinforced slabs without shear reinforcement. However, the model is open for further development, e.g. by introducing forces from shear reinforcement and forces from prestressing tendons.

Another limitation concerns the geometry. For thick slabs with short shear spans, e.g. footings, another type of punching failure is believed to occur. This type of failure is probably more similar to the shear failure of beams with short shear spans. To consider this in the proposed model, other failure criteria would have to be implemented. Further research on this issue is currently in progress.

Previous research by others, e.g. Braestrup (1989), has indicated that the punching shear strength is influenced by the ratio between the column width B and the effective depth d of the slab. This is not directly considered in the present proposal. Hence, further research is needed in order to include the effect of the B/d ratio in the mechanical model.

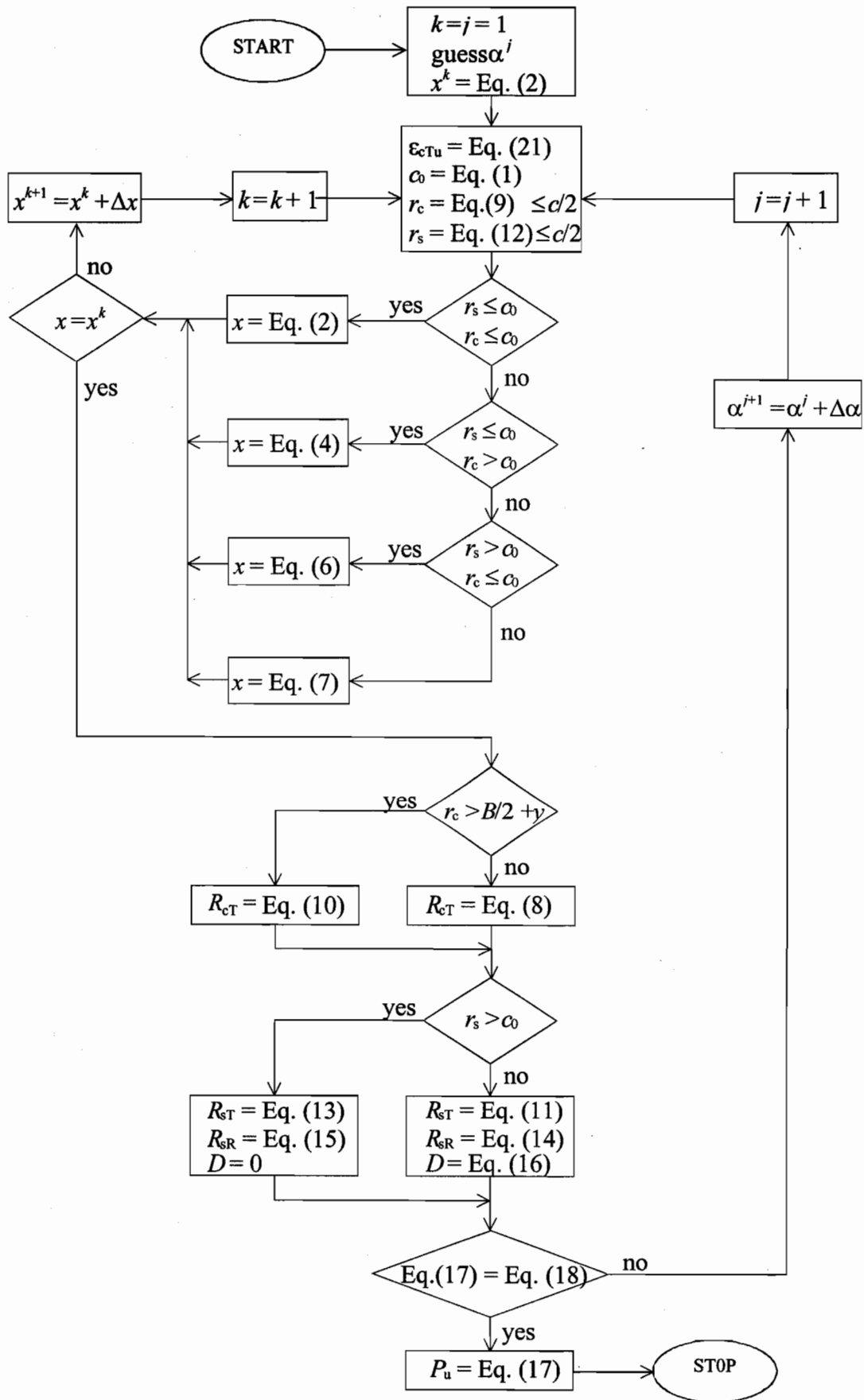


Fig. 4.8-9: Flow-chart of the algorithm of the proposed model

4.9 Empirical model by Staller

Markus Staller

The development of a new empirical formula for the determination of the punching shear capacity of reinforced concrete slabs is introduced in this section. By associating the new formula with the statistical procedure of Zilch/Staller/Brandes (2000), it is possible to derive an design equation, which is optimised under economical and theoretical safety perspectives.

4.9.1 Empirical Modelling

The empirical models development is based on evaluating experimental data with the help of linear or non-linear, single or multiple, regression analysis. Therefor the parameters known to have substantial influence on the load bearing capacity and deemed suitable for mathematical expression as functions are included in the consideration. The load bearing characteristic and failure mechanism must not necessarily be expressed in general terms. This is advantageous for such a complex combination of multiple mechanisms involved in punching and shear problems. The disadvantage of lack of general applicability – the determined formulae only apply to an existing parameter array – can be moderated because even mechanical models should not be used beyond verified applications.

The development of empirical formulae was used several times in the area of reinforced concrete members, i.e. beams and slabs without shear reinforcement. Thereby, the decisive factors are expressed as a product of independent partial functions. The multiple, non-linear problem can be reduced to a simple, linear, multiple regression by using suitable transformations, like the power function $y = \alpha x^\beta$ that has already used several times is suitable for this purpose.

The aim of a regression analysis is to determine the independent parameters of a mathematical relationship, in such a manner that the available number of experimental data are best approximated. In general, this adjustment is determined using the Gaussian "method of least squares". Thereby the choice of a suitable type of function, on which the regression analysis is to be based, is of crucial importance.

The following investigations are based on a calculated shear stress along a critical perimeter at a distance of $2d$ from the column face – as required in Model Code 90. The choice of shape and size for this critical perimeter must be done before the regression analysis in order to obtain a representative relative variable. As illustrated in the regression analysis introduced in the following section, the choice of this comparatively large perimeter leads to the smallest variation coefficients in comparison to the perimeters with a distance of $1.5d$ or $2.5d$.

4.9.2 Choice of suitable data by engineering criteria

For the purposes of the analysis, only those experimental results are used whose failure type can be traced back exclusively to a punching failure of the slabs. With regard to engineering criteria (e.g. $d \geq 80$ mm, $\rho_1 \leq 4$ % and $f_{ic} \geq 12$ MPa) the total number of experimental data is decreasing from 224 to 182 of 22 different test sites. The evaluation procedure and the compilation of the experimental data is described in detail by Staller (2000).

4.9.3 Selecting the parameters

From the observations made during experimental tests and mechanical models as well as analysis with the help of the finite element method, the following parameters, decisive material properties and geometrical dimensions can be emphasised as significant:

- **Geometry**
To the group of geometrical parameters belong the absolute dimensions of the slabs on the one hand (slab thickness and effective depth, total dimensions and span as well as the dimensions and shape of the supports), and relative variables, such as slenderness or the related support dimensions on the other hand.
- **Material properties**
Regarding the material properties of concrete and reinforcing steel, strength properties and material properties that influence the deformation of the construction materials are to be distinguished. To the first group, one has the compressive and tensile strength of concrete as well as the yielding point and tensile strength of reinforcing steel. Material properties, such as the modulus of elasticity and the fracture energy (or characteristic length) are decisive for the deformation properties in the elastic state or when the ultimate tensile strength is exceeded.
- **Longitudinal reinforcement**
The effect of longitudinal reinforcement depends in the first place upon the available geometrical or mechanical ratio of reinforcement, but also upon the diameter of bars and their arrangement. The existence of compressive reinforcement is of subordinate importance.

For the development of the mathematical function for the significant parameters, a clear mutual influence can be assumed. For this reason the selection of a multiplicative approach of the individual parameters seems to be adequate. From the above listed parameters partial functions are formed which can be compiled for the selected product formula. By means of extensive observations the significant parameters can be reduced, sharpened and finally defined:

- size effect (quotient of effective depth d and characteristic length l_{ch}): $\xi = F(d/l_{ch})$
- mechanical ratio of reinforcement: $\omega = \rho_1 \frac{f_y}{f_{lc}}$
- slenderness of the slab (symbols are explained in Chapter 6):
$$\lambda = \frac{1}{2} \frac{c_1 - c_2}{d} \text{ and } \lambda = \frac{1}{2} \frac{a_1 - a_2}{d}$$
- tensile strength of concrete: f_{ct}

The shear strength v , defined as the shear stress in a defined perimeter, results from the multiplication of a free constant C and the chosen partial functions:

$$v = C F(\xi) F(\omega) F(\lambda) F(f_{ct}) \quad (4.9-1)$$

In the following sections, individual parameters will be explained and quantified in detail.

4.9.3.1 Effective depth and size effect

One of the strongest influences on the punching load is the thickness of the slabs or the effective depth of the slabs, respectively, and the size effect. The size effect law $v \sim 1 + d^{-1/2}$ developed by Nielsen/Braestrup et al. (1978) for beams without shear reinforcement, resulted in the best approximation of the experimental data, when regression analysis was applied on the punching problem. Following a suggestion by Gustafsson/Hillerborg (1988) the effective depth is replaced by a dimensionless quotient of effective depth and characteristic length d/l_{ch} . This means that one of the most important fracture mechanics parameters for the description of ductility or brittleness of the concrete is included in the empirical approach. The partial function $F(\xi)$ for taking into account the size effect is:

$$F(\xi) = 1 + (d/l_{ch})^{-1/2} \quad \text{with: } l_{ch} = \frac{G_f E_c}{f_{1ct}^2} \quad (4.9-2)$$

The dependence of the punching shear capacity on the effective depth in relation to the characteristic length is demonstrated by Fig. 4.9-1. The close agreement with the applied, normalized experimental values is clearly visible in the whole range including higher d/l_{ch} -values, thus, as a tendency, in thicker slabs in connection with high strength concrete.

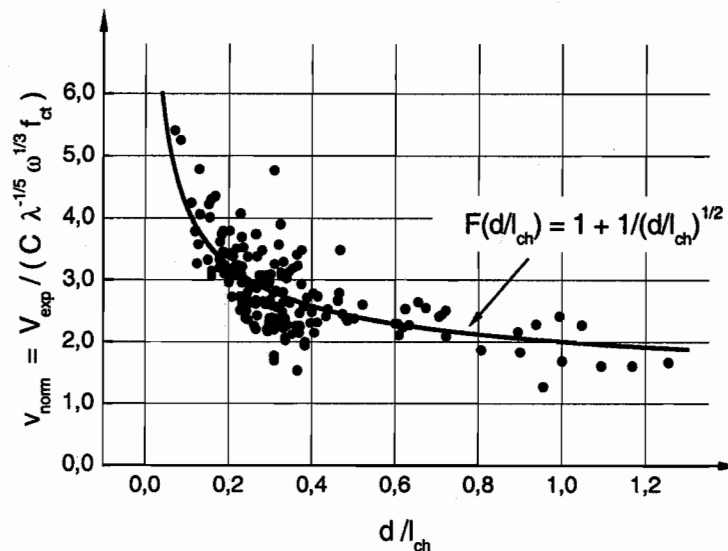


Fig. 4.9-1: Dependence of the punching shear capacity of normalized experimental data on the effective depth related to the characteristic length

4.9.3.2 Geometrical and mechanical ratio of reinforcement

The influence of different types of arrangement and bar diameters of the reinforcement is introduced in a simplified form via the ratio of reinforcement in the formula. Increasing the ratio of longitudinal reinforcement leads to an increased load bearing capacity like that of beams without shear reinforcement.

The mechanical ratio of reinforcement ω is introduced into the regression calculation through the geometrical ratio of reinforcement ρ_1 . The partial function according to eq. (4.9-3) with the value $C_1 = 0.3557$ is derived from the regression calculation and simplified for further examination:

$$F(\omega) = \omega^{\frac{1}{3}} \quad (4.9-3)$$

This functional relationship is depicted in Fig. 4.9-2, showing the dependence of the punching shear capacity along the ordinate, in form of normalized test values v_{norm} , versus the mechanical ratio of reinforcement ω .

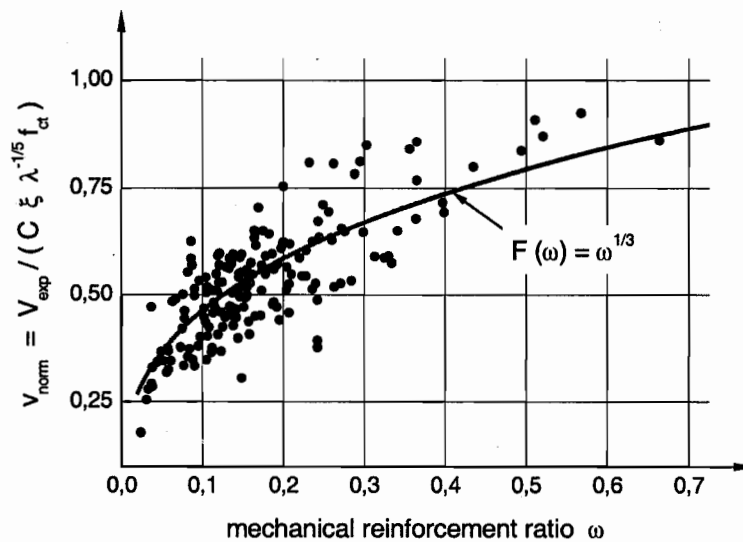


Fig. 4.9-2: Dependence of the punching shear capacity of normalized test values versus the mechanical ratio of reinforcement.

4.9.3.3 Slenderness

An investigation of geometrical variables indicated negligible influence on the punching shear capacity of the corresponding diameter c_2/d of the support. In contrast, a substantial improvement can be made by adding the slenderness to approximate the calculated values to the test values. The slenderness λ depicts the ratio of the span to the effective depth of the slab, similar to beams under transverse loads.

The regression analysis for the partial function resulted in an exponent $C_2 = -0.1879$. The influence of slenderness is simplified for the approximating function and introduced according to eq. 4.9-4:

$$F(\lambda) = \left(\frac{1}{\lambda} \right)^{\frac{1}{5}} \quad (4.9-4)$$

In Fig. 4.9-3 the normalized test values v_{norm} are depicted as a function of the slenderness. A satisfactory agreement is apparent in the entire range of values.

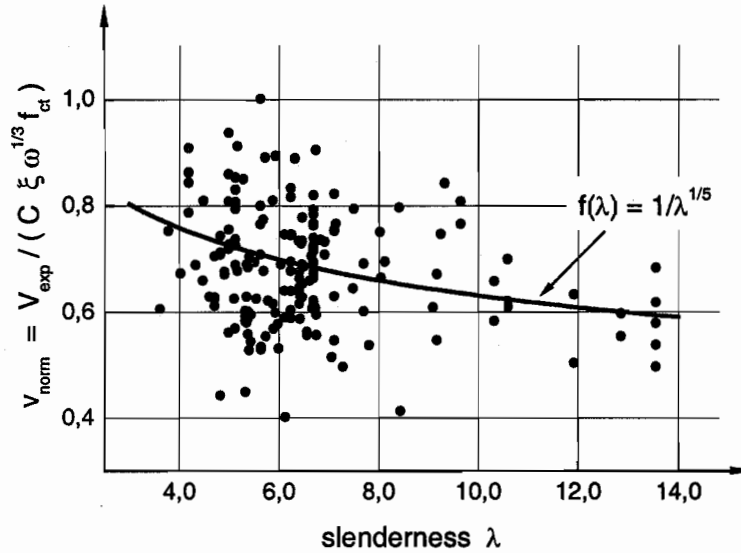


Fig. 4.9-3: Dependence of the punching shear capacity of normalized test values upon slenderness.

4.9.3.4 Material Properties of Concrete

The influence of concrete strength is the most important parameter for the punching shear capacity of reinforced concrete slabs. Code rules compiled in Section 2 are all based on the compressive strength of concrete (except DIN 1045 (7.88) and SIA 162). Theoretical investigations, however, show crucial dependence on the ultimate tensile strength [Men  trety (1994)] or on ductility (dependent upon E_c , G_f , f_{1ct}) [Hallgren (1996)]. Therefore the uniaxial tensile strength f_{1ct} of concrete is chosen for the partial function to account for concrete strength, besides the ductility indirectly introduced into the size-effect via the characteristic length. The result of regression calculation for $F(f_{1ct}) = f_{1ct}^{C_4}$ shows a value near one ($C_4 = 0.9843$) for the exponent C_4 . The simplified partial function is approximately:

$$F(f_{1ct}) = f_{1ct} \quad (4.9-5)$$

In case no measured tensile strength values of the experimental investigations are available, a calculated value according to section 6.4.3 that directly depends upon the compressive strength of concrete has to be determined for f_{1ct} .

4.9.4 Determination of punching shear capacity

After the derivation and verification of the individual partial functions, the final regression analysis, under the condition that the mean value of the relationship of v_{exp} / v_{cal} is one, leads to the prefactor $C = 0.5325 \approx 0.53$. The following equation then enables the mathematical determination of the average shear stress in failed condition along the critical perimeter at a distance of $2d$ (form acc. to Model Code 90). By multiplication with the critical perimeter u and the effective depth d the punching shear capacity is:

$$V_{cal} = C \xi \sqrt[3]{\omega} \lambda^{-\frac{1}{5}} f_{1ct} u d \quad \text{with: } \xi = 1 + (d/l_{ch})^{-\frac{1}{2}} \quad (4.9-6)$$

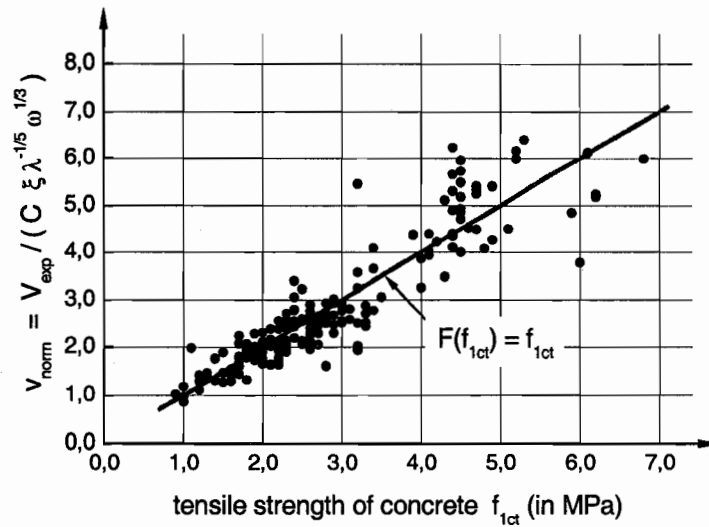


Fig. 4.9-4: Dependence of the punching shear capacity of normalized test values versus the tensile strength of concrete.

In Fig. 4.9-5 the experimental results are compared with the results of the calculations by the empirical approach. In this context one can clearly see that the range of the experiments, characterised by the distance of the quantile value to the experimental results, is negligible. This quantile value corresponds to the simple standard deviation and is exclusively for evaluating the results, without concerning any reliability background. The comparison of the calculated values with the experimental data leads to a variation coefficient $v = 0,17$. The presented empirical approach shown in Fig. 4.9-5 describes the behaviour very realistically and with less scattering in comparison with mechanical models [Staller (2000)].

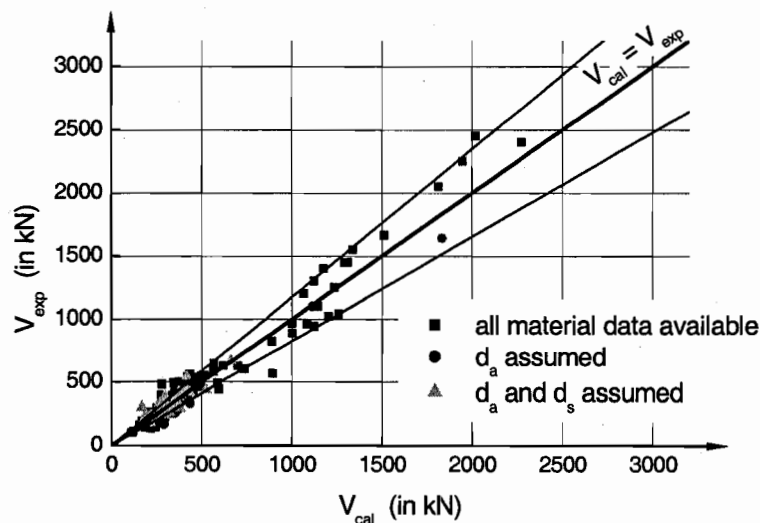


Fig. 4.9-5: The comparison of the calculation results of punching shear capacity and the experimental results which meet engineering criteria.

The empirical approach developed here represents a very practical way for the prediction of the ultimate load of punching shear failure. The parameters presented are significant for determining the punching shear capacity and the selection is thus confirmed.

4.10 Discussion of mechanical models

Karl - Heinz Reineck

Summarising the preceding description of models and theories in the following a brief comparison is made between the proposed models. The intention is to outline the main assumptions and indicate the limits of application of the different models.

The general criteria any model must be measured against first are of course:

- equilibrium.
- assumed constitutive laws, material strengths and failure criteria; thereby especially the role of the concrete tensile strength is examined.
- consideration of compatibility.

First it is obvious from this list, that empirical methods cannot be included in such a comparison of models against these criteria, of course, and this reminds on their limitations especially in the vital aspect of equilibrium. Only the strength criterion for the total load or shear force is fulfilled within the range of experimental evidence. However, even this is not guaranteed because of inevitable simplifications of empirical relations for code purposes and due to lack of any or much evidence, like for example it was and still is the case for members with large depths. The equilibrium and the relation to the loads and the section effects is only kept through the global analysis, but this is normally carried out for linear elastic material behaviour and not for the real behaviour corresponding to the punching failure.

The above remarks certainly are not meant to discriminate empirical methods, which have played a decisive role in the development of structural concrete and still are indispensable in codes for certain problems, like especially for the punching design. Most models also contain empirical aspects or parameters in one or the other way, e.g. by assuming the geometry of the failure surface or by using simplified constitutive laws or simple failure criteria. Generally, it has also to be conceded that mechanical models require higher efforts for the analysis and for the definition of the different parameters involved so that simpler empirical methods are often preferred in design practice.

The above criteria have been covered to some extent by the description of the different models in the previous sections. However, in many cases the load transfer could not fully be recognised and thus the assessment was difficult how the criterion equilibrium was fulfilled. Therefore, with respect to the first two criteria equilibrium and failure criterion, in the following the shear transfer actions are especially looked at, which to some extent explain the load transfer and especially describe to what extent the concrete tensile strength is considered or even decisive. As stated already in section 3.1, the shear transfer actions of one-way members without transverse reinforcement are also valid for symmetrical punching, because no load is transferred in the ring direction, which however is highly stressed. The following Table 4.10-1 shows a comparison of some of the previously discussed models with respect to the shear transfer actions utilised in each model. The selected models are meant to be representative for different other proposed models, like e.g. the model by Hallgren is the last proposal for a modified Kinnunen/Nylander model and also represents the main features of the models proposed by Broms or Shehata.

Shear transfer action		Kinnunen / Nylander (1960) (sect. 3.2)	Moe (1960) (sect. 3.2)	Braestrup / Nielsen et al. (1976) (sect. 3.2)	Georgopoulos (1990) (sect. 4.3)	Alexander / Simmonds (1992) (sect. 4.6)	Menetrey (1994) (sect. 4.7)	Hallgren (1996) (sect. 4.8)
in tension zone	cantilever action of concrete teeth	No	No	No	No	Yes (beam shear)	No	No
	friction stresses along cracks	No	No	No	No	Yes (beam shear)	No	No
	dowel action of tension chord	No	No	No	No	Yes (beam shear)	Yes (V_{dow})	Yes
	residual tensile stresses across cracks	No	No	No	Yes	No	Yes; (V_{ct})	No
in compression zone	shear stresses in compression zone	see below	see below	?	see below	see below	No	see below
	inclined compression chord	inclined strut	inclined curved strut	?	inclined strut	Arch loaded by cross-beams	No	inclined strut
Vp of prestressing tendon		No	No	No	No	No	Yes	No
shear reinforcement		No	No	No	No	No	Yes	No
failure criterion		- tangential ϵ_{cu} - f_{sy} long. steel	flexural failure	Mod. Coulomb: - $f_{cu} = 0.835 f_c$ - $f_{ct} = f_c / 400$ or $f_{ct} = 0$	tangential ϵ_{cu} or radial σ_c	shear V_c acc. to ACI 318 of cross-beams	- f_{ct} - f_{yw}	tangential ϵ_{cu} based on fracture mech.

Table 4.10-1: Comparison of representative models with respect to the shear transfer actions utilised in each model

The comparison in Table 4.10-1 reveals that the different shear transfer actions in the tension zone employed in one-way shear were not modelled in the classical models for punching and only partly considered in the new models, and these are the actions directly related to the concrete tensile strength. In several research works the Kinnunen/Nylander approach was further developed and modified, where the failure is mainly related to the highly stressed compression zone and the load is obviously transferred to the column by an inclined strut. However, increasingly in the last decade models based on fracture mechanics have been proposed, and this means that researchers acknowledge concrete tension by accepting small residual tensile stresses across cracks, like by Menétrey or by Georgopoulos. Likewise, in nonlinear Finite Element Methods the concrete tensile strength is modelled (see Chapter 5), respectively more precisely the concrete tensile properties, i.e. strength and fracture energy.

Therefore it can be concluded that there is still no agreement on the role of the concrete tensile strength, and this may be visualised by the two models shown in Fig. 4.10-1 a and Fig. 4.10-1 b. However, there is surely an increasing awareness for the role of the concrete tensile strength in modelling the punching failure. It is noteworthy that the new developments since the CEB Bulletin 168 was published in 1985 include the concrete tensile strength respectively concrete tensile properties. This is a step forward to unifying the models for punching failures and shear failures in one-way members, because for the latter concrete tension plays a decisive role in most models for slender members by means of the shear transfer actions friction along cracks or residual tensile stresses across cracks.

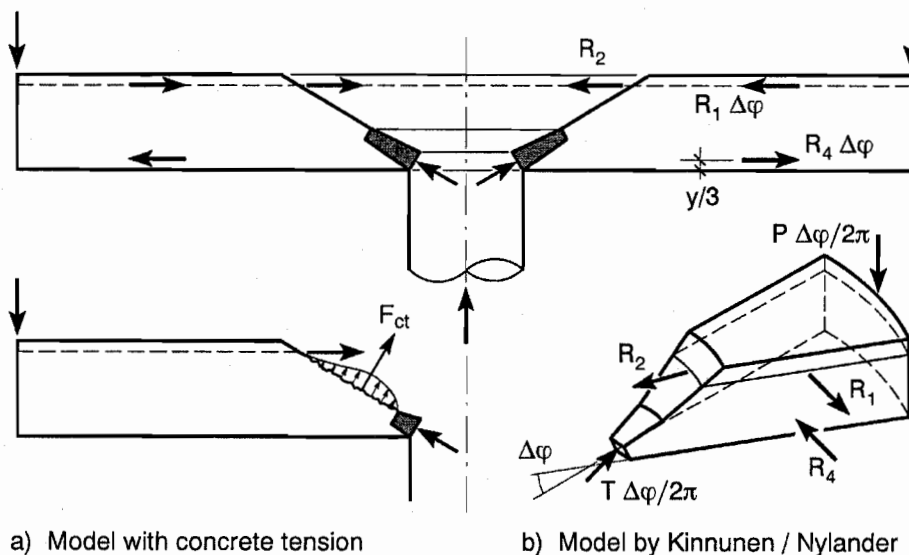


Fig. 4.10 - 1: The two basic models under discussion for punching

The failure criteria also show a wide scatter of assumptions and frequently are the field for further improvements of an existing model, like that of Kinnunen/Nylander. It should be noted that not all failure criteria are related to the shear transfer actions, which is the case when the ring direction is assumed to govern the punching failure.

It must be pointed out that the shear transfer actions depend very much on the crack pattern and failure surface. For punching mostly a very flat inclined shear crack is either calculated or assumed, i.e. the location and inclination of which is orientated by the failure cone observed after the punching failure. Such a flat crack almost opens perpendicular to the crack surface so that friction cannot play any role in lack of a slip of the crack surfaces. Conse-

quently, in many models like that of Kinnunen/Nylander no shear transfer actions are applied along the crack surface, and in some only the dowel action, like by Hallgren (see sect. 4.8). In these models like in most others for punching the steeper flexural shear cracks, i.e. the ring cracks, are mostly ignored, and this is in contradiction to many models proposed for one-way slender members, where the steeper cracks were looked at. These steeper cracks develop from flexural cracks at early load stages and certainly influence the state of stress near the crack tips and the appearance of the flat shear crack at failure.

However, there are some models where almost the same flat crack shapes are assumed, but the load is transferred by means of tensile stresses across this flat crack, based on fracture mechanics as stated above. This corresponds to using a truss or strut-and-tie model with concrete ties, if these approaches were consequently followed up with respect to the load transfer in the whole D-region and not only in the crack surface.

This aspect of a design model is important, and that is to define and describe the load transfer in the whole member, so that the flow of forces can be followed up from the point of load application until the support. This has often been ignored, because in case of the mostly used failure mechanism approach only the forces and shear transfer actions in the predefined or calculated crack surface were analysed, but rarely are the forces followed up in the adjacent regions. Such a consequent follow-up of all forces is the main advantage of truss models or strut-and-tie models, and when doing this very often concrete ties have to be applied "to close gaps in the load path" or to achieve a complete load transfer, and this is demonstrated by the following Figures 4.10-2 and 4.10-3.

In Fig. 4.10-2 the model proposed by Kinnunen/Nylander is supplemented by following-up the forces assumed along the failure crack surface into the adjacent column region and into the "rigid body" towards the applied loads. A simple strut-and-tie model with a direct strut between the load point and the support at the column is not possible (see Fig. 4.10-2 a), because the assumed inclined strut carrying all the applied loads into to the support (see Fig. 3.2-1) is too steep and does not directly point towards the points of load application. Therefore, a truss is required to close the gap, and in lack of stirrups in this region far away from the column the truss must contain concrete ties in order to keep equilibrium (see Fig. 4.10-2 b).

Therefore, even in this model by Kinnunen/Nylander concrete tensile stresses are required although it is regarded as a classical model only utilizing concrete in compression, but this only applies to the failure region itself. The modified model by Hallgren requires a more refined strut-and-tie model, because additional tensile forces within the failure region have to be included for the dowel action of the longitudinal reinforcement. Of course it is clear that these are simple models where the ring forces are only indicated, but the main transfer of the vertical load is visualized and the utilisation of concrete tension is made clear.

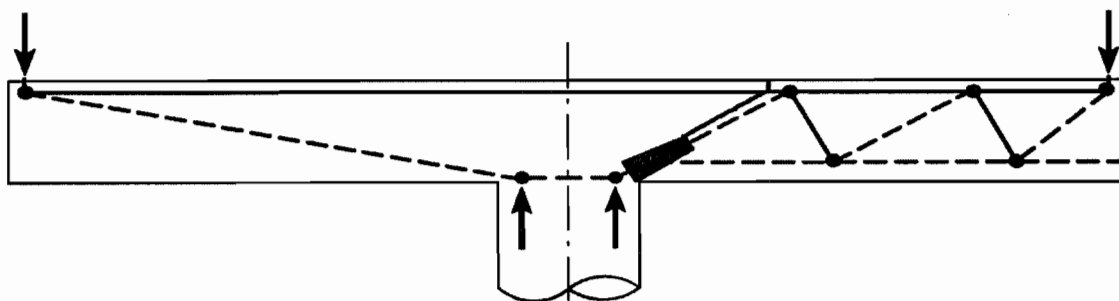
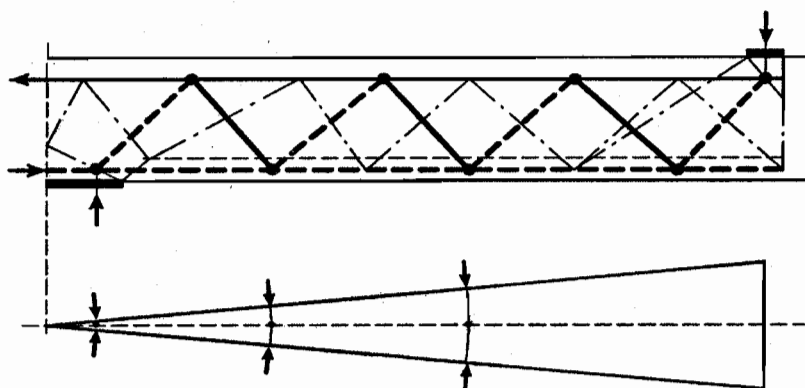


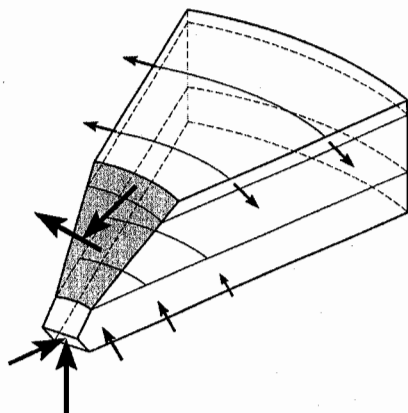
Fig. 4.10-2: The model by Kinnunen/Nylander (see Fig. 3.2-1) supplemented by two different strut-and-tie models to show the complete load transfer

Likewise can a strut-and-tie model be shown for the models where residual concrete tensile stresses are assumed to transfer the loads across the inclined crack, and this model is simply continuing the truss with concrete ties up to the D-region at the support (Fig. 4.10-2 a). The angle of the inclined struts respectively that of the concrete ties is about 45° , which is steeper than in one-way members without transverse reinforcement, where the struts are inclined at 30° according to Reineck (1990, 1991). The reason is that the slab is uncracked outside the D-region where punching occurs and then a truss with struts inclined at 45° represents the flow of forces, as already proposed earlier by Pralong (1982) and Andrä (1982). This Fig. 4.10-2 b, like the Fig. 3.2-1 by Kinnunen/Nylander, demonstrates that the slab is modelled as a sandwich consisting of three panels, i.e. bottom and top chord and the web, as already proposed by Baumann (1972) or Marti (1990). The load is transferred in the web to the column, and the chords carry the moments in the radial as well as in the tangential or ring direction and they are additionally stressed by the tangential forces from the web due to the shear force.

This presentation of strut-and-tie models was addressed here finally to demonstrate the need for describing the complete load transfer for a better understanding of the phenomenon punching by researchers as well as by designers. It hopefully encourages future researchers not only to present their refined theories and models but also use such simpler models.



a) section through radial sector with strut-and-tie model with concrete ties



b) spatial view on failure surface and sector with forces

Fig. 4.10-3: The models with tensile stresses across cracks (see Fig. 4.3-1 by Georgopoulos or Fig. 4.7-1 by Menétrey) supplemented by a strut-and-tie model to show the complete load transfer

5 Numerical Investigations

Finite element simulations have become an important tool to analyse and to predict the behaviour of reinforced concrete structures. The detailed treatment of the theory and of the latest developments of the finite element method is subject of the *fib* Task Group 4.6 “Modelling of reinforced concrete structures using finite elements”. However, the results of current numerical simulations might be helpful for the understanding of the punching phenomenon. Therefore, in the present chapter 5 of this bulletin an overview of the application of the finite element method for the simulation of punching failure will be given.

5.1 Simulation of punching using FEM

5.1.1 Introduction

In previous years several numerical investigations were carried out applying the finite element method. Among these, studies with two-dimensionally modelled systems using rotationally symmetric elements by Loseth/Slatto/Syvertsen (1982), Andrä (1982), van Foeken (1983), de Borst/Nauta (1985), González-Vidosa et al. (1988) and Menétrey (1994) as well as Hallgren (1996) must be distinguished from the three-dimensional systems by Dyngeland et al. (1994), Ozbolt et al. (1999) and Staller (2000). A separate position is taken by those investigations applying degenerated shell elements which are placed in layers by Marzouk/Chen (1993) and Polak (1998).

Before some of these investigations will be presented in detail, the dominant techniques which are necessary to apply the finite element method to quasi-brittle and strain-softening materials such as concrete will be outlined. For this purpose, the characteristics of the finite elements types as well as the basic concepts for the modelling of the material (micro and macro models) and for the modelling of the fracture (smeared and discrete crack approach) will be explained.

5.1.2 Element types

In general, three element types are used for the simulation of the punching failure:

- rotational symmetric (2-D-) continuum elements (applied in chapters 5.4 and 5.6);
- spatial (3-D-) continuum elements (applied in chapters 5.2, 5.3 and 5.7).
- shell elements (applied in chapter 5.5);

The advantage of 2-D-elements is that rotational symmetric problems can be simulated with a small number of elements and therefore a minimum numerical effort. However, slabs with orthogonal reinforcement which are generally found in practical applications as well as punching with moment transfer or punching at edge and corner columns can not be modelled using 2-D-elements.

In contrast, 3-D-elements offer high flexibility and accuracy in the modelling of reinforced concrete structures and generally lead to the most realistic results. On the other hand, using 3-D-elements the pre- and post-processing of the FE-mesh becomes more difficult and the numerical effort is high. As a result, the application of 3-D-elements is up to now limited to smaller structures.

Shell elements were applied for the simulation of punching to allow for the application of

the FEM to larger structures because of the relatively small number of degrees of freedom per node with these elements. However, shell elements require a transformation of the strains perpendicular to the plane of the elements and, therefore, lead to less accurate results compared to a continuum analysis.

5.1.3 Modelling of the material

In the last decade a large number of constitutive laws to describe the deformation behaviour as well as the damage and cracking of concrete has been developed. The existing material models can be classified into two groups: (1) micro models and (2) macro models. In micro models the structure of the material is modelled while the material behaviour on the micro level and the interaction between the micro components is described by means of relatively simple laws. The macroscopic material behaviour results automatically from the behaviour of the micro components and from their interaction. The fundamental disadvantage of these micro models is the enormous numerical effort of the analysis. Therefore, realistic structures generally have to be modelled using macro models. Macro models are based on continuum mechanics and have to correctly reproduce relatively complex mechanisms on the micro level, e. g. cracking, cohesion, friction or interaction between microcracks. In principle, the damaged zones (cracks) might be modelled using a smeared crack approach or a discrete crack approach. Generally, the constitutive relation is formulated in terms of the macroscopic continuum stress and strain tensors and their invariants. For concrete, different material models for macroscopic analyses have been developed, e. g. models based on the theory of plasticity, plasticity-damage-models, continuum damage mechanics, endocronic theory or microplane models. A detailed discussion of material models for concrete will be given in the *fib* bulletin "Modelling of reinforced concrete structures" which is currently in preparation by the *fib* Task Group 4.6.

5.1.4 Smeared crack approach

The widely used smeared crack approach is applied in all investigations presented in the subsequent chapters 5.2 to 5.7.

5.1.4.1 General

In a smeared crack model, cracks are modelled by changing the constitutive (stress-strain) relations of the elements in the vicinity of the crack. The advantages of the smeared approach are its convenience for computation and the fact that microcracks are often neither straight nor discrete but tortuous and distributed. However, because of the strain-softening nature of concrete the results of a smeared analysis may depend significantly of the mesh size [Bažant and Cedolin (1979), Ožbolt (1995)] The zones of damage tend to localise in a limited area (volume in case of 3-d simulations) whose size depends on the dimensions of the finite elements. Fig. 5.1-1 demonstrates that in a coarse mesh the damaged area (volume) and the energy consumption due to cracking are larger than in a fine mesh, i. e. the energy consumption depends on the element size. Theoretically, a reduction of the element size to zero means that the energy consumption due to cracking tends to zero, too. Therefore, an appropriate "localisation limiter" must be introduced which guarantees a localisation of the damage in a mesh-independent area (volume).

Currently there are two mayor concepts to control the localisation of a smeared analysis.

First, the relatively simple crack band approach by Bažant and Oh (1983) and second, the methods of higher degree, e. g. the Cosserat-continuum by Eringen (1965), the nonlocal integral approach [Pijaudier-Cabot and Bažant (1987), Ožbolt and Bažant (1996)] and the nonlocal gradient approach by de Borst (1991).

5.1.4.2 Crack band approach

The fundamental assumption of the crack band approach by Bažant and Oh (1983) is that the damage will localise in one line or band of finite elements. In order to keep the dissipation of energy due to cracking constant and equal to the fracture energy of concrete G_F , the constitutive strain-softening law of the material has to be adapted in such a way that:

$$G_F = A_f \cdot h = \text{constant}$$

where: A_f = area under the constitutive stress-strain curve in tension (fig. 5.1-1)
 h = element size and width of the crack band, respectively

The same condition is applied for the strain-softening in compression assuming that the fracture energy in uniaxial compression G_C is approximately 100 times larger than the fracture energy in tension G_F , i. e. $G_C = 100 \cdot G_F$. As a result, the constitutive law is adapted to the element size for strain softening both in tension and in compression.

Even though with the crack band method the energy consumption of the concrete is modelled correctly, the calculations still may depend on the size and the orientation of the elements [Ožbolt (1999)].

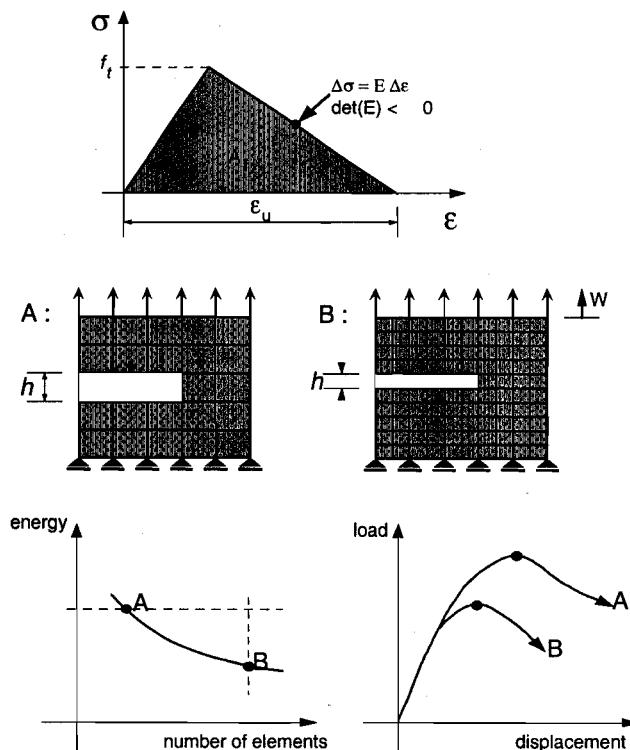


Fig. 5.1-1: Sensitivity of a smeared analysis on the element size

5.1.4.3 Nonlocal continuum

In order to avoid a mesh sensitivity and a localisation of the fracture process in quasi-brittle materials the concept of nonlocal continuum has been developed as a general approach. The basic idea of the nonlocal approach is to take into account the stress and deformation interactions over a distance in the continuum. That is, the strain at a point depends not only on the strain at the same point but on the entire strain field in a certain neighbourhood of the point. As a result, a localisation of damage can be avoided.

An effective form of the nonlocal concept where all variables affected by strain-softening are nonlocal while the other variables are local has been proposed by Pijaudier-Cabot and Bažant (1987) as well as Bažant and Ožbolt (1990). The fundamental parameter of this concept is the so-called characteristic length l_{ch} over which the strains are averaged.

In the beginning, l_{ch} was supposed to be a material property of the concrete that correlates with the maximum aggregate size d_a ($l_{ch} \approx 3 \cdot d_a$). However, experience has shown that l_{ch} also depends on other parameters and that the optimum values of l_{ch}/d_a for the application in numerical simulations vary for different problems (e. g. tensile tests and shear tests). Moreover, theoretical investigations by Bažant (1991) have proven that l_{ch} is not a material property but depends on the stress and strain field in the vicinity of the crack process zone. Therefore, a new nonlocal concept has been developed that is based on the interaction of the microcracks by Ožbolt and Bažant (1996).

Theoretically, a nonlocal analysis is more powerful than the relatively simple crack band method and leads to mesh-independent solutions. In practice, the layout of the FE-mesh still may influence the results of a nonlocal analysis if the mesh is too coarse in zones with strong damage or discontinuities. Finally, the numerical effort of the method is very high.

5.1.5 Discrete crack approach

The discrete crack approach is generally based on the fictitious crack model by Hillerborg (1976) which describes the tension softening fracture process zone through a fictitious crack ahead of the pre-existing crack whose faces are acted upon by certain closing stresses such that there is no stress concentration at the tip of the extended crack. The fictitious crack model assumes that the fracture process zone is of negligible thickness hence the cracks are modelled in a discrete fashion.

In a finite element analysis of a concrete structure, the basic assumptions of the fictitious crack model, namely that the material behaves elastically until the attainment of the tensile strength f_{ct} and that the fracture energy G_F is expended in a process zone of negligible thickness, are realised as follows [Karihaloo (1994)]. When the normal tensile stress in an element attains the level f_{ct} at a particular external load, the corresponding interelement boundary nodes are released and the element is subjected to the assumed tension softening stress-displacement distribution $\sigma(w)$. This procedure is repeated with every load increment until failure occurs. The insertion of $\sigma(w)$ at the released ("fractured") nodes requires a complete modification of the element topology in the vicinity of the "fractured" element. This procedure is called remeshing and is particularly difficult if the crack propagation direction is not known a priori [Karihaloo (1994)].

5.2 Three-dimensional Numerical Analysis of Punching Failure

Joško Ožbolt, Henrik Vocke

5.2.1 Introduction

At the Institute of Construction Materials of the University of Stuttgart a finite-element code which is based on the microplane material model has been developed. In the present chapter a brief overview of the program is given. Subsequently, the results of a numerical simulation of a punching test are presented and compared with the test results. Finally, a parametric study is carried out in which the influence of material and geometrical properties on the punching behaviour is studied. The results are discussed and compared with design code recommendations.

5.2.2 Finite-Element-Code

The used FE-Code is based on the “microplane” material model [Taylor (1938), Bažant and Prat (1988), Ožbolt and Bažant (1992), Ožbolt, Li and Kožar (2000)] and a smeared crack approach. To avoid a spurious mesh sensitivity either the improved crack band approach (stress relaxation method) or the nonlocal integral approach are applied as “localization limiters” (see chapter 5.1).

The spatial discretisation of the quasibrittle material (concrete) is performed by eight node solid finite elements. The reinforcement is modeled by discrete bar elements or smeared within the concrete elements. Moreover, particular contact elements (friction or spring elements) are available, which transfer load in one or both directions, respectively. The analysis is carried out incrementally, i. e. the load or displacement is applied in several steps.

In the microplane model the material properties are characterised separately on planes of various orientations within the material. On these microplanes there are only a few uniaxial stress and strain components and no tensorial invariance requirements need to be considered. The tensorial invariance restrictions are satisfied automatically since the microplanes to some extent directly simulate the response on various weak planes in the material (interparticle contact planes, interfaces, planes of microcracks, etc.). The constitutive properties are entirely characterised by relations between the stress and strain components on each microplane in both, normal and shear directions (fig. 5.2-1).

It is assumed that the strain components on the microplanes are projections of the macroscopic strain tensor (kinematic constraint approach). Knowing the stress-strain relationship of all microplane components, the macroscopic stiffness and the stress tensor are calculated from the actual strains on the microplanes by integrating the stress components on the microplanes over all directions. This integration is carried out numerically based on the method of virtual work.

The simplicity of the model is due to the fact that only uniaxial stress-strain relationships are required for each microplane component and the macroscopic response is obtained automatically by integration over the microplanes. The model used in the present studies has been verified for various applications by Ožbolt, Mayer, Vocke and Eligehausen (1999). It has been demonstrated that the model is able to realistically predict the behaviour of concrete for any stress-strain history.

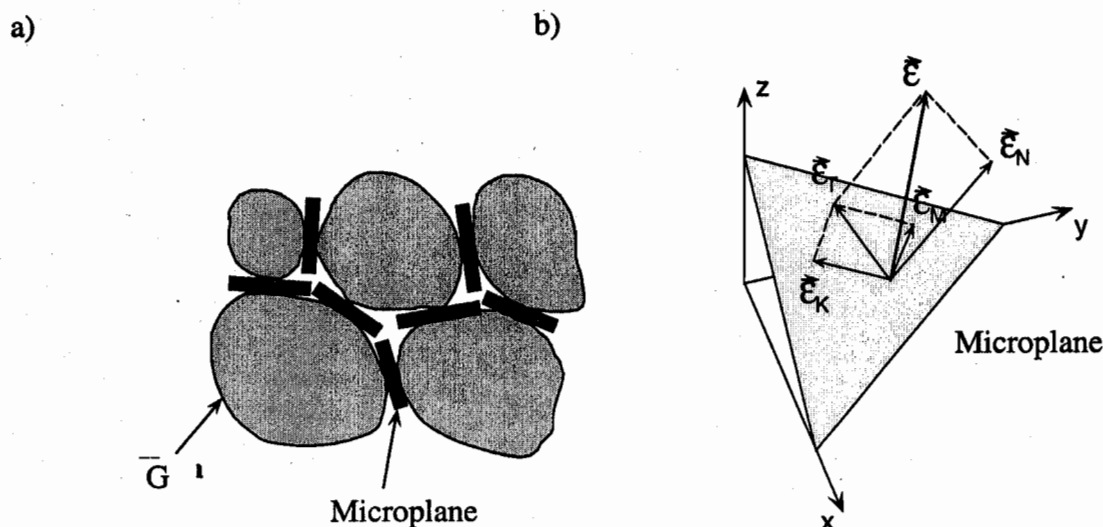


Figure 5.2-1: The concept of the microplane model: a) interparticle contact planes (microplanes) in the material and b) strain components

5.2.3 Verification of the FE-code

In order to verify that the employed FE-code MASA is able to realistically predict the punching failure of reinforced concrete flat slabs, numerous punching tests from the literature have been simulated by Ožbolt and Vocke (1999). In the following the calculation of a punching test from Hegger and Beutel (1998) is presented and the calculated results are compared with the test results.

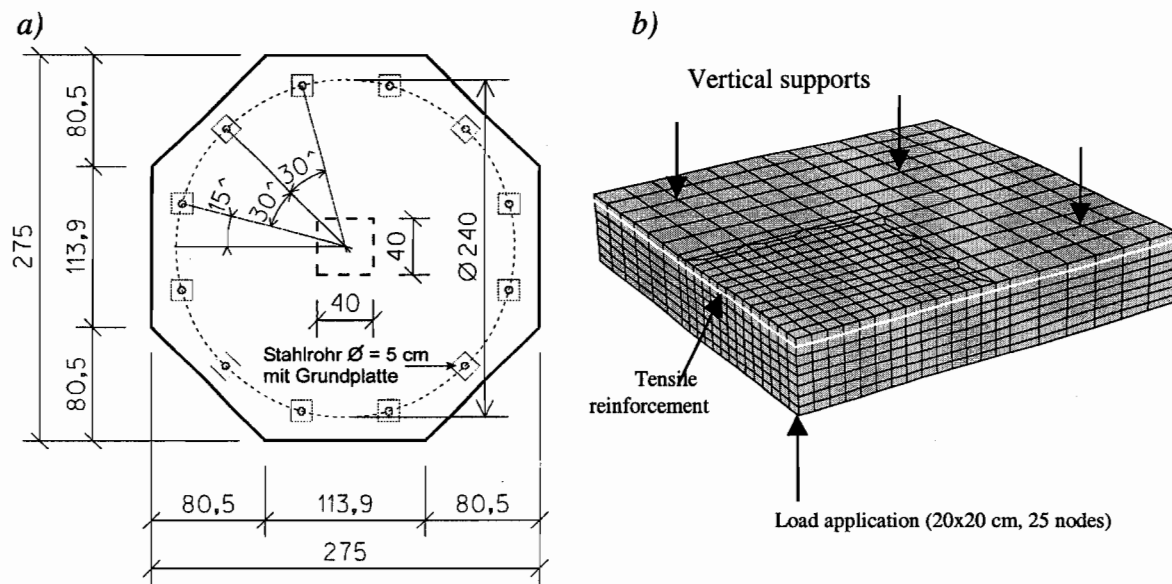
5.2.3.1 Geometry, spatial discretisation and material properties

The tested geometry of the slab is shown in fig. 5.2-2a. The hexagonal specimen was vertically supported by 12 tie rods and loaded in the column with a hydraulic jack. After reaching the service load the slab was unloaded and reloaded twice. The material properties and the arrangement of the reinforcement are summarised in Table 5.2-1. The reinforcement ratio was chosen as $\rho_1 = 0,8 \%$. The slab was provided with no shear reinforcement.

The finite-element mesh used in the simulation is shown in Fig. 5.2-2b. For reasons of symmetry only a quarter of the test slab was modelled. In the region of punching the model was optimised by local mesh refinement. The spatial discretisation of concrete was performed by 8-node solid elements. The reinforcing steel was modelled by 2-node bar elements, which were connected with the concrete elements via common nodes. The steel was represented by uniaxial ideally elasto-plastic stress-strain relationship. The load was applied by controlling the vertical displacement of the nodes at the bottom of the column area. To account for the confining effect of the column, these nodes were fixed in the horizontal directions. The support points were taken the same as in the experiment. To prevent local damage, the finite elements around the supporting nodes were defined as linear-elastic.

Concrete				Tensile Reinforcement			Compr. Reinforcement		
f_{cm}	f_{ctm}	E	G_F	f_{ym}	Number	ρ_1	f_{ym}	Number	ρ_1
[MPa]	[MPa]	[MPa]	[N/mm]	[MPa]	[-]	[%]	[MPa]	[-]	[%]
21,0	1,73	21697	$\approx 0,10$	569	Ø14/10	0,8	569	Ø10/17,5	0,24

Table 5.2-1: Material properties of the test slab



Slab thickness $h = 230$ mm, effective depth $d = 195$ mm

Total: 3141 nodes, 3160 elements

Figure 5.2-2: Punching of a flat slab: a) dimensions of test specimen and b) FE-mesh

5.2.3.2 Numerical results and comparison with test results

The same as in the experiment, the failure obtained in the analysis is due to punching shear with the formation of the characteristic punching cone. Fig. 5.2-3a shows the crack pattern in the post-peak regime obtained in the analysis. The cracks are plotted in terms of maximal principal strains (dark zones in fig. 5.2-3a). The comparison with the corresponding test result (see fig. 5.2-3b) shows that the numerical simulation predicts the same shape of the punching cone as observed in the experiment.

The calculated and the experimentally measured load-deflection curves are plotted in fig. 5.2-4. The load corresponds to the vertical column force and the deflection is monitored on the column loading surface. As can be seen the agreement between the numerical and measured data is reasonably good, the peak load and the corresponding displacement are very well estimated. However, the analysis exhibits a more ductile behaviour in the post-peak regime. This is probably due to numerical reasons (convergence difficulties in the post-peak regime) and to the fact that the experiment was carried out load-controlled.

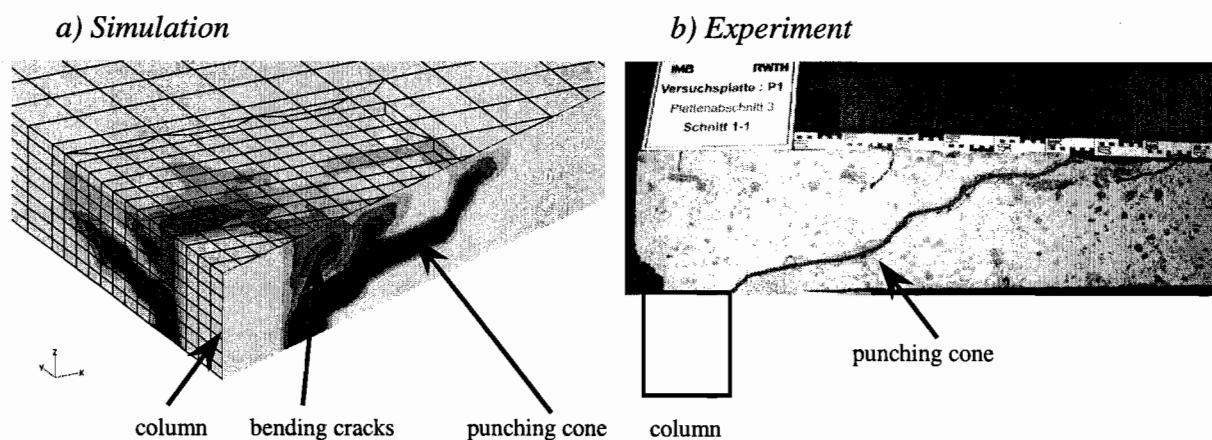


Figure 5.2-3: Crack pattern in the post-peak regime: a) simulation in terms of maximum principal strains (dark zones) and b) experiment

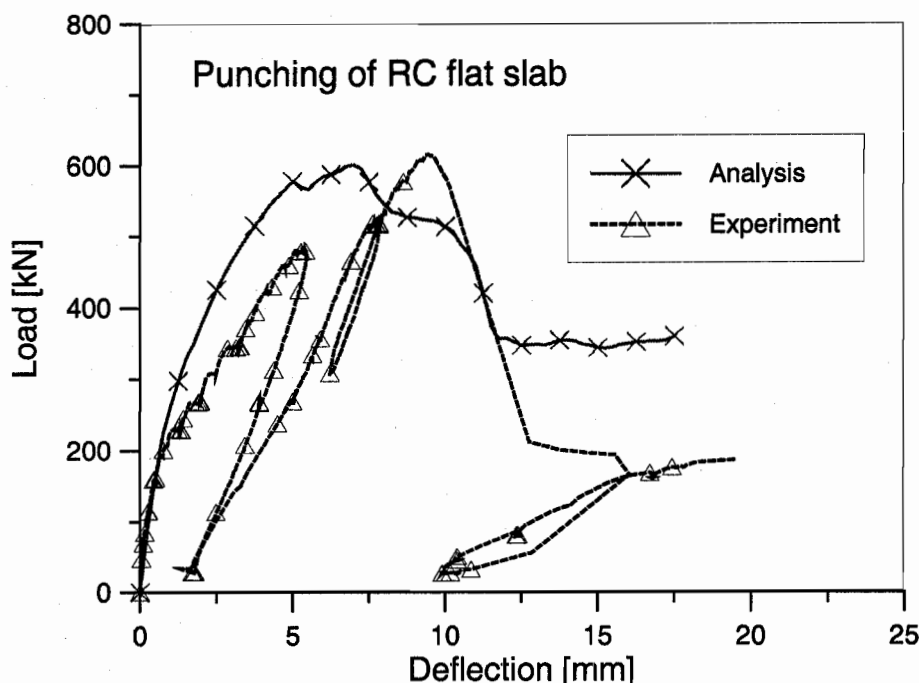


Figure 5.2-4: Calculated and measured load-deflection curves

In fig. 5.2-5a the calculated and measured vertical deflections of the slab tensile surface are plotted for different load levels. Furthermore, fig. 5.2-5b shows the calculated and measured strains in the tensile reinforcement. The tangential strains and the radial strains in the concrete compression zone around the column for different load levels are shown in fig. 5.2-5c and fig. 5.2-5d, respectively. Fig. 5.2-5a shows that the analysis correctly predicts the distribution of vertical displacements, which is characterised by a pronounced rotation at the column face and small rotations ahead. While at peak load the calculation and the experiment give approximately the same maximum deflection, at service load the calculated values are slightly smaller. An improved prediction of the deflections at service load could be achieved by using a finer FE-mesh which would allow a more realistic modelling of the cracking in the tensile zone.

A very good agreement between measured and predicted strains in the tensile reinforcement is demonstrated in fig. 5.2-5b. It can be seen that in particular at peak load the predicted distribution agrees well with the experimental data. As mentioned above, the relatively low calculated strains at the level of the service load correspond to the small calculated deflections at this level (fig. 5.2-5a). The tangential and radial strains in the concrete compression zone are shown in figs. 5c and 5d. To avoid a distortion of the calculated results due to an excessive localisation of the strains in the first concrete elements around the column, the strains at peak load ($P_U = 615$ kN, $r = 235$ mm) are derived from the strains in the compressive reinforcement. These steel elements are connected with the nodes of the concrete elements and their strains represent an average strain of the concrete compression zone. From figs. 5.2-5c and 5.2-5d can be seen that the simulation and the experiment exhibit good agreement. Compared to the test results, the calculated radial concrete strains are slightly underestimated while the strains in the tangential direction are slightly larger than in the test.

From the presented comparison between calculated and test results can be concluded that the used finite element code is able to realistically predict the load bearing behaviour, the deformations and the failure mechanism of flat slab-column connections.

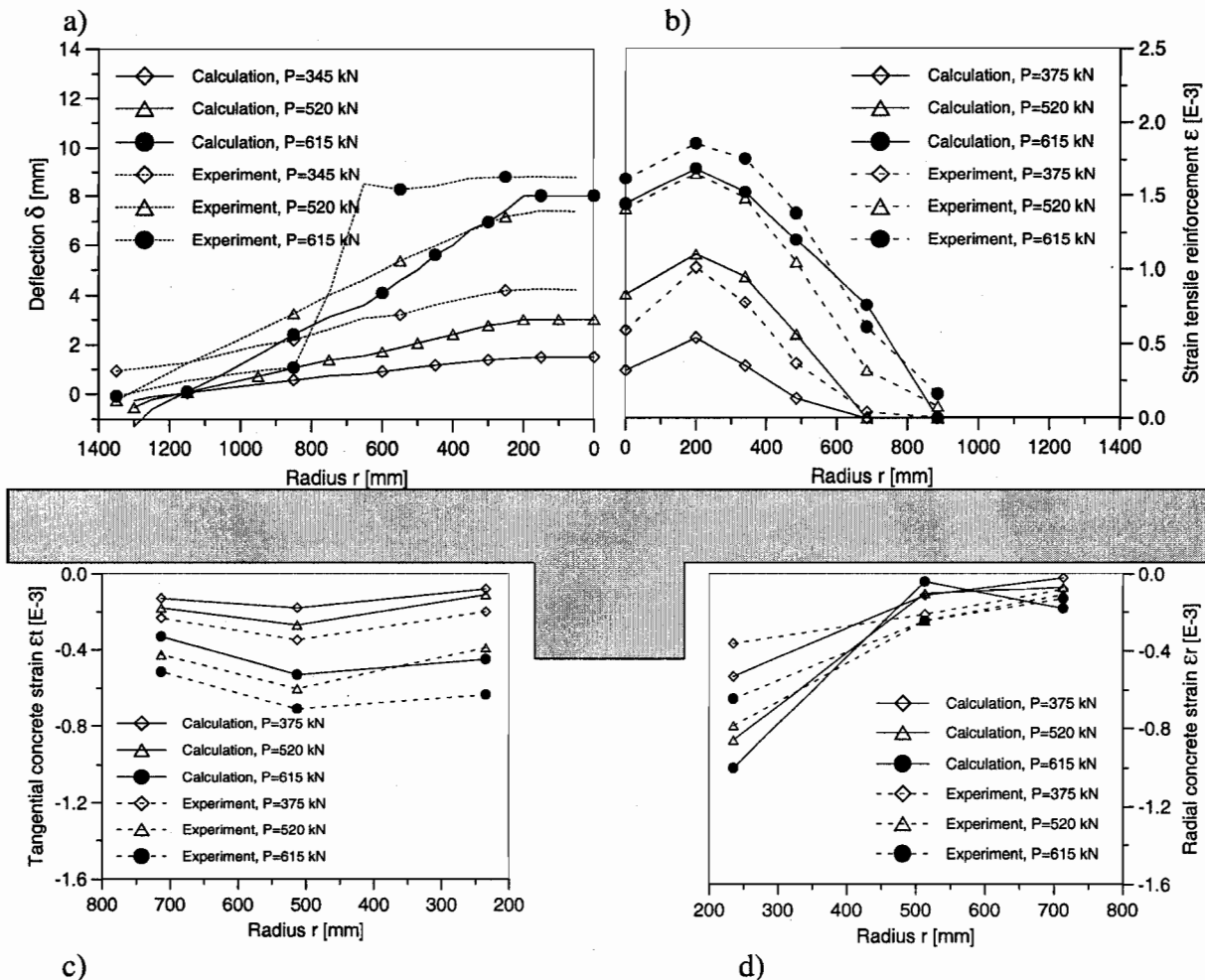


Figure 5.2-5: Comparison of the deformations in simulation and experiment: a) deflections, b) strains in the tensile reinforcement, c) tangential strains and d) radial strains in the concrete compression zone

5.2.4 Parametric study

In the following section the influence of relevant material and geometrical parameters on the punching behaviour of the test slab (fig. 5.2-1) is studied using the finite element code verified above. In general, the calculations are carried out by varying one parameter while all other parameters, which were the same as in the experiment (see section 5.2.3), were kept constant.

5.2.4.1 Influence of material parameters and reinforcement

The influence of the uniaxial cylinder compressive strength f_c , the concrete tensile strength f_t , the fracture energy G_F and the reinforcement ratio ρ_l on the punching capacity is summarised in fig. 5.2-6. For each varied property, three calculations were carried out. The results shown in fig. 5.2-6 are normalised to the peak load obtained for the middle value of the three varied parameters. The figure shows that the fracture energy G_F and the reinforcement ratio ρ_l have a similar and relatively strong influence on the maximum load. It is shown that the punching resistance of the slab increases approximately as the cube root function of G_F and ρ_l . High values of G_F and ρ_l lead to a more stable growth of the cracks in the tensile cord. As a result, the height of the compression zone increases what leads to the higher punching load and more brittle failure.

In contrary, for smaller values of G_F and ρ_l the strong damage of the tensile cord induces a smaller height of the compression zone and consequently the punching capacity is lower.

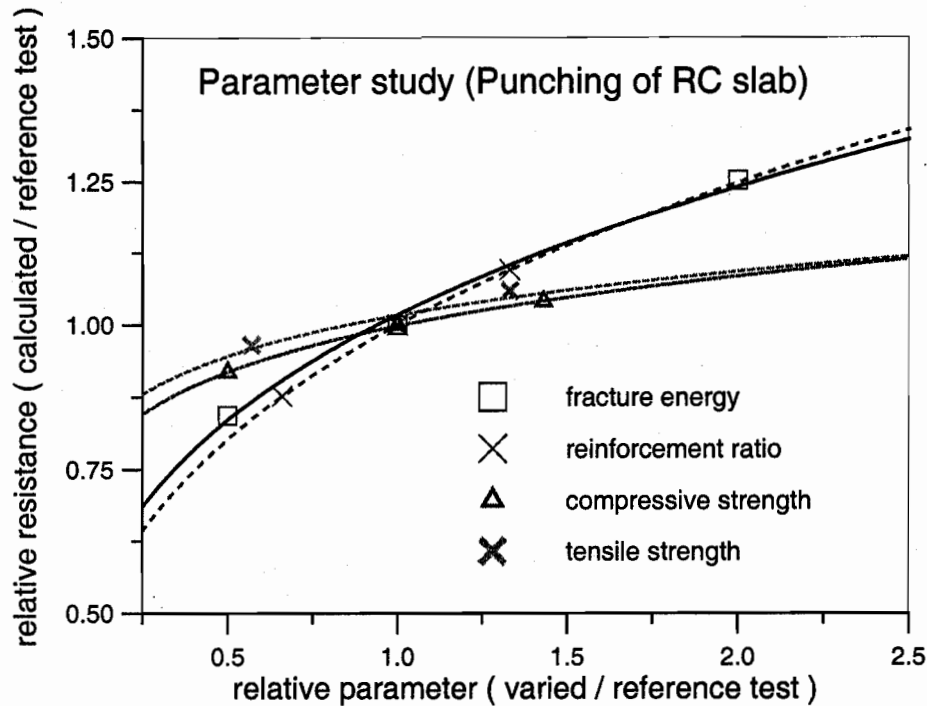


Figure 5.2-6: Influence of material properties and reinforcement ratio on the punching load

Contrary to the influence of the fracture energy G_F and the reinforcement ratio ρ_l , the numerical results indicate a relatively small effect of the compressive and tensile concrete strength on the punching capacity (fig. 5.2-6). A high tensile strength f_t does not lead to a more stable propagation of shear cracks in the concrete which is of major importance for the punching response. Depending on the fracture energy G_F , the crack propagation at peak load might even become less stable if the tensile strength increases. Therefore, the positive effect of the tensile strength remains relatively small while the brittleness of the structural response tends to increase with higher tensile strengths. Provided that the reinforcement ratio is sufficient to avoid a flexural failure (yielding of reinforcement), the compressive strength f_c does not lead to significant increase of the peak load (fig. 5.2-6). This is due to the fact that the shear capacity of the compression zone grows with the compressive strength, however, it does not contribute to more stable growth of the cracks of the tensile zone.

5.2.4.2 Influence of the slab size

The phenomenon that the nominal strength σ_N decreases with an increase of the structural size is known as the size effect [Bažant and Cao (1987)]. To investigate the influence of the size of the slab and the dimensions of the column on the punching load capacity, a parametric study was carried out. In addition to the reference slab (fig. 5.2-2), two slab thickness $h = 115$ and 460 mm and five column dimensions $c = 10, 20, 30, 50$ and 60 mm were analysed. By geometrical scaling, the span of the slab and the amount of reinforcement were related to the thickness of the slab. For $h = 115$ and 460 mm additional finite element meshes were generated using approximately the same element sizes as in original mesh (see fig. 5.2-2). In all cases the concrete properties and the reinforcement ratio were kept constant and the same as specified in table 5.2-1.

h [mm]	d [mm]	column size c (square cross-section) [mm]					
		100	200	300	400	500	600
115	90	110 kN	150 kN	183 kN	212 kN	-	-
230	205	-	475 kN	568 kN	652 kN	688 kN	728 kN
460	435	-	-	-	1788 kN	-	2018 kN

Table 5.2-2 Calculated punching loads for different slab thickness and column dimensions

The calculated peak loads for all varied geometrical parameters are summarised in Table 5.2-2. These results can be evaluated from the different point of view. Under size effect one generally considers the change of the nominal strength with an proportional increase of all structural dimensions (thickness of the slab, column dimensions, span of the slab and amount of reinforcement). However, from the practical point of view it would also be important to know how the variation of a single geometrical parameter influences the nominal strength.

For the proportional scaling of the slab and column geometry, in fig. 5.2-7 the nominal strength $\sigma_N = P_U/d(4c + \pi \cdot 3d)$ is plotted as a function of the effective thickness of the slab. The nominal strength is scaled with a value of σ_N for $d = 205$ mm. The calculated results are fitted by the Bazant's size effect formula:

$$\sigma_N = B \cdot f_t \cdot (1 - d/d_0)^{-1/2} \quad (5.2-1)$$

where B and d_0 (characteristic size) are material and geometry dependent constants. They were obtained from a regression analysis of the calculated data ($B = 0.46$; $d_0 = 320$ mm).

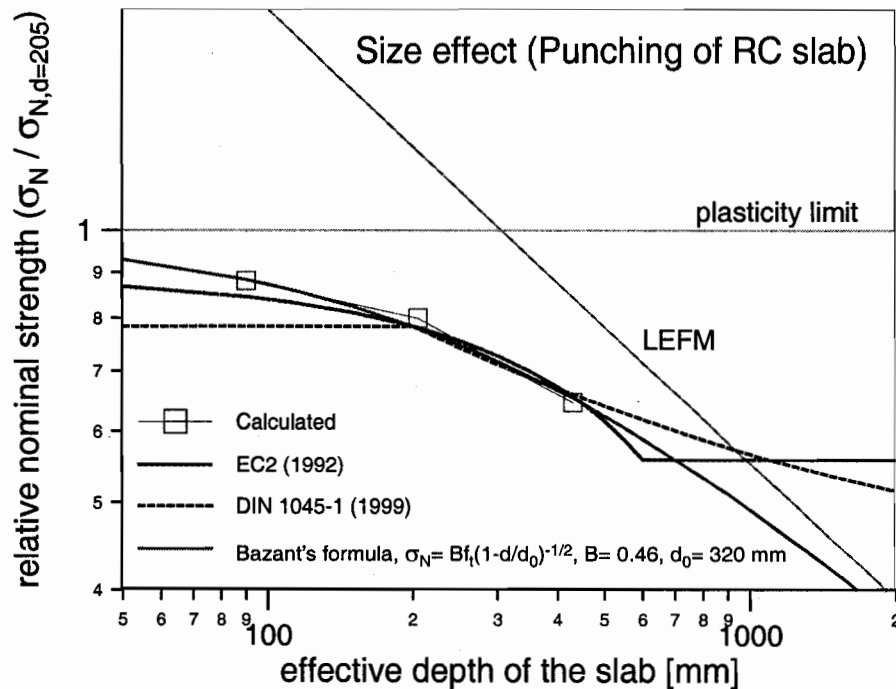


Figure 5.2-7: Size effect on the nominal strength - proportional scaling of the slab and column geometry

In fig. 5.2-7 the size effect curves according to EC2 and according to DIN 1045-1 are plotted as well. For the investigated size range the calculated data exhibits a relatively strong size effect on the nominal strength. The data are well fitted by the Bažant's type of the size effect formula. The relatively strong size effect could be expected since the parametric study (see section 5.2.4.1) indicated a strong sensitivity of the nominal strength on the variation of the fracture energy of concrete. The comparison with both design formulas, which are based on experimental results, shows relatively good agreement with the calculated data. For smaller sizes ($d < 200$ mm) both EC2 and DIN 1045-1 are conservative. Extrapolation of the numerical results according to (1) up to $d = 1000$ mm indicates that both design codes overestimate the nominal strength for very thick slabs. To confirm or disconfirm this, numerical studies for larger slabs will be carried out.

5.2.5 CONCLUSIONS

Based on the results presented in the paper the following conclusions can be drawn: (1) The three-dimensional finite-element code used in the present study is based on the microplane material model for concrete and a smeared crack approach. The comparison between experimental results and results of the numerical simulation demonstrate that the code is able to realistically predict the load-bearing and deformation behaviour as well as the failure mode of slab-column connections; (2) The numerical analysis shows that for the investigated slab geometry the fracture energy and the reinforcement ratio give a dominant influence on the punching capacity which increases approximately proportionally to the cube root of these parameters. In contrary, the influence of the concrete compressive and tensile strength is relatively small. The structural response becomes more brittle when the values of the varied parameters increase; (3) The results of the parametric study confirm that the maximum load and the fracture mechanism of reinforced concrete structures which exhibit complex cracking (i.e. no fracture upon first formation of a crack) do not primarily depend on the tensile strength of the concrete but on the entire tensile behaviour which is characterised by the interaction of tensile strength and fracture energy; (4) Finally, it is demonstrated that increasing the effective depth d leads to a decrease of the nominal shear capacity of the slabs. For the investigated size range the numerical prediction shows good agreement with the design code recommendations.

5.3 3D Numerical Punching Analysis of Shear Reinforced Flat Slabs - Variation of Quantity and Arrangement of Stirrups

Rüdiger Beutel

5.3.1 Purpose of the performed numerical investigation

Shear reinforced flat slabs can fail due to punching in the following modes: a) within the shear reinforced area, b) at the column face and c) outside of the shear reinforced zone (Fig. 5.3-1). In order to investigate the parameters of these different failure modes non-linear finite-element simulations of punching tests have been carried out.

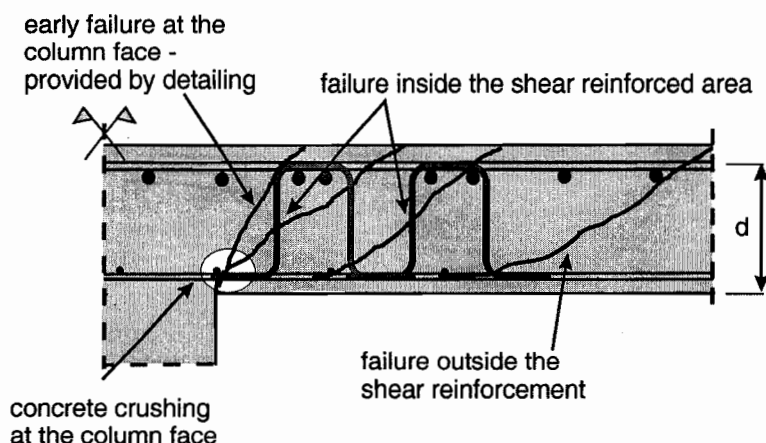


Figure 5.3-1: Failure modes due to punching of shear reinforced flat slabs.

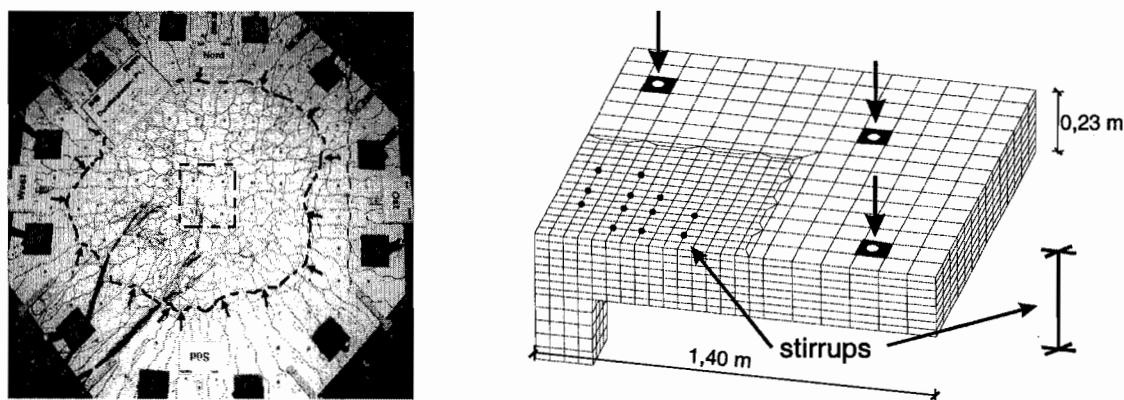


Figure 5.3-2: A typical test specimen and the finite-element mesh.

The tests and the numerical simulations represent the full scale part of a flat plate structure extending from an interior column to the line of contra flexure for a concentric loading of the column (Fig. 5.3-2).

The model is a quarter of a slab-sector made of 8-node brick-elements considering the symmetric conditions. The flexural reinforcement was simulated with truss bar elements. In order to anchor the stirrups in a realistic way several geometric forms of anchor systems were simulated with bar elements (Fig. 5.3-3).

Anchorage types (a) to (c) result either to stirrups carrying no load or to an early concrete shear failure. The early shear failure was caused by an unrealistic concrete damage in the region of the anchorage system. Only type (d) described the anchorage behaviour in a realistic way. Anchorage system type (d) is characterised by flexural restraint truss bar elements (Fig. 5.3-3d).

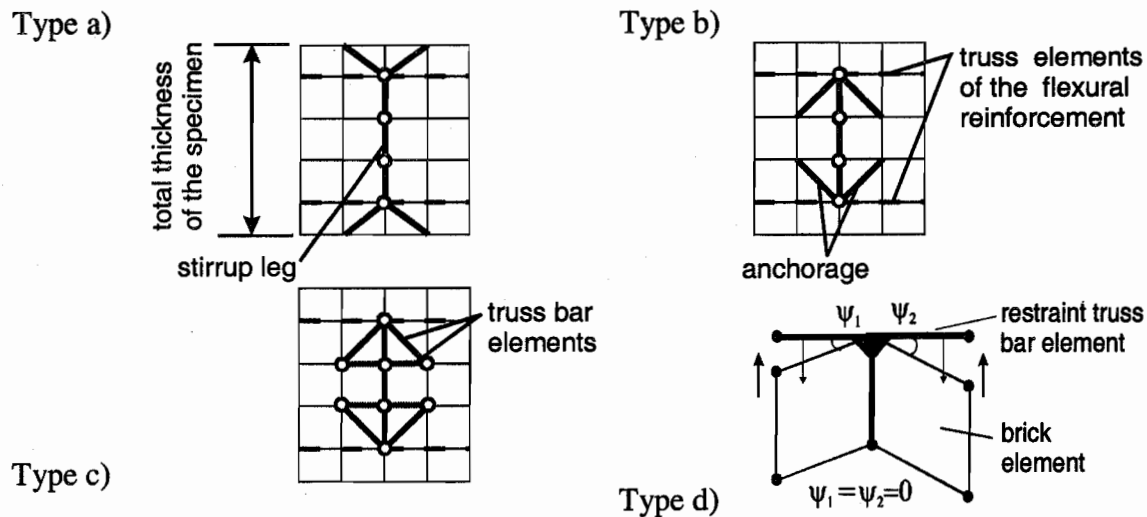


Figure 5.3-3: Different numerical stirrup anchorage types with bar elements. Type a), b), c) made of truss bar elements, type d) made of restraint truss bar elements.

5.3.2 Comparison between tests and simulations by using FEM

In the past all punching simulations only worked in an axisymmetric way with non-discrete stirrups. Therefore it was not possible to simulate realistic stirrup arrangements. For the first time MASA gives the opportunity to simulate shear reinforced flat slabs in a 3D structure with discrete stirrups (Fig. 2, right). The following punching failure modes of shear reinforced flat slabs could be simulated:

- 1) Failure within the shear reinforced area (test specimen P2-II).
- 2) Concrete crushing at the column face (test specimen P6-I).
- 3) Failure outside of the shear reinforced area.

All simulations of tests are based on the measured material properties. Only the fracture energy was assumed regarding Model Code 90 as 0,1 N/mm.

5.3.3 Failure of stirrups – Comparison with test P2-II

Fig. 5.3-4 shows the load-deflection curve of the test in comparison with the simulation. At the serviceability load level the simulation responds with a higher stiffness than the test. The calculated failure occurs at 90% of the test failure load. Taking into account that the concrete behaviour and the stirrup forces of the simulations correspond very well to the test results (Fig. 5.3-4 and Fig. 5.3-5), the difference between the maximum loads can only be explained by the fact that dowel action of the longitudinal reinforcement was not simulated. Therefore, the additional shear force due to dowel action has to be considered.

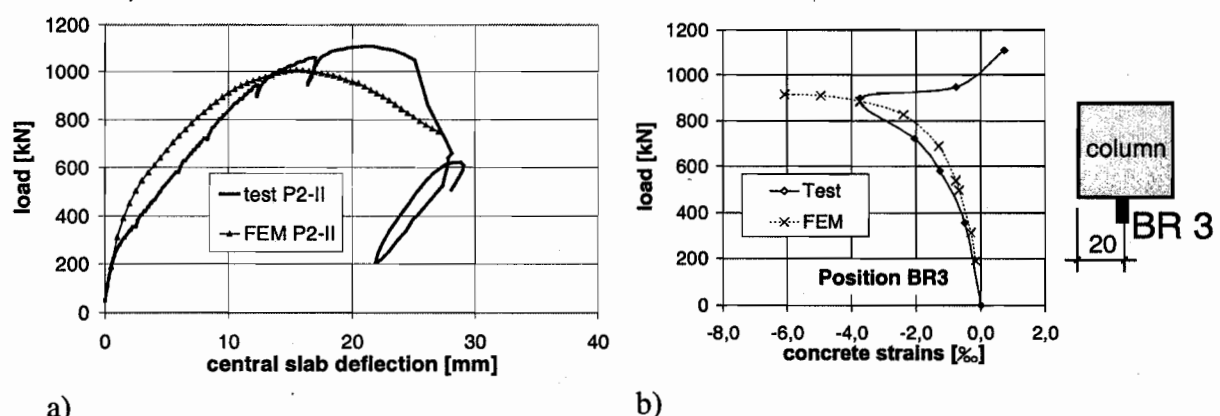


Figure 5.3-4: Load-deflection curve and concrete strains in the compression zone at the column face.

Looking at the descending branch of the load-deflection curve it becomes clear that the FEM-simulation acts semi-ductile. This behaviour of the simulation was not surprising as the failure was localised in the area of the stirrups. In contrast to this the test failed more brittle at a higher load level in a combination of modes (1) and (3).

The short introduction of the anchorage problem induces special attention to the stirrup behaviour. Figure. 5.3-5 shows the excellent correspondence of the measured and the calculated stirrup stresses in each row. In addition the sum of the stirrup forces fits well with the measured one in terms of dimension and development. It has to be pointed out that now judgements about contradictory stirrup arrangements of different design codes (e.g. ACI, EC2) are possible. Parametric studies in order to optimise the arrangement of the stirrups are presented in Chapter 5.3.4.

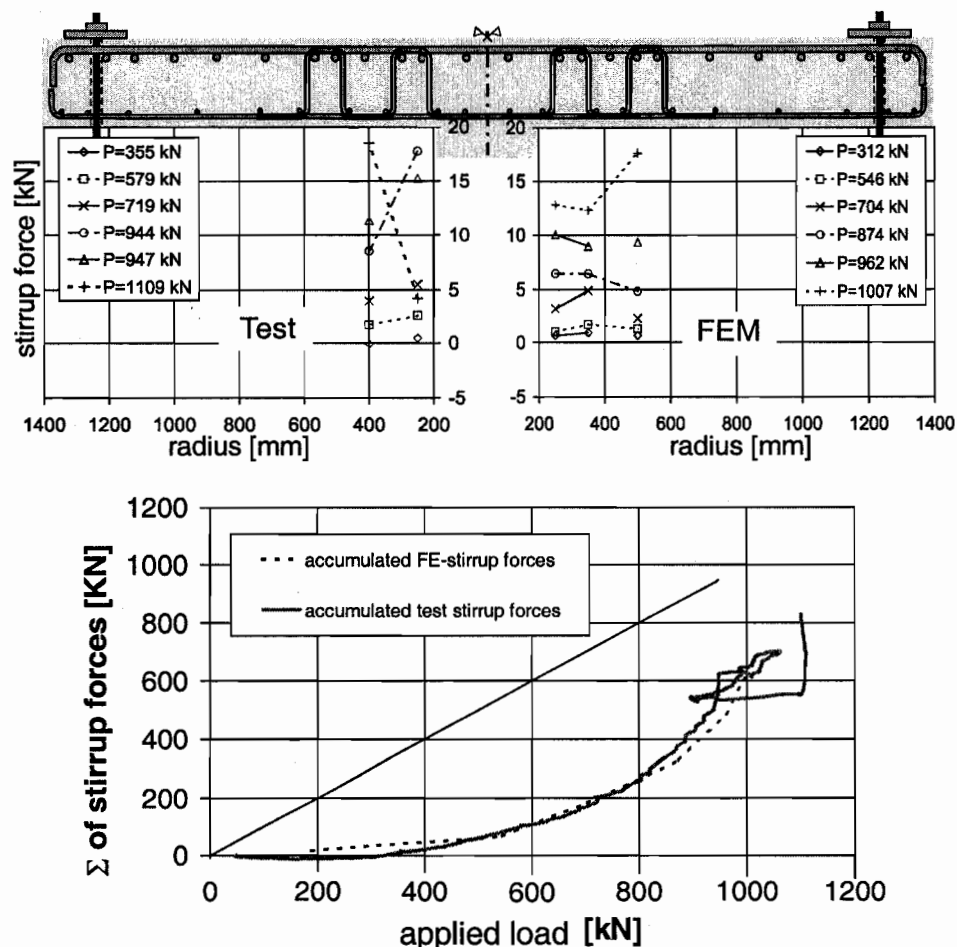


Figure 5.3-5: Stirrup stresses in different rows and total stirrup forces ($\rho_l = 0.8\%$, $f_c = 29.8 \text{ N/mm}^2$, $f_t = 3.7 \text{ N/mm}^2$, $E = 24900 \text{ N/mm}^2$, $G_f = 0.1 \text{ N/mm}$, $f_{y,flexure} = f_{y,stirrups} = 549 \text{ N/mm}^2$).

5.3.4 Concrete crushing at the column face – Comparison with test P6-I

Two types of concrete failure at the column face are possible: Semi-ductile concrete failure occurs at relatively low load levels if the position of the first stirrup row exceeds a critical value ($0.5-0.7 \cdot d$). A brittle failure at the column face is caused by crushing of the compression zone at the highest load level without yielding of the longitudinal and shear reinforcement. The simulation of specimen P6-I deals with the latter failure.

The calculated load deflection curve of P6-I (Fig. 5.3-6) shows the same behaviour as the simulation of P2-II up to the peak load. In contrast to P2-II the descending branch of the simulation and the test indicates the expected brittle punching failure. The difference between

the failure loads increases in comparison to P2-II. In the case of P6-I it is about 480 kN. This fact confirms the missing dowel action in the simulation.

The dowel strength is limited by the splitting of the concrete cover, the concrete crushing below the bar and the interaction of the axial forces and the dowel shear force. Taking this mechanical behaviour into account, the dowel force of P2-II results in 126 kN, the one of P6-I in 509 kN. Therefore, the accuracy of the simulations is quite exact which could be shown by the following calculation comparing the total load of the simulation with the test load.

$$\text{P2-II: } V_{\text{calc}} / V_{\text{test}} = (1000 + 126) / 1109 = 1,02$$

$$\text{P6-I: } V_{\text{calc}} / V_{\text{test}} = (1870 + 509) / 2349 = 1,02$$

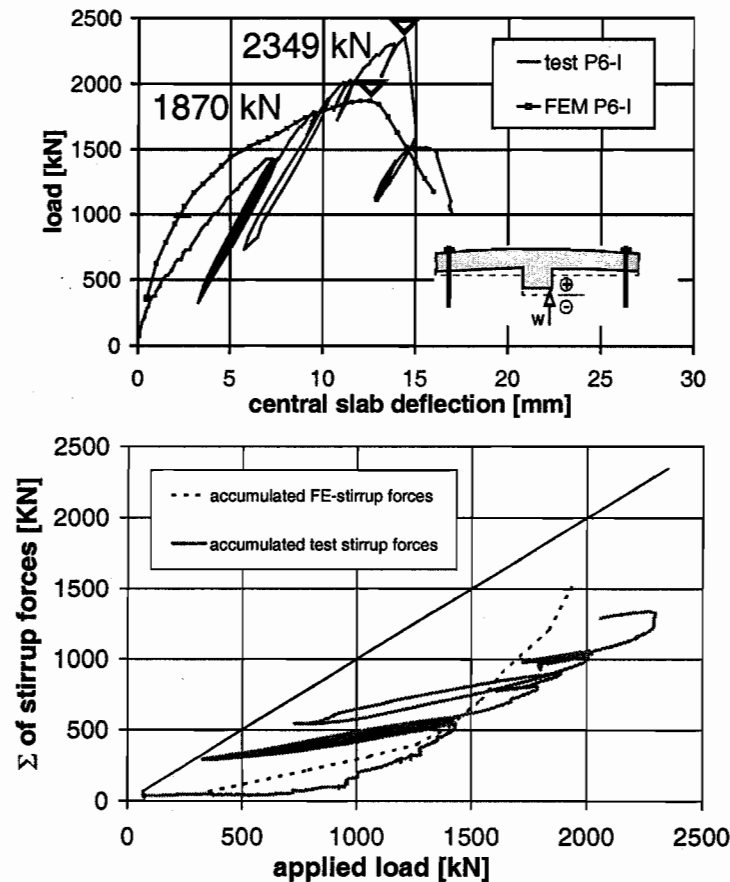


Figure 5.3-6: Comparison of the load deflection curve and the total stirrup force ($\rho_l = 1.75\%$, $f_c = 46.3 \text{ N/mm}^2$, $f_t = 5.0 \text{ N/mm}^2$, $E = 27300 \text{ N/mm}^2$, $G_f = 0.1 \text{ N/mm}$, $f_{y,\text{flexure}} = 557 \text{ N/mm}^2$, $f_{y,\text{stirrups}} = 563 \text{ N/mm}^2$)

In the simulation, the total stirrup forces were always calculated with the highest stirrup stresses. The stirrup stresses of test P6-I are based on 16 strain gauges placed in the middle of the stirrups. This position was not necessarily hit by the shear cracks and therefore the measured strain is not the maximum value. At specimen P2-II the measurement was more precise as the same number of strain gauges were used for less stirrups. Taking into account that all punching design rules are based on tests, the necessity of numerical simulations is obvious as the test results are unsafe.

The development of the punching part of the new German Design Code DIN 1045-1 (1998-1999) for shear reinforced flat slabs, which has been worked out at the Institute for Structural Concrete in Aachen, was based on both tests and simulations with MASA.

5.3.5 Failure outside of the shear reinforced area

The test program [Beutel and Hegger (1998)] includes two tests which failed outside the shear reinforced area. When simulating those tests varying the number of stirrup rows, it could be shown that a failure outside the stirrups only occurs if the number of stirrup rows is less than three (Fig. 5.3-7). The low increase of the punching resistance, in the case of two stirrups only, indicates the observed behaviour of these tests. The complete shear resistance of the stirrups can only be activated if more than two rows are placed.

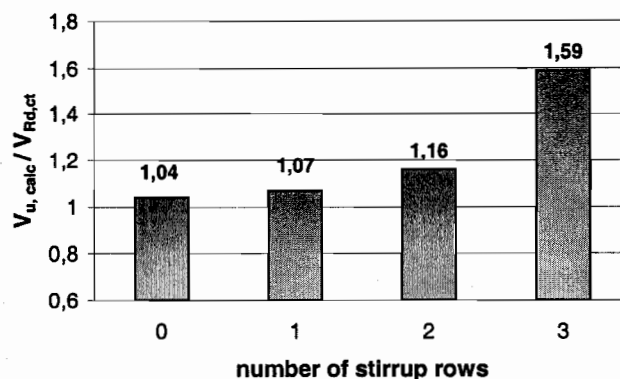


Figure 5.3-7: Influence of the number of stirrup rows ($\rho_l = 1.75\%$, $f_c = 46.3 \text{ N/mm}^2$, $f_t = 5.0 \text{ N/mm}^2$, $E = 27300 \text{ N/mm}^2$, $G_f = 0.1 \text{ N/mm}$, $f_{y,flexure} = 557 \text{ N/mm}^2$, $f_{y,stirrups} = 563 \text{ N/mm}^2$)
 $V_{Rd,ct}$ = punching resistance without shear reinforcement according to DIN 1045-1.

5.3.6 Parametric Study

After having verified the significant punching failure modes with MASA, a parametric study was possible. It is based on the FE-Model of P6-I with a flexural reinforcement ratio of 1.75 %. Therefore, the failure mode is always characterised by punching due to concrete crushing at the column face. This parametric study investigates different stirrup arrangements in a 3D simulation of a flat slab sector. The following parameters were varied:

- The radial distance between the column face and the first stirrup row.
- The radial distance between two stirrup rows.
- Longitudinal reinforcement ratio in order to produce different strut and tie models.

5.3.6.1 Variation of the radial distance between the first stirrup row and the column

In a first study the distance between the column face and the first stirrup row was varied. The second and third row always had a constant distance to the column face. This led to strut angles between the column face and the first stirrups varying between 33° to 77° . An additional simulation was performed without any first stirrup-row (strut angle: 22°). Every simulation had the same shear reinforcement area and the same stirrup cross section in every row.

As shown in Fig. 5.3-8 the punching resistance is influenced by the location of the first row and is varying within a range of 10% with respect to the maximum load. The highest load can be carried by a slab with the first stirrup perimeter at $0.8 \cdot d$ (strut angle approx. 45°). If the strut angle is steeper than 45° the first stirrups are less activated as the main shear crack decreases the anchorage zone at the bottom of the first stirrup row. For a flatter angle the strut force increases and concrete crushing occurs earlier (similar to the well known strut and tie model for uniaxial shear according to Reineck (1990)).

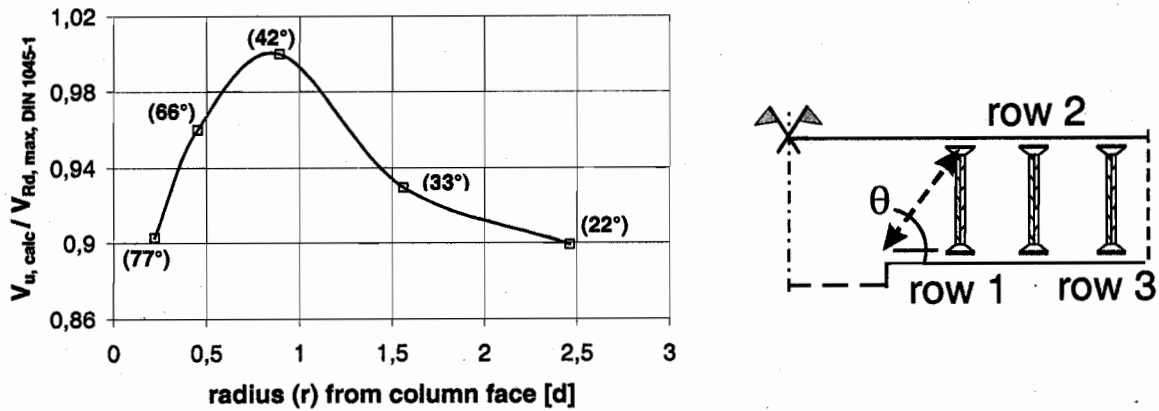


Figure 5.3-8: Influence of the location of the first shear reinforcement row on the punching resistance

5.3.6.2 Variation of the radial distance between two stirrup layers

The distance between the first and second stirrup row was varied between $0.8 \cdot d$ and $2.5 \cdot d$. The first stirrup row, the shear reinforced area and the stirrup cross-section remained constant.

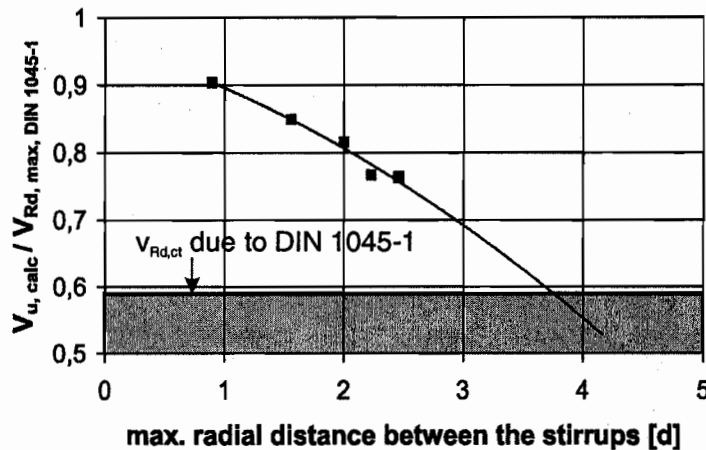


Figure 5.3-9: Influence of the distance between the stirrup perimeters on the punching resistance

It can easily be observed from Fig. 5.3-9 that an increase of the perimeter distance led to a decrease in the punching strength which was generally expected (similar to Fig. 5.3-8). It has to be pointed out that the maximum load could not be reached by a stirrup distance of $1.0 \cdot d$. This behaviour indicates that the punching shear crack at the column face is steeper than 45° and therefore the stirrup rows have to be placed closer than $1.0 \cdot d$. A distance of $0.5 \cdot d$ for the first row and $0.75 \cdot d$ for the following ones is recommended.

5.3.6.3 Strut and tie models influenced by the longitudinal reinforcement ratio

Comparing the main compression stress distribution of the simulation P2-II ($\rho_l = 0.8\%$) and P6-I ($\rho_l = 1.75\%$) important differences can be realised. For specimen P2-II high compression fields occur between the stirrup rows as it is shown in Fig. 5.3-10a. This is in agreement with the strut and tie model including struts between each stirrup row and ties represented by the shear reinforcement (Fig. 5.3-10b). This can finally be verified by the crack pattern of the saw cut of specimen P2-II (Fig. 5.3-10c).

The simulation of P6-I ($\rho_l = 1.75\%$) does not show high local compression fields between the stirrup rows. Moreover the compression field is smeared from the column face to the bearing zone forming a direct strut. This model with a strut fan is shown in Fig. 5.3-10e. The

measured high tensile forces of the flexural reinforcement at the bearing zone support this model. The data shown in Fig. 5.3-9 also lead to a strut and tie model with low strut angles. Even at a distance between the stirrup rows greater than $1,5 \cdot d$ the punching resistance is much higher than that for slabs without shear reinforcement. Thus, the direct strut carries a significant part of the load.

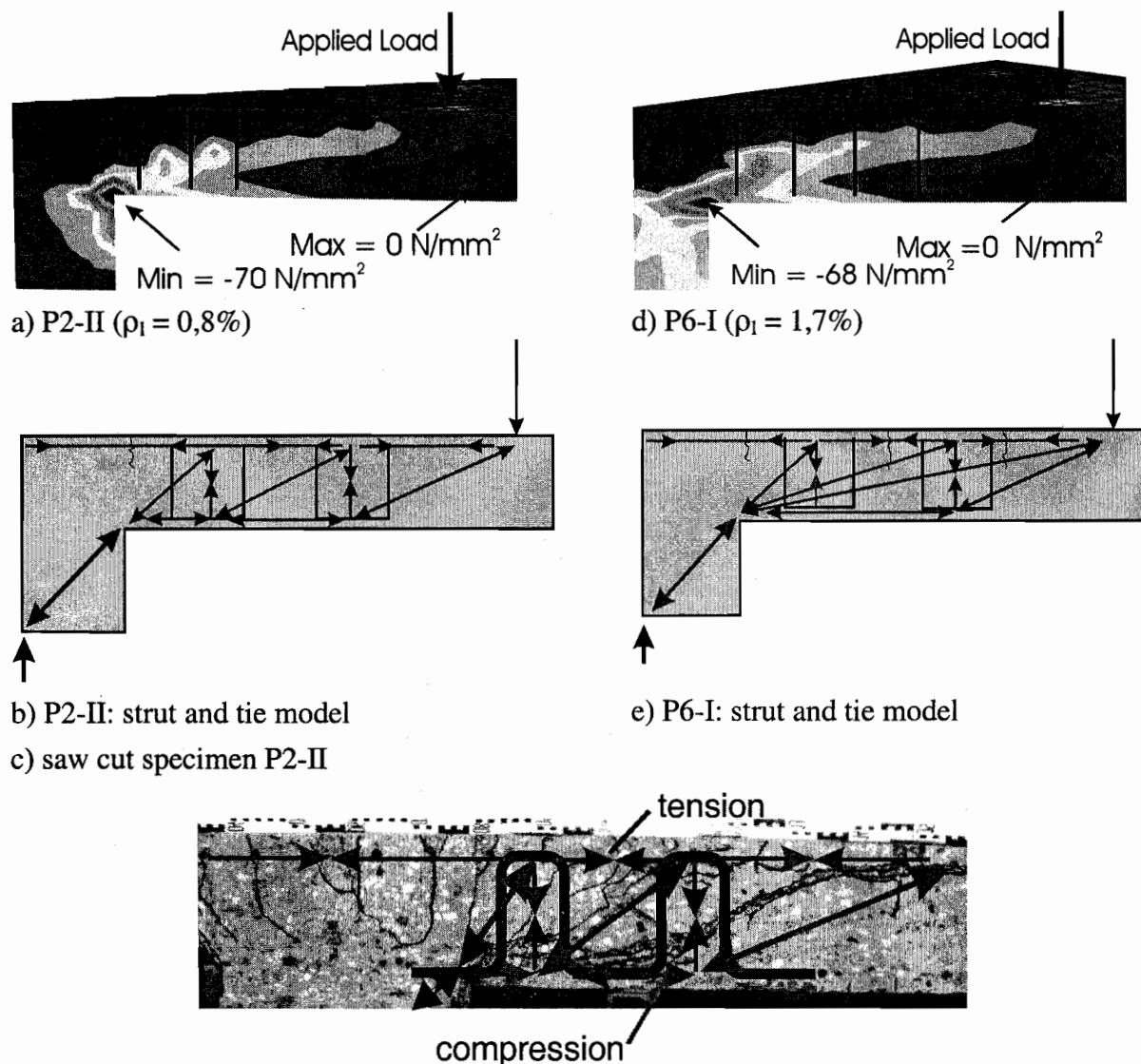


Figure 5.3-10: Comparison of the simulations of P2-II and P6-I with strut and tie models and with the tests

5.3.7 Conclusions

In this contribution 3D non-linear finite-element analyses of flat slabs with the program MASA are presented. Simulations of two different punching tests indicate that MASA is an excellent instrument to simulate punching failure of shear reinforced flat slabs. Now it is possible to study the details of the following punching failure mechanisms: 1) failure inside the shear reinforced area, 2) concrete failure at the column face, 3) failure outside the shear reinforced area.

Finally, a parametric study was performed in order to investigate the arrangement of the shear reinforcement in terms of position and as well as the influence of the reinforcement ratio on strut and tie models in the punching zone.

5.4 Axisymmetric simulation of punching failure

Ph. Menétrey

5.4.1 Preliminaries

The approach adopted here is concerned with the finite element simulation of the punching failure mechanism using a triaxial concrete formulation as presented by Menétrey et al. (1994) and (1997). The finite element simulation of punching failure in reinforced concrete structures is a promising approach but in order to reproduce the punching failure process, based on a review of the experimental tests published in the literature, the following requirements has to be satisfied:

- 1) The punching failure involves tangential cracks as well as radial cracks. Consequently the numerical model must be able to manage these two types of cracks.
- 2) Shear and flexural effects interact and cannot be considered separately. Consequently, a continuum model should be developed as opposed to a structural one.
- 3) Punching failure is a highly localized failure and is not a diffuse failure. Therefore, the numerical model should capture strain localization.
- 4) Punching failure exhibits size-effects which can be properly accounted for by non-linear fracture mechanics concepts.
- 5) The influence of flexural reinforcement is prominent. Therefore, a discrete idealization of the reinforcement as opposed to embedded composite models should be considered so that yielding and debonding can be captured.

The following assumptions were adopted:

- the punching failure is a three-dimensional failure phenomenon which may be simulated for an axisymmetric geometry
- the dowel action (shear resistance transferred by reinforcing bars crossing a concrete interface) which influences the response of reinforced concrete slabs near the peak load is neglected.

5.4.2 Description of the numerical model

The reinforced concrete behavior is described at the constitutive level within the framework of the incremental flow theory of plasticity as presented by Chen (1982) assuming that the elastic and plastic strains increments are uncoupled: $\Delta\epsilon = \Delta\epsilon_e + \Delta\epsilon_p$. The actions of steel reinforcement and concrete are uncoupled. The steel model is characterized by a bi-linear stress-strain response and symmetric response under tension and compression.

The concrete failure criterion is described with the triaxial strength criterion developed by Menétrey and Willam (1995). It is a three-parameter failure criterion which is expressed in terms of the three invariant of the stress tensor (ξ, ρ, θ) . This failure criterion was developed for the punching failure simulation in order to reproduce the various states of stress characterizing the punching failure process. The apex of the failure criterion results in a grey region in the stress space where the stress return algorithm

is not defined. If the stress state is located in the grey region, it is returned to the apex of the failure criterion.

The evolution of the plastic deformation is described with a flow rule derived from the plastic potential g . The plastic potential satisfies the following two conditions: (1) for uniaxial tensile test, the flow direction corresponds to the loading path for uniaxial tension, (2) at the uniaxial ultimate compressive strength, the flow direction is given by the dilatancy angle ψ_c , introduced as a new material parameter.

The concrete cracking phenomenon is described with the smeared crack model using the strain-softening formulation which refers to a gradual decrease in tensile strength with additional deformation. The fictitious crack model developed by Hillerborg et al. (1976) and (1983) is considered for which the reduction of the tensile stress σ_t is controlled by the crack opening w along the line of an exponential decohesion process:

$$c = \frac{\sigma_t}{f_t} = \exp \left\{ -a \frac{w}{w_r} \right\}, \quad (5.4-1)$$

where a is a parameter and w_r is the crack rupture opening. The gradual decrease in tensile strength with additional deformation allows to fit the tensile experimental data which dominates the post-peak response of concrete failure and especially punching failure. The cohesive parameter is uncoupled in the expression of the concrete failure criterion resulting in an isotropic loss of strength due to reduction of the cohesion.

The fracture energy $G_f = \int_0^{w_r} \sigma_t dw$ (amount of energy absorbed per unit area in opening the crack from zero to w_r) is invariant with the finite element size. Therefore, the mapping between the crack opening w used for the definition of the constant fracture energy, and the cracking strain ϵ_c used at the constitutive level leads to the definition of the finite element size h^e normal to the crack following the idea of the crack band model by Bazant and Oh (1983) so that: $w = h^e \epsilon_c$. The simulation of localized failure like punching failure requires this dependence on the finite element size which plays the role of localization limiter.

As already mentioned, punching failure is characterized by distinct states of stress. The treatment of these different state of stress is accounted for with the failure criterion. In addition and associated to these different state of stresses, different brittleness behaviors are observed. These different brittleness behaviors are included in the model by considering the number of micro-cracks N formed in a specimen following the experimental observations that splitting in compression as well as shear results from various micro-cracks, and in the opposite failure in tension and biaxial compression result in one single crack.

The combination of radial and tangential cracks is accounted for in an average form by computing the increment of crack opening Δw as the positive norm of the plastic strain $\|\langle \Delta \epsilon_p \rangle\|$ (being the internal variable) where $\|\cdot\|$ denotes the norm and $\langle \rangle$ are the Macauley brackets which extract positive components of the argument such that $\langle x \rangle = \sqrt{0.5(x_i + |x_i|)x_i}$. Consequently, the increment of crack opening may be expressed as

$$\Delta w = h^e \Delta \gamma \left\| \left\langle \frac{\partial g}{\partial \sigma} \right\rangle \right\| \frac{1}{N}, \quad (5.4-2)$$

where $\Delta \gamma$ is the plastic multiplier.

The stiffness degradation due to cracking is assumed to occur only in the softening range of the response. The elastic constitutive matrix is modified isotropically with a degradation of the initial Young's modulus E_0 so that: $E = cE_0$ where c is the cohesion given in eq. 5.4-1.

The stress integration algorithm is based on the elastic-predictor, plastic-corrector strategy. The elastic-predictor step is elastic and the plastic corrector step is evaluated with the cutting plane algorithm developed by Ortiz and Simo (1986). A relaxation method is coupled with the cutting plane algorithm. The non-linear solution is advanced in incremental load steps and requires iterations. A modified Newton-Raphson algorithm is implemented with a special strategy adapted to capture localized failure.

Circular reinforced concrete slabs are considered which are modeled with four node quadrilateral axisymmetric elements (quad-axi). The regularization parameter of the quad-axi (h^e) which appears in the softening formulation is the square root of the cross section area so that the radial direction is not taken into account. This results in a constant crack spacing along the perimeter as observed in punching failure experiments.

Mesh locking occurs as a result from near incompressibility. For the developed concrete constitutive model, the flow rule often allows only for little volume changes, so if plastic strains become large, the response becomes nearly incompressible. This difficulty is overcome by the treatment of incompressibility developed by Hughes (1980) using the mean-dilatation formulation.

5.4.3 Punching failure mechanisms

The circular slab tested by Kinnunen and Nylander (1960) and reinforced with ring reinforcement (denoted by IB15a) is simulated because of its perfect axially symmetric geometry. It has a total diameter of 1840 mm and a thickness of 150 mm. Experimentally, the load is applied to the column (150 mm in diameter) by means of a hydraulic jack and transferred to the floor by mean of tie rods along a radius of 855 mm.

The finite element mesh used for the simulation is chosen such that the punching crack is aligned with the mesh without predefining its orientation. The mesh is refined around the corner of the slab-column intersection as illustrated in fig. 5.4-1. The load is applied by controlling the vertical displacement to follow an eventual softening response.

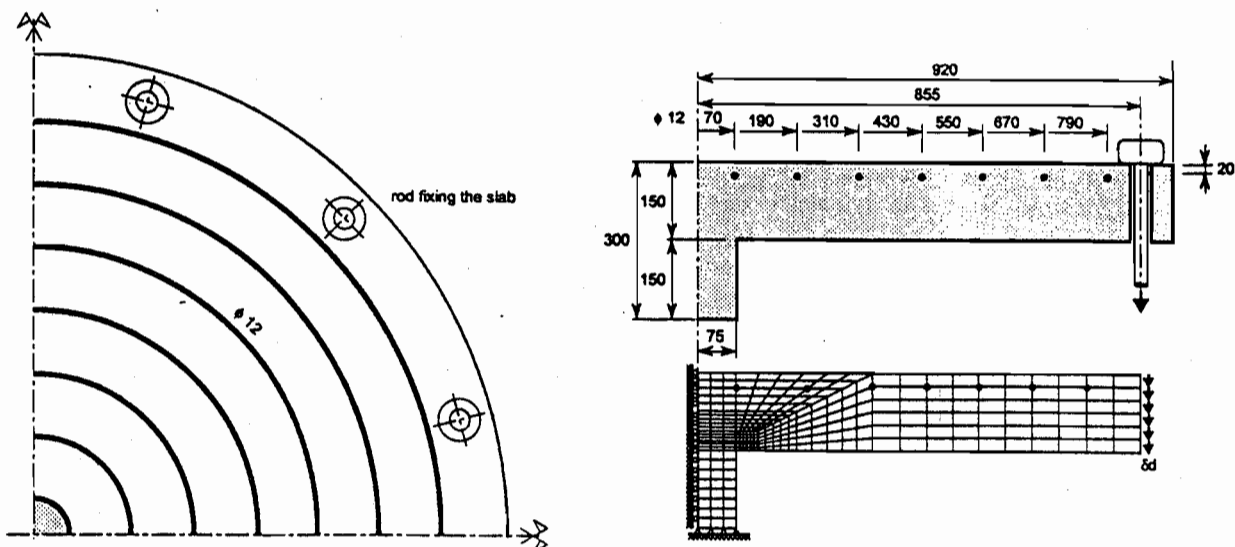


Figure 5.4-1: Overview, cross-section and mesh of the simulated axisymmetric slab

The concrete is characterized by a mean compressive strength of $f_c=28$ MPa. The tensile strength is taken as $f_t=3$ MPa. The fracture energy—according to the CEB-FIP model code (1990)—depends on the maximum aggregate size (32 mm) and on the tensile strength so that $G_f=120$ N·m/m². The following parameters are assumed:

Young's modulus $E_0=25000$ MPa, Poisson's ratio of 0.2, number of cracks in compression $N_b=10$, and dilatancy angle at the ultimate uniaxial compressive strength $\psi_c = 10^\circ$. The reinforcement is made of Swedish Kam steel ribbed bars of 12 mm in diameter which are characterized by Young's modulus of 210'000 MPa and a uniaxial yield strength of 450 MPa. The positions of the ring elements are approached so that they are located at the nodes of the concrete finite element mesh. The position of the ring element controls the crack inclination, but this is the case numerically and also experimentally.

The response of the slab is illustrated in fig. 5.4-2 where the load-displacement curve of the point located at the perimeter of the slab is presented. It can be observed that the predicted punching load (202 kN) agrees with both experimental failure loads (188 and 208 kN). The numerical simulation does not reproduce the five horizontal branches monitored experimentally due to pauses during the load-controlled test without closed-loop systems. This implies that the numerical model predicts a stiffer response than the one monitored experimentally.

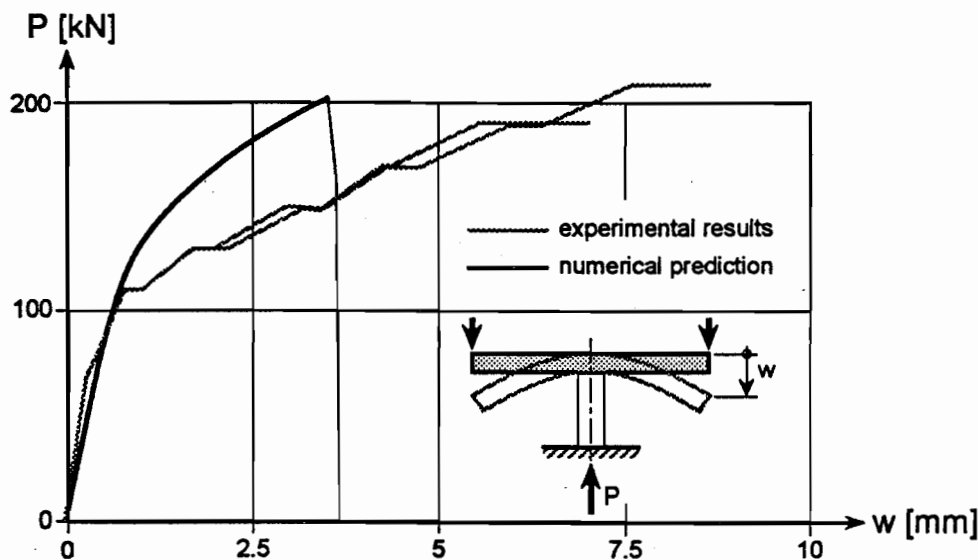


Figure 5.4-2: Experimental and numerical responses of the slab with ring reinforcement

The deformed mesh of the simulated crack is shown in fig. 5.4-3. A punching mode of failure is reproduced for which the deflections are localized along an inclined band of element. The punching failure simulation illustrates the cracking phenomenon in the vicinity of the column as shown in fig. 5.4-4 for three load steps. The tangential cracks appear if the principal strains are larger than the rupture deformation in uniaxial traction: f_t/E_0 . The tangential cracks are symbolized with a straight line for which the length is computed based on the cohesive parameter and the size of the finite element such that: crack length = $0.33h^e(1 - c)$. The crack orientation is similar to the principal strains orientation. A stress-free crack is symbolized with a thicker line and is represented if $c < 0.007$ (corresponding to $w > w_r$).

It can be observed in fig. 5.4-4 for a vertical displacement of 3.1 mm corresponding to a stress-free tangential flexural crack opened through half the slab thickness that the first inclined stress-free crack appears inside the slab thickness, just below the reinforcement. The numerous micro-cracks located around the stress-free crack are at that time closing. This shows that the punching crack is initiated by micro-cracks coalescence at the top of the slab. This coalescence phenomenon is justified experimentally by the tests of Regan (1983) reporting that micro-cracks are formed across the slab thickness before failure occurs and Moe (1961) who observed visually the formation of inclined cracks across the

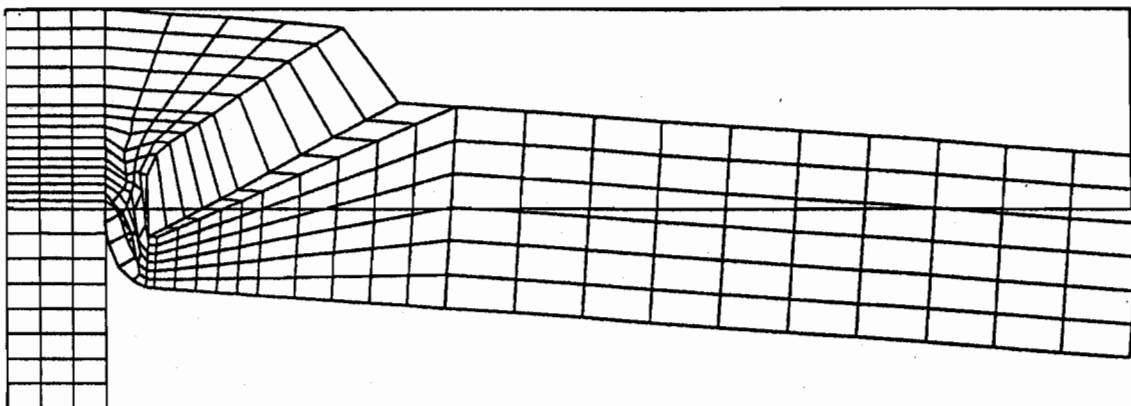


Figure 5.4-3: Deflections under vertical displacement of 3.3 mm

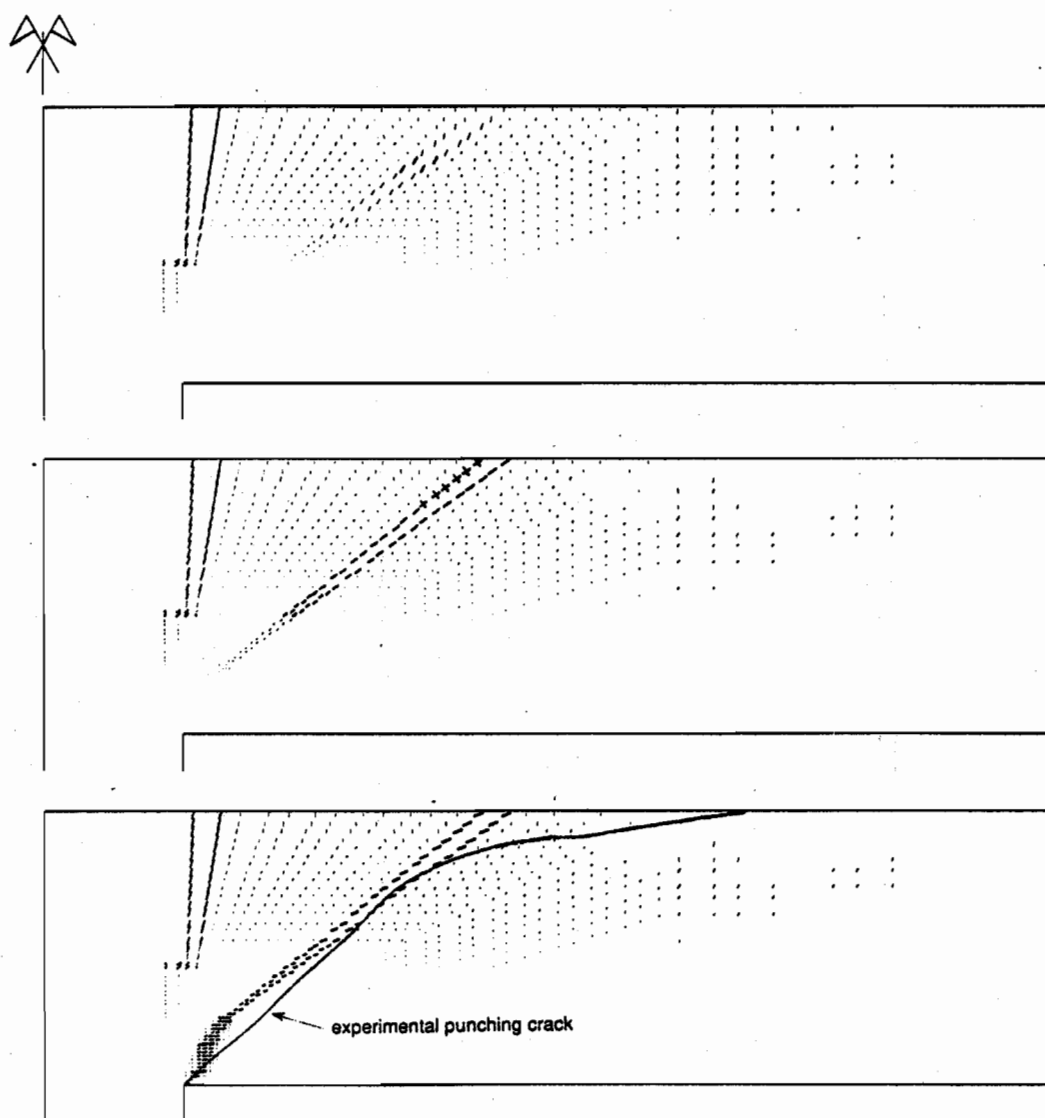


Figure 5.4-4: Tangential cracks for three values of vertical displacement: (1) 3.1 mm, (2) 3.2 mm, and (3) 3.3 mm

slab thickness before failure occurs. By increasing the vertical displacement this inclined crack expands toward the corner of the slab-column intersection. The others inclined micro-cracks are at the same time closing. At failure, the punching crack has reached the corner of the slab-column intersection. It can be observed that the punching crack orientation is close to the experimental one except at the top of the slab.

Based on this simulation, it can be inferred that *the punching failure is initiated by micro-cracks coalescence and is followed by a crack propagation*. The ACI Committee 446 (1992) suggested that punching failure results only from a crack propagation but the proposed direction of propagation is contrary to the one observed here as the punching crack is propagating from the upper part of the slab to the bottom.

5.4.4 Parametric analysis

The reference punching failure simulation is performed on a similar slab than the previous one except that the concrete uniaxial compressive strength is $f_c=28.1$ MPa, the uniaxial tensile strength is $f_t=3$ MPa and the slab is reinforced with orthogonal bars 12 mm in diameter and spacing $s = 115$ mm which are replaced by an equivalent thin steel plate. The percentage of reinforcement ρ —defined as the ratio of the steel and the effective concrete area—leads to the equivalent thickness t of this thin steel plate. The percentage of reinforcement of the reference slab is $\rho = 0.8\%$ which is replaced by a thin steel plate of thickness: $t = \rho d = 0.8 \cdot 125 = 1$ mm. This equivalent thin steel plate is modeled with ring and axisymmetric bar elements. The ring elements have approximately the same spacing as the one of the orthogonal reinforcement. They are located at the nodes of the concrete finite element mesh and have an area which is a function of this spacing s in the form of: $t s$. The axisymmetric bar elements are characterized by their thickness which is equal to the thickness of the equivalent thin steel plate and are fixed rigidly to the concrete only at the extremity of a fictitious fastening length l_F . This fictitious fastening length is determined by analogy with the crack spacing observed experimentally during a tensile test in reinforced concrete so that the condition

$$h^e < l_F < 2s_{r,m}, \quad (5.4-3)$$

is satisfied, where $s_{r,m}$ is the mean crack spacing in tension and h^e is the finite element size. The numerical simulation of the reference slab is performed by assuming a fastening length of $l_F=75$ mm along the axisymmetric bar elements.

The punching failure in the reference slab is successfully simulated allowing the parametric analysis to be performed. The concrete parameters such as the tensile and compressive strengths are known to be interrelated. However, for the following parametric analysis, they are considered independently in order to determine their effects on the punching failure process. The influence of the concrete's uniaxial tensile strength is investigated by simulating three slabs with different tensile strengths ($f_t=2.1, 3$, and 3.9 MPa). The cracking mechanism is analogous for the three slabs but the response curves are distinct as shown in fig. 5.4-5a). The load at which the first stress-free tangential flexural crack appears (illustrated by a discontinuity of the response curves) increases with increasing value of the tensile strength. Once the tangential flexural crack has formed, the slope of the response is similar for all slabs. The failure occurs first in the slab with the lowest tensile strength.

The uniaxial concrete compressive strength does little influence the punching failure as neither the cracking mechanism nor the response curve are modified for slabs with

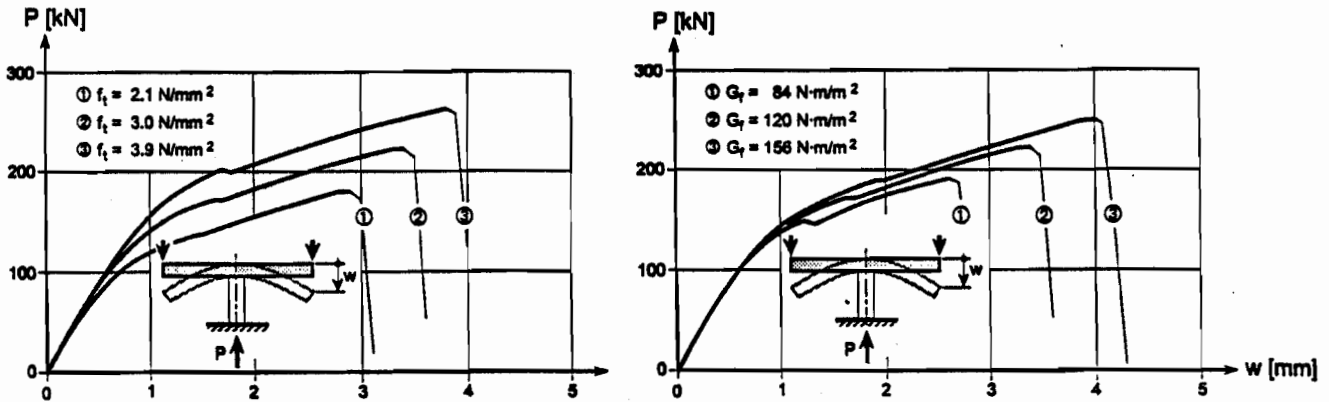


Figure 5.4-5: Influence on the response of the concrete: a) uniaxial tensile strength; b) fracture energy

different uniaxial compressive strengths: $f_c=22.5, 28.1$, and 33.7 MPa (when the tensile strength is held constant). It can be concluded that *the punching failure is due to tensile failure of concrete along the inclined punching crack and is not due to compressive failure of concrete*. The influence of the tensile strength was already suggested by Moe (1961) who mentioned that the punching failure is very often of a splitting type, and it is comparable to the type of failure observed in specimens under tension. This influence of the tensile strength of concrete was further considered for the development of the analytical model by Menétrey (1996) (presented in sec. 4.7.2). The influence of the concrete fracture energy is investigated showing that it influences the failure load and the ductility of the response as they are increased with increasing fracture energy as illustrated in fig. 5.4-5b).

The influence of the percentage of reinforcement is studied by simulating slabs with different percentages: $\rho=0.2, 0.4, 0.8, 1.2, 1.6$, and 2% . A similar cracking mechanism is observed for all these slabs. The corresponding response curves are presented in fig. 5.4-6 in addition to the one of a flexural failure generated for a plain concrete slab. It can be observed that after a similar initial elastic behavior, the response of the slabs varies tremendously depending on the percentage of reinforcement. *By increasing the percentage of reinforcement, the value of the punching load is increased but the ductility is decreased*. It is observed that by increasing the percentage of reinforcement, the state of internal cracking decreases. It should be mentioned that the influence of the dowel force is not considered in the numerical model. These results are comparable to the experimental results obtained by Elstner and Hognestad (1956).

The size-effect is investigated by simulating four slabs of different sizes but with a similar scaling factor which applies to the concrete geometry and the steel area. Except for these dimensions, the slabs have similar boundary conditions and material characteristics. The finite element mesh is refined for large structures to avoid unstable response as the softening slope is controlled by the finite element size. The steel fastening length ($l_F=75$ mm) is constant from one slab to another. The nominal shear stress is computed as

$$\tau_n = \frac{F_{failure}}{\pi(2r_s + d)d}, \quad (5.4-4)$$

where the radius of the column is denoted by r_s and d is the slab effective depth. It

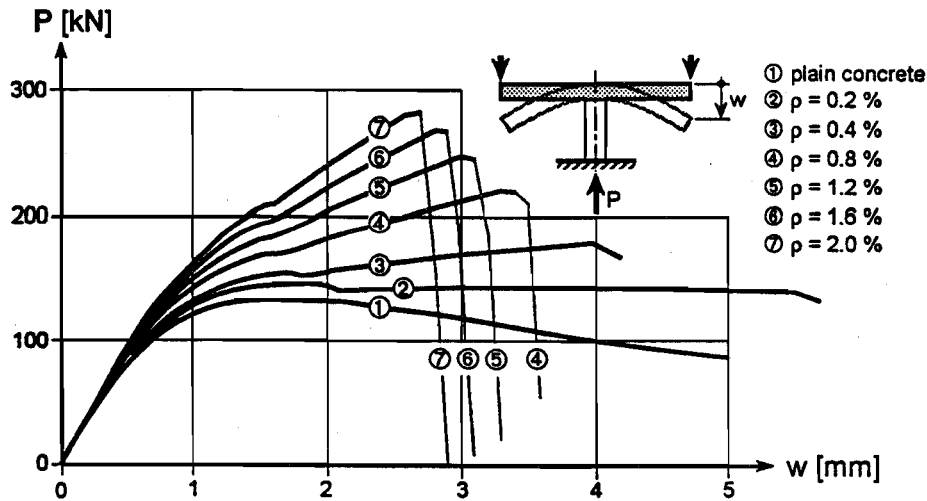


Figure 5.4-6: Influence of the percentage of reinforcement on the response

was observed that the nominal shear stress decreases with increasing slab thickness illustrating size-effect.

In assuming a constant fracture energy, Bažant (1984) derived a size-effect law which was shown to describe the size-effect in punching failure by Bažant and Cao (1987). It is adjusted based on the four slab simulations (without having the experimental scatter) following the RILEM Recommendations (1990) by linear regression which gives

$$\tau_n = 1.55f_t(1 + d/34)^{-1/2}, \quad (5.4-5)$$

where f_t is the uniaxial tensile strength of concrete. This relation is plotted in fig. 5.4-7 in which the two asymptotes: horizontal one (strength criterion) and inclined one (linear elastic fracture mechanic) are distinguished. Another adjustment of the size-effect law is obtained in forcing the strength criterion given by the upper bound load proposed by Marti and Thürlimann (1977). This leads to the following size-effect law

$$\tau_n = 1.8f_t(1 + d/24)^{-1/2}. \quad (5.4-6)$$

This relation is also plotted in fig. 5.4-7. It can be checked that for both proposed laws, the effective depth of the slab is divided by a value which is close to the standard maximal aggregate size of 32 mm. The obtained size-effect law is considered in the analytical model developed by Menétrey (1996) with the parameter μ given in eq. 4.7-7.

This size-effect law was derived for a constant ratio $h/r_s = 2$ where h is the slab thickness and r_s is the radius of the column. However, experimentally the radius of the column or similarly the radius where the punching crack is initiated modifies the size-effect law. The influence of the column radius is investigated by simulating numerically the punching failure in slabs with different radii. The influence of the column radius r_s is clearly shown by computing the value η as follow:

$$\eta = \frac{F_{num}}{\pi(r_1 + r_2) s f_t^{2/3} \xi \mu}, \quad (5.4-7)$$

recalling that r_1 and r_2 are function of r_s . F_{num} is the punching load obtained with the numerical simulation. The results are presented in table 5.4-1 where it can be observed that the parameter η is reduced when the column size is increased. Consequently, the nominal shear strength diminishes when the radius of the punching crack initiation increases. This effect can be considered as an adjustment of the size-effect law in terms

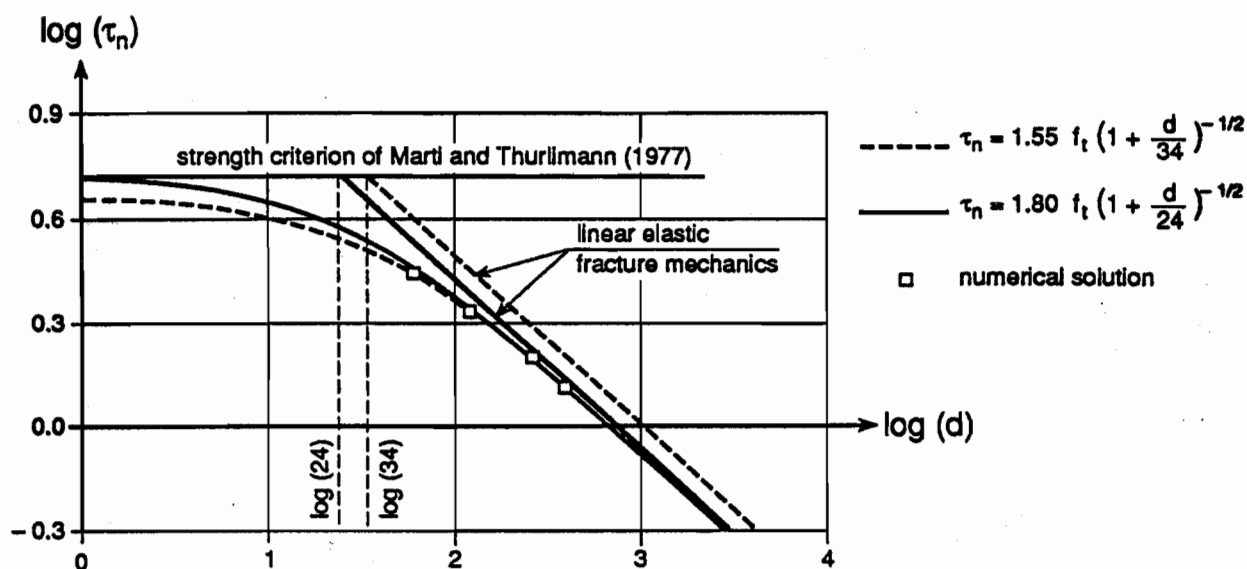


Figure 5.4-7: Size-effect law derived from the numerical simulations

h mm	r_s mm	r_s/h mm	F_{num} kN	η
150	75	0.5	222	1.045
150	150	1	265	0.871
150	225	1.5	295	0.745

Table 5.4-1: Influence of the column size on the punching strength

of the radius of the column. The influence of the column size or more generally the radius of the punching crack initiation r_{sc} is usually not considered in analytical model. The omission of this parameter could lead to unsafe punching load prediction for slabs with large radius of punching crack initiation (punching failure outside the shear reinforcement, punching outside shearhead perimeter). The model developed by Menétrey (1996) includes the parameter η as given in eq. 4.7-8.

5.5 Shell Finite Elements For Punching Analysis Of Reinforced Concrete Slabs

Maria Anna Polak

5.5.1 Introduction

This chapter presents and discusses an analysis procedure which is based on layered shell finite elements, accounts for punching shear, and is suitable for the global analysis of structures.

In the global analysis the behaviour is modelled by considering the structure as a whole, with all actual boundary and loading conditions included in the finite element model. For such analyses of slab systems, plate or shell elements are generally used. The finite element formulation based on the theory equivalent to the Mindlin plate bending theory allows to include transverse shear strain energy in the element formulation [Hinton and Owen (1984)]. The presented formulation incorporates such elements. With the aid of the developed finite element model, it becomes possible to assess whether the given slab is likely to fail in flexure or in punching shear. This makes it useful for practical design situations where it can detect the undesired punching shear failure mode.

5.5.2 The Finite Element Formulation

The presented finite element formulation was incorporated into degenerate, quadratic, isoparametric shell elements (Fig. 5.5-1). These elements have been derived from three-dimensional elasticity and have five degrees of freedom per node: three translations (u, v and w) and two rotations (θ_x, θ_y). The two rotations are not dependent on translations and thus the element allows transverse shear deformations. The assumptions adopted in the element formulation are first that stresses normal to the midsurface are negligible and second that normals to midsurface remain straight, but not necessarily normal after deformation. This implies a linear distribution of axial strains and constant transverse shear strains through the thickness of an element.

Considering the above assumptions, at every point within the element there exist 5 non-zero stress components: ($\sigma_x, \sigma_y, \tau_{xy}, \tau_{xz}, \tau_{yz}$) and therefore five independent strains, [$\epsilon_x, \epsilon_y, \gamma_{xy}, \gamma_{xz}, \gamma_{yz}$]. In the proposed formulation, strain ϵ_z is found from the condition $\sigma_z=0$. Thus, three-dimensional states of stress and strain exist within the element. The iterative process of analysis proceeds as follows:

1. From the previous iteration the five strains, [$\epsilon_x, \epsilon_y, \gamma_{xy}, \gamma_{xz}, \gamma_{yz}$], are known,
2. The sixth strain is found from:

$$\epsilon_z = \frac{-[D_{31}\epsilon_x + D_{32}\epsilon_y + D_{34}\gamma_{xy} + D_{35}\gamma_{xz} + D_{36}\gamma_{yz}]}{D_{33}} \quad (5.5-1)$$

3. From the six strains, principal strains and directions are found. These are not necessarily parallel and normal to the midsurface of the shell element. In fact, the larger the transverse strains, the more rotated the principal systems with respect to midsurface.
4. From principal strains, principal stresses are found using appropriate constitutive model for concrete. Any rational model for reinforced concrete can be used in the presented formulation for shell elements. In the presented analyses, a secant formulation was used with the following moduli:

Young's moduli:

$$E_i = \frac{f_{ci}}{\varepsilon_i}, \quad i = 1, 2, 3 \quad (5.5-2)$$

and shear moduli [Vecchio (1990)]:

$$G_{ij} = \frac{E_i E_j}{E_i + E_j}, \quad i, j = 1, 2, 3 \quad (5.5-3)$$

where f_{ci} are concrete stresses in the principal directions, calculated using appropriate constitutive models.

5. Assuming the Poisson's ratio, after cracking, is zero, the material stiffness matrix for the concrete layer, with respect to the principal axes, has the form:

$$[D_{cp}] = \begin{bmatrix} E_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & E_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & E_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & G_{12} & 0 & 0 \\ 0 & 0 & 0 & 0 & G_{13} & 0 \\ 0 & 0 & 0 & 0 & 0 & G_{23} \end{bmatrix} \quad (5.5-4)$$

6. The strains in the directions of reinforcement are found. The material matrices for transverse and longitudinal reinforcements are formulated in the reinforcement directions as:

$$[D_{st}] = \begin{bmatrix} \rho_s \frac{f_s}{\varepsilon_s} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (5.5-5)$$

where ρ_s is the reinforcement ratio, f_s and ε_s are reinforcement stress and strain, respectively. Reinforcement layers are used to model in-plane reinforcement. The transverse reinforcement is modelled as a property of a concrete layer (Fig. 5.5-2).

7. The material stiffness matrices for all concrete layers, and for transverse and longitudinal reinforcements are transformed to the x-y-z coordinate system and added together to form the new material stiffness matrix of a structure, [D].
8. This new material stiffness matrix is then used in the subsequent iteration. The process continues until convergence is achieved.

In the proposed formulation the nonlinear solution algorithm is based on an iterative, full-load, secant stiffness formulation. The convergence criteria used are based on changes in deformations where displacements and rotations are examined separately. The analysis accounts also for nonlinearities due to changing structural geometry (Total Lagrangian Formulation).

5.5.3 The Material Model

The features of the constitutive model used in the presented analyses are as follows [Polak and Vecchio (1993), Polak (1998^{1&2})]. The uncracked concrete is treated as linear elastic material. After cracking, the concrete becomes orthotropic, with the principal directions of strain defining planes of orthotropy. All material moduli are first calculated in these principal directions (Eq. 5.5-2 and Eq. 5.5-3). The formulation is based on the modified compression field theory [Vecchio and Collins (1986)] and adopts the assumption that the principal directions of

stress and strain coincide. The smeared, rotating crack approach is used, with the crack orientations assumed to be perpendicular to the principal tensile strain directions. The stress-strain relationship for compression and tension response of concrete were developed by Vecchio and Collins (1986), Thorenfeldt et al. (1987), Collins and Porasz (1989) and Izumo et. al (1992). The constitutive model for in-plane reinforcement is linear elastic-perfectly plastic with strain hardening. For out-of-plane (transverse) reinforcement the bilinear model is used: linear elastic until yielding and then perfectly plastic.

The analysis done using the described model can detect whether the failure of a structure is due to flexure or to transverse shear. For slabs without stirrups failing in shear, the failure of concrete is initiated before yielding of longitudinal steel. For slabs with stirrups, the shear failure is initiated by yielding of transverse reinforcement which causes a substantial loss of stiffness of a concrete layer, in the direction of transverse reinforcement.

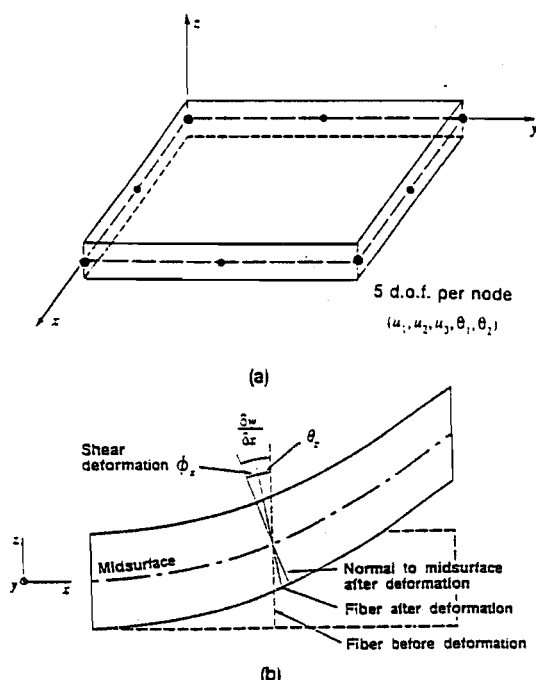


Fig. 5.5-1: Degenerate shell element
a) reference system and degrees of freedom
b) assumptions

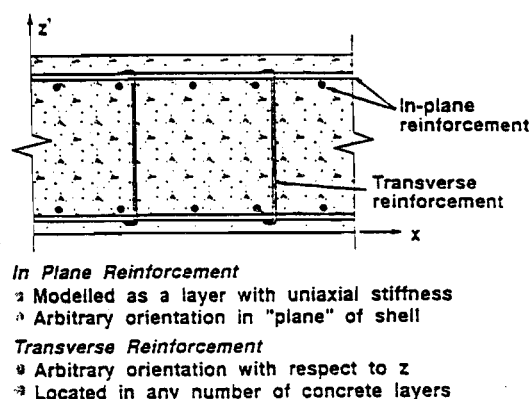


Fig. 5.5-2: Model for transverse and longitudinal reinforcements

5.5.4 Experimental Corroboration

The experimental corroboration presented in this paper concentrated on the analysis of slabs and shells subjected to high transverse (out-of-plane) stresses. The formulation was also tested before, by analysing slabs and shells in bending and in-plane shear [Polak and Vecchio (1993)].

5.5.4.1 Slabs Subjected to Punching Shear- Test Series I

Reinforced concrete slabs with and without shear reinforcement were tested by Yamada et al (1992). The slabs were supported on central columns and subjected to transverse load applied at eight, symmetrically distributed, locations. The specimens were reinforced with orthogonal, isotropic, in-plane reinforcement in the tension and compression zones of the plates. The slabs were discretized by 25 elements modelling one quarter of a slab [Polak (1998²)]. The specimens details and the results of the analyses are given in Table 5.5-1. The shear ratio ($V_{\text{predicted}}/V_{\text{test}}$) versus the amount of transverse reinforcement is plotted in Fig. 5.5-3. The distribution of

longitudinal strains in transverse reinforcement for slabs K4 and K7 is shown in Fig. 5.5-4. The predicted values refer to the maximum strains in the z-direction (ϵ_z), in the compression zone of the slabs at the gauss points which were located closest to the actual stirrup locations. The proposed model was able to trace the strains, ϵ_z with adequate accuracy.

Specimen	f'_c [MPa]	Flexural reinforcement			Transverse reinforcement		$V_{u,t}$ [KN]	$V_{u,p}$ [KN]	$V_{u,p}/V_{u,t}$	Failure Type
		ρ_t [%] X,Y dir (*)	ρ_c [%] X,Y dir	f_{yf} MPa	ρ_x [%]	f_{yt} MPa				
K1	26.0	1.53	1.53	568	0.00		658	450	0.68	shear
K2	27.17	1.53	1.53	568	0.25	347	950	750	0.79	shear
K3	25.90	1.53	1.53	568	0.50	347	1183	950	0.80	shear
K4	27.37	1.53	1.53	568	0.55	317	1153	950	0.82	shear
K5	26.00	1.53	1.53	568	1.11	317	1440	1125	0.78	flexure
K6	26.39	1.53	1.53	568	0.99	330	1274	1100	0.86	flexure
K7	27.76	1.53	1.53	568	1.98	330	1498	1250	0.83	flexure
T1	21.58	1.23	0.62	811	0.00		441	350	0.79	shear

(*) $\rho = A_s/A_c$

Table 5.5-1: Test series I - slabs tested by Yamada, Nanni and Endo (1992)

5.5.4.2 Slabs Subjected to Punching Shear- Test Series II

Six slabs tested by Elstner and Hognestad (1956) were chosen to corroborate the analytical model. The specimens were square (1829x1829x152.6 mm) (72 x 72 x 6 in.), supported along the edges and loaded with a central load uniformly distributed over an area of 254 x 254 mm. The reinforcement was provided only in the tension zone. Specimens B2, B4, B9 and B14 were without stirrups and with different amounts of longitudinal reinforcement. Specimens B12 and B16 were reinforced with inclined stirrups at 45° (B16) and at 90° (B12) angles. The finite element model consisted of one quarter of a slab modelled by 25 unequal elements [Polak 1998²].

The results of the analyses are presented in Table 5.5-2 and Fig. 5.5-5. The analyses predicted the failure modes and the failure loads of the slabs. The capacity of the slab with inclined stirrups (B16) and the capacity of the slab with vertical stirrups (B12) were predicted with good accuracy.

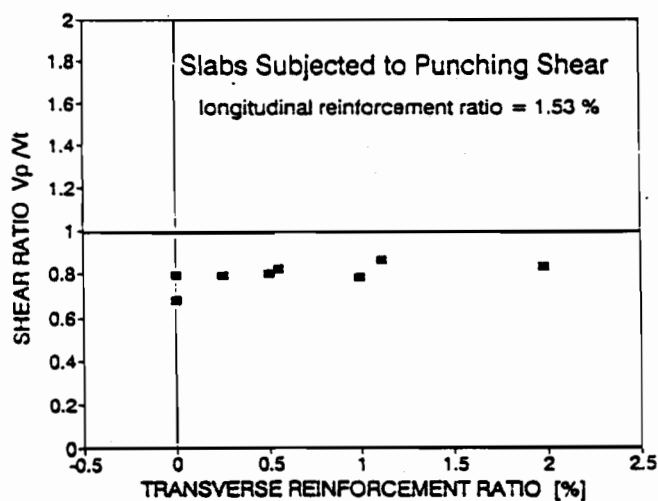


Fig. 5.5-3: Test series I: shear ratio ($V_{predicted}/V_{test}$) versus the transverse reinforcement ratio.

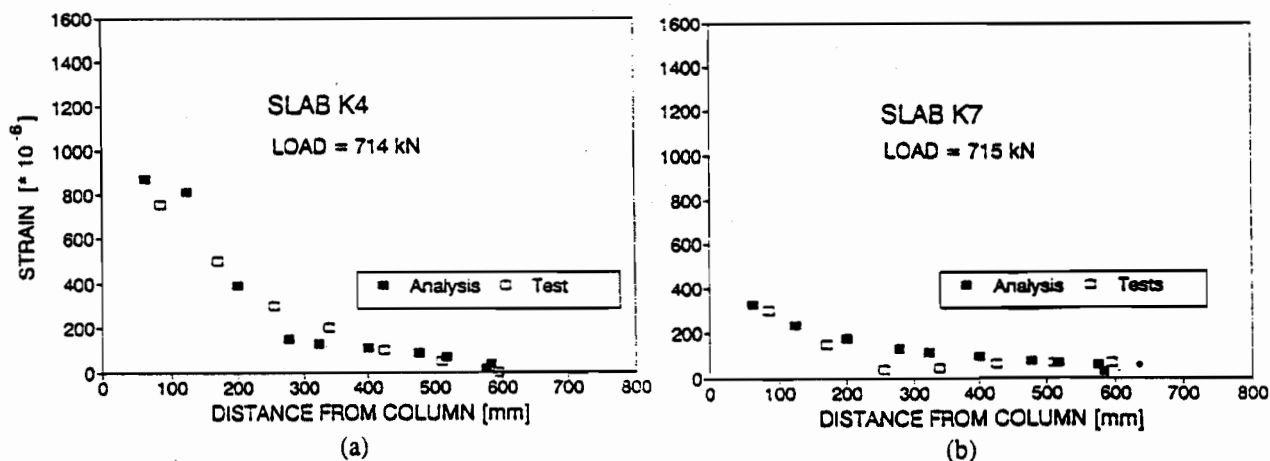


Fig. 5.5-4: Test series I: longitudinal strains in transverse direction a) slab K4 b) slab K7

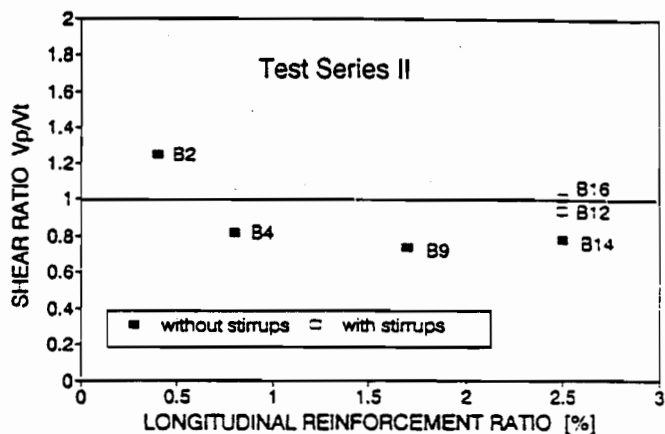


Fig. 5.5-5: Test series II: shear ratio ($V_{predicted}/V_{test}$) versus the longitudinal reinforcement ratio

Slab	Material Properties			Reinforcement				P _u [kN]			Type of failure
	f _c [MPa]	f _{y,l} [MPa]	f _{y,t} [MPa]	longitudinal		transverse (stirrups)		test	analysis	Analysis/ test	
				ρ _x [%]	ρ _y [%]	ρ _t [%]	angle [deg.]				
B2	47.6	321		.4	.36			200	225	1.125	flexure
B4	47.7	303		.81	.7			334	275	0.82	flexure
B9	43.9	341		1.67	1.36			505	375	.74	shear
B14	50.5	325		2.5	2.0			578	450	.78	shear
B12	45.9	332	331	2.5	2.0	.51	90	787	800	1.01	shear
B16	45.3	332	331	2.5	2.0	0.36	45	747	700	0.94	shear

$$^{(*)} \rho = A_s / A_c$$

Table 5.5-2: Test series II - slabs tested by Elstner and Hognestad (1956)

5.5.4.3 Shell Elements Subjected to Transverse Shear -Test Series III

Five tests on shell panels subjected to different combinations of transverse shear and in-plane forces [Adebar and Collins (1994)] were chosen for the analysis. The specimens SP3, SP4, SP7, SP8 and SP9 were 1524 x 1524 mm, with a thickness of 310 mm. The in-plane reinforcement consisted of two orthogonal layers of bars with equal amount of reinforcement in both directions. The reinforcement was oriented at a 45° angle with respect to the specimens' sides. The specimens were also reinforced in the out-of-plane direction (T-headed bars placed in a 120x120 mm grid). The applied loading consisted of uniform, out-of-plane shear, equilibrating bending moments and various combinations of in-plane forces. The details are given in Table 5.5-3. The specimens were modelled using a mesh of 8 elements [Polak (1998¹)]. The shell was divided into ten concrete layers. The out-of-plane reinforcement was placed in the inner 8 layers.

Specimen	Concrete Properties		Reinforcement Properties							Applied Loading:	Shear Strength - Test	Shear Strength - Predicted
	f' _c [MPa]	ε' _c	In-Plane				Transverse			Ratio of in-plane shear to transverse shear		
			ratio* ρ _x =ρ _y	f _y [MPa]	f _{ult} [MPa]	φ [mm]	ratio ρ _z	f _y [MPa]	φ [mm]		Transverse / In-Plane [MPa]	Transverse / In-Plane [MPa]
SP3	50	.0022	.0358	480	660	20	.0008	460	8	Transverse shear only	1.64 / 0.00	1.6 / 0
SP4	52	.0023	.0358	480	660	20	.0008	460	8	4	1.90 / 7.60	2.0 / 8.0
SP7	54	.0020	.0375	536	637	20	.0008	460	8	-4	1.60 / 6.40	1.1 / 4.4
SP8	53	.0021	.0375	536	637	20	.0008	460	8	In-plane shear only	0.0 / 16.75	0.0 / 16
SP9	50	.0026	.0375	536	637	20	.0008	460	8	-8	1.22 / 9.76	.9 / 7.2

* - ratio per one layer

Table 5.5-3: Test series III

The presence of in-plane shear significantly alters the transverse shear strength of a shell element (Table 5.5-3). The interaction diagram between in-plane and out-of-plane shear for a typical shell panel is shown in Fig. 5.5-6. The proposed model accurately predicted the capacity of SP3 and SP4, specimens subjected to pure transverse shear and transverse shear plus compressive force. For specimens SP7 and SP8, subjected to transverse shear with in-plane tension, the predicted strengths were lower than observed. The model also predicted, with reasonable accuracy, the strains in the transverse reinforcement (Fig. 5.5-7).

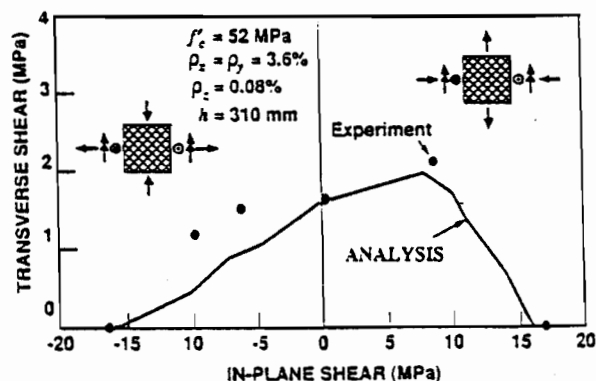


Fig. 5.5-6: Test series III: transverse shear versus in-plane shear for SP shell specimens.

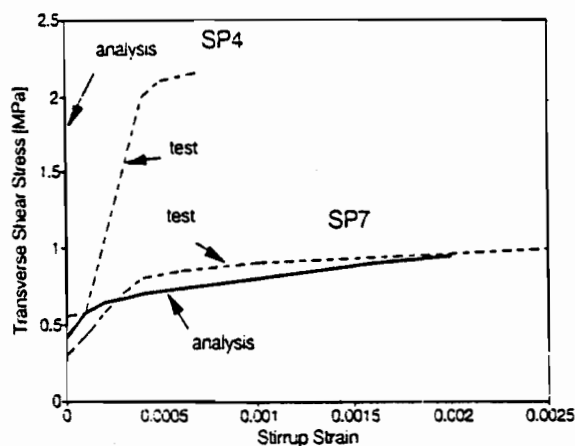


Fig. 5.5-7: Test series III: stirrup strains versus transverse shear stresses for SP-shell specimens.

5.5.4.4 Reinforced Concrete Shells Subjected to Punching Shear -Test Series IV

The test specimens were curved reinforced concrete shells subjected to concentrated central loading [McLean et. al (1990)]. The specimens were approximately one-sixth-scale representations of typical exterior walls for Arctic offshore structures. These were single span shells, simply supported along two edges and with two different curvatures: $R/t = 6$ and $R/t = 12$. Table 5.5-4 summarises the specimens properties and the results of the tests and the analyses [Polak (1998¹)]. The ratios of the predicted to the observed shear strengths of all five specimens are presented in Fig. 5.5-8.

For the specimens with transverse reinforcement and $R/t = 6$, there was little benefit from increasing the amount of transverse reinforcement from 0.24% to 0.48%. Also, for shells with the same amount of stirrups (0.24%) and two different curvatures (AS2 and AS6, Table 4), the normalised shear strength was almost the same ($V_{test}/(b_0 d f'_c)$). This suggests that these specimens (AS2, AS3 and AS6) failed by reaching the limit in concrete compressive strength rather than by yielding of stirrups. The predicted shear strength capacities suggest the same characteristics. Specimens with 0.24 and 0.48% of transverse reinforcement (AS2, AS3 and AS6) failed at the same load level. For the specimens without shear reinforcement (AS1 and AS5), the finite element formulation was able to correctly predict lower shear strength of the specimen with higher R/t ratio.

Specimen	R/t	Shear Reinforcement [%]	f_c [MPa]	V_{test} [MPa]	$\frac{V_{test}}{b_o d f_c}$	V_{pred} [MPa]	Shear Ratio $\frac{V_{pred}}{V_{test}}$
AS1	12	0.0	48	553	10.1	300	0.54
AS2	12	0.24	54	848	14.7	650	0.77
AS3	12	0.48	50	856	15.4	650	0.76
AS5	6	0.0	50	676	12.1	600	0.89
AS6	6	0.24	41	757	14.9	600	0.79

Table 5.5-4: Test series IV: shell specimens

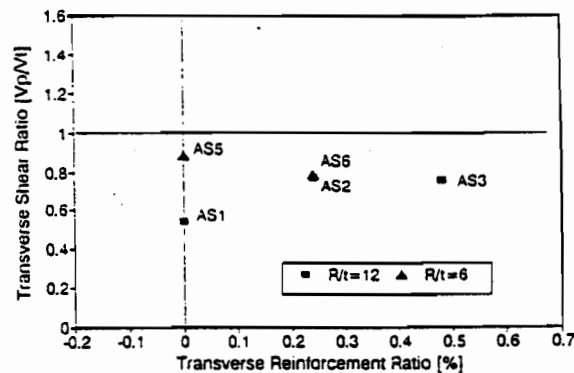


Fig. 5.5-8: Test series IV: shear ratio versus transverse reinforcement ratio for curved shell specimens

5.5.5 Discussion

In all the analyses done in this study the mode of failure was predicted correctly. However, there was a general tendency to underestimate the capacity (around 20%) for slabs that failed due to transverse shear stresses. This can be due to the underestimate of the concrete contribution to shear carrying capacity. For example, for the slab B2 which failed in flexure the result of the analyses overestimated the slab capacity. In the proposed formulation, any change in the constitutive relationships and in the method for calculating shear modulus influences the predicted shear capacity of reinforced concrete members. For example, when the shear modulus was assumed to be 10* the shear modulus described by the Equation 5.5-3, the ultimate load results for the slab K1 tested by Yamada was 650 kN (instead of 450 kN, Table 5.5-1). Further research is needed to formulate a model for the shear transfer in cracked reinforced concrete slabs. The development of such shear transfer model must be directed towards its implementation into a specific element, in this case the degenerate shell element.

5.5.6 Conclusion

The most significant conclusion that comes from this research is that it is possible to use layered plate and shell elements for modelling structures where transverse shear can be responsible for the failure mechanism. The higher order elements (quadratic) make the formulation practical for global analyses of structures. Thus, by incorporating the proposed formulation, punching shear behaviour can be addressed in the global analyses of reinforced concrete shells and slabs.

5.6 Numerical analysis with the finite element program MARC

Markus Staller

The calculations were made using the general purpose finite element program MARC (Version K7.3.2) which is suitable for linear as well as for geometric and physical, non-linear static and dynamic calculations. Detailed background information to the results see Staller (2000).

Some experimental investigations performed by Hallgren (1996) and Tolf (1988) were selected for the numerical studies. These slabs were modelled using three-dimensional, isoparametric elements with eight nodes. Special rebar-elements were used for the reinforcement (assuming perfect bond). For reasons of symmetry, the modelling of one quarter of the slab is sufficient. The load on the slab is performed deformation-controlled. The solution algorithm used was the Newton-Raphson method.

The numerical model is based on the smeared crack concept with a fixed orthogonal crack model. As a material model the von Mises criterion was used for reinforcing steel and the Drucker/Prager criterion with isotropic hardening for the concrete. A bilinear approach was set for the concrete on the tensile side for the tension softening. To allow for the shear transfer in the cracks, a shear reduction factor dependent on crack strains was implemented in the program by special subroutines.

The perspective illustration of the crack pattern in Figure 5.6-1 shows the initial radial and tangential cracks at the end of the last load increment. The punching cone can be recognised here in very close agreement with the experiment. But the diameter of the cone, influenced by the too steep formation of the punching crack, is somewhat too small on the top side of the slab. This can be traced back to smeared crack approach which, in the period just prior to failure, is no longer able precisely to simulate the cracks.

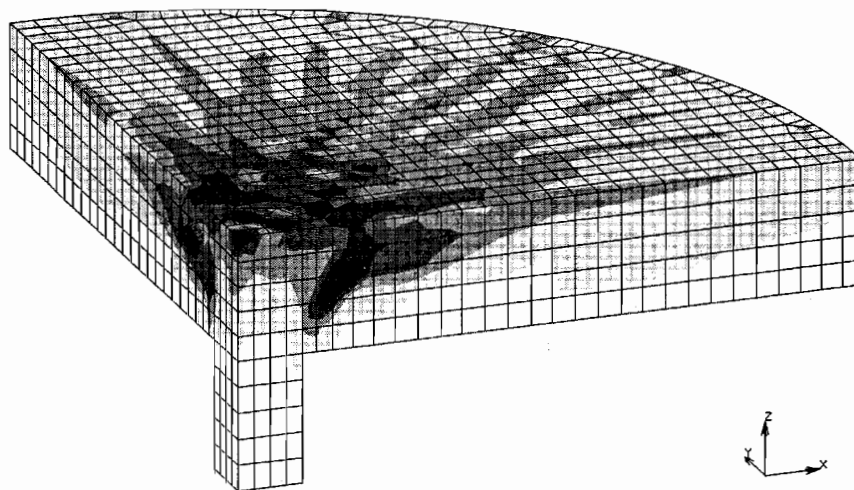


Fig. 5.6-1: Three dimensional illustration of crack formation in one of the investigated slabs in the last load increment (performed by MARC)

The qualitative progression of the load-deflection curve could be reproduced pretty well. During the quantitative determination of the failure loads and deflections, the results deviated from the measured values when using higher strength concrete with a very brittle behaviour.

The effects of different values for the tensile strength and the fracture energy essentially concern the behaviour at the start of crack formation and the following behaviour, respectively, but also lead to a greater punching load, especially for increasing tensile strengths.

5.7 Studies with the FE-Program ABAQUS

Mikael Hallgren

The tests of high strength concrete (HSC) slabs and normal strength concrete (NSC) slab were simulated by using the non-linear finite element method [Hallgren (1996)]. The purpose of the finite element analyses was to reproduce the structural behaviour of the slabs numerically, using the measured material strengths and fracture mechanical properties, and to clarify the mechanism leading to punching shear failure. Furthermore, the aim was also to study the influence of material properties on the structural behaviour. The numerical simulations were performed with the general-purpose finite element program ABAQUS (1994), version 5.4, which offers a non-linear material model based on the smeared crack approach. The analysis was limited to slabs without shear reinforcement.

With the axi-symmetrical model used in the finite element calculations, it was possible to simulate the structural behaviour, including flexural cracking and inclined shear cracking, of the test slabs, see Fig. 5.7-1. For the slabs which failed in brittle punching in the tests, the simulations indicated a brittle failure at loads which are in good agreement with ultimate loads observed in the tests.

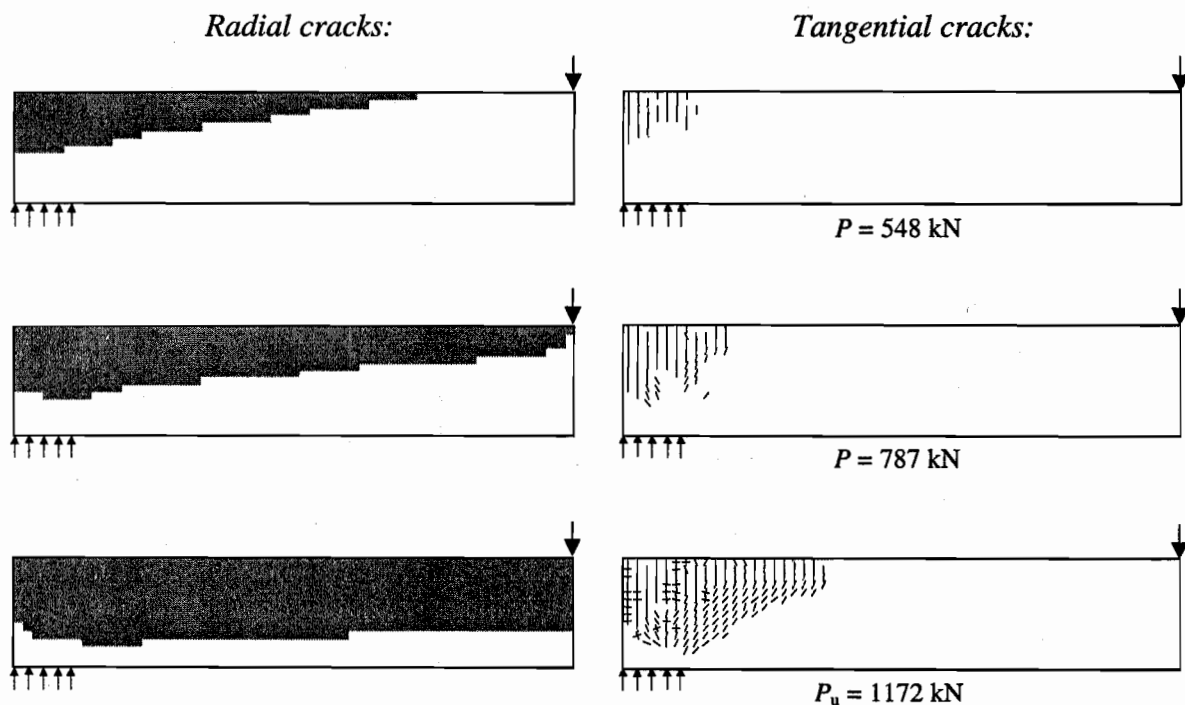


Fig. 5.7-1: Crack patterns at various load levels obtained from the finite element analysis of a HSC slab

In the finite element analyses of the slabs, the concrete between the tip of the shear crack and the slab-column root was found to be in a tri-axial state of compressive stress. The strain corresponding to the smallest compressive stress was tensile and orientated approximately

normal to the shear crack. The compressive stress in this direction had a confining effect, preventing the shear crack to progress through the compression zone. The punching shear failure is assumed to occur when the confining stress is reduced due to horizontal cracking in the tangential compression zone at the distance slightly longer than one depth of the compression zone from the column face. Hereby, the shear crack breaks through the radial compression zone, causing a sudden loss of the load capacity.

A parametric study showed that punching shear strength is not linearly proportional to the concrete tensile strength. The higher brittleness of higher concrete grades reduces the rate of increase of the punching shear strength. A linear correlation was found between the numerically obtained ultimate loads at punching and the concrete tensile strength multiplied with the ratio between the characteristic length of concrete, i.e. ductility of concrete, and the depth of compression zone.

5.8 Brief review of other numerical studies

Rüdiger Beutel

The purpose of this chapter is to give a short review of other punching studies using the finite element method. This review puts the main emphasis on the structural behaviour of the punching simulations, because detailed information are usually complicated and not focused in this bulletin. However, it is well known that the structural results of finite element simulations are influenced by the element-type and the material behaviour. To give an idea of the behaviour of the individual finite element analysis, basic information are given by a subject catalogue.

program: ADINA

model: isolated slab-column connection; axis-symmetric volume elements for the concrete; truss elements for the tangential loop reinforcement

steel stress strain relationship: elastic-plastic including the tension stiffening effect

concrete stress-strain relationship in compression: elastic-plastic, the failure criteria depends on the ratio of the principal stresses

concrete stress-strain relationship in tension: elastic behaviour up to the tensile stress, without tension softening, smeared crack approach

results:

- Reduction of the height of the radial compression zone (height $\approx d/5$) in the vicinity of column face at a load level of 70% of the punching failure load because of tangential bending cracks.
- Inclination of the main punching crack: $\alpha \approx 25^\circ$
- The tangential strain of the flexural loop reinforcement increases by increasing the load before any shear cracks occur. If the first shear crack appears within a radius of $1,5 d$, the tangential strains of the loop reinforcement seems to be constant. Therefore, this flexural reinforcement does not participate any more at the flexural resistance.
- The compression zone of the flat slab is highly stressed in the radial direction because of inclined struts of a strut and tie shear mechanism.

The tangential concrete strain distribution across the slab depth is almost linear.

Table 5.8-1: Numerical investigation of Andrä (1982) - Punching without shear reinforcement at interior columns

program: own development for research model: isolated slab-column connection; axis-symmetric volume elements for the concrete; smeared approach for the tangential loop reinforcement within the volume elements steel stress strain relationship: elastic-plastic including the tension stiffening effect concrete stress-strain relationship in compression: elastic-plastic, the failure criteria depends on the endochronic theory concrete stress-strain relationship in tension: elastic behaviour up to 50 % of the tensile stress, linear tension softening up to two times of the elastic maximum tensile stain, smeared crack approach
results: • Reduction of the height of the radial compression zone (height $\approx d/6$) at the column face because of tangential bending cracks • Inclination of the main punching crack: $\alpha \approx 20^\circ$ • Crack formation up to 25% of the deflection at failure, constant structure stiffness from 25% to 75% of the deflection at failure, fast decreasing of the structure stiffness above 75% of the deflection at failure The ductility depends on the flexural reinforcement ratio. It decreases if the flexural reinforcement increases.

Table 5.8-2: Numerical investigation of Løseth, Slåtto, Syvertsen (1982) - Punching without shear reinforcement at interior columns

program: own development for research model: isolated slab-column connection with holes close to the area of the column; slabs with enlarged column head, layered shell elements; smeared approach for the orthogonal flexural reinforcement within the shell elements steel stress strain relationship: elastic-plastic including the tension stiffening effect concrete stress-strain relationship in compression: elastic-plastic, the failure criteria depends on theory by Figueiras (1983) concrete stress-strain relationship in tension: elastic behaviour up to the tensile stress, linear tension, smeared crack approach
results: • Large holes close to the area of slabs with enlarged column heads do not increase the total deflection significantly on the serviceability load level, because the shear forces were carried direct by struts to the column. The punching failure is caused by crushing of the concrete at the column face. The failure criteria is fixed at the maximum concrete compressive strength in the radial direction. This admissible compressive stress depends on the principal stresses σ_1 and σ_3. Therefore, punching is a combined failure in tension and compression.

Table 5.8-3: Numerical investigation of Gastmeyer (1994) - Punching without shear reinforcement at interior columns

program: NLFEA model: isolated slab-column connection; axis-symmetric volume elements for the concrete smeared approach for the tangential loop reinforcement within the volume elements steel stress strain relationship: elastic-plastic including the tension stiffening effect concrete stress-strain relationship in compression: elastic-plastic, the failure criteria depends on the ratio of the principal stresses concrete stress-strain relationship in tension: elastic behaviour up to the tensile stress, smeared crack approach
results: • Reduction of the height of the radial compression zone at the column face because of tangential bending cracks • Large triaxial concrete stresses within a distance of $\frac{1}{3}d$ or $\frac{1}{2}d$ to the column face • Inclination of the main punching crack: $\alpha \approx 15^\circ$ • The punching failure of flat slabs without shear reinforcement was induced by a splitting of the compression zone at distance between $0,4d$ and $1,2d$ to the column face. Due to the fact that this failure was induced outside the triaxially stressed region. It was concluded, that the triaxial stress state has improved the concrete shear resistance and therefore the biaxial stress state determinates the punching shear capacity. First, stirrups are able to control the punching crack width and they improve the ductility of the punching failure. Secondly, stirrups are able to decelerate the splitting of the compression zone but they were not able to avoid this failure mode.

Table 5.8-4: Numerical investigation of Vidosa, Kotsovos, Pavlovic (1988) - Punching with shear reinforcement at interior columns

program: ABAQUS model: isolated slab-column connection; volume elements for the concrete, truss elements for the orthogonal reinforcement, high strength concrete steel stress strain relationship: elastic-plastic including the tension stiffening effect concrete stress-strain relationship in compression: elastic-plastic, the failure criteria depends on the ratio of the principal stresses concrete stress-strain relationship in tension: elastic behaviour up to the tensile stress, strain softening, smeared crack approach
results: • The maximum shear stresses were evaluated in a distance of $1,5d$ to the column face in the concrete compression zone. • The shear cracks start in the tension surface and continue in the direction of the concrete compression zone. The structural behaviour and the failure loads of flat slabs including stud-rails or double headed studs was satisfying. The failure loads of stirrups and bent-up bars were 31% over-estimated.

Table 5.8-5: Numerical investigation of Marzouk, Jiang (1996) - Punching with shear reinforcement at interior columns

5.9 Summary

In the present chapter 5 the progress in modelling the punching failure using nonlinear FEM achieved in the past decade has been outlined. It was demonstrated that the failure mode as well as the punching load and the deformations of flat slab column connections can realistically be predicted in numerical simulations.

In all investigations presented in chapter 5 a smeared crack approach was applied in combination with the crack band method as localisation limiter. However, different finite elements and material models for concrete were used in the presented formulations.

Menétrey and Hallgren applied rotational symmetric continuum elements, Ozbolt and Vocke, Beutel as well as Staller introduced spatial continuum elements while Polak based her simulations on shell elements. The numerical results of Menétrey and Hallgren show that rotational symmetric continuum elements are appropriate for the simulation of punching failure. However, 2-D-elements can not be used for important practical applications like orthogonal reinforcement or unsymmetrical punching. Therefore, 3-D-elements are becoming state-of-the-art in recent investigations by Ozbolt and Vocke, Beutel, Staller as well as Marzouk and Jiang. This development is also supported by strongly increasing computer capacities and powerful pre- and postprocessors which allow for a convenient application of spatial (3-D-) elements. Nevertheless, the analysis of large structures remains difficult using spatial (3-D-) elements. Therefore, the application of shell elements according to Polak can be reasonable for a global analysis of larger structures. This is especially true if some concession to the accuracy of the numerical results is acceptable, e. g. for the practical design of concrete structures.

All material models used in chapter 5 show a strong influence of the concrete tensile properties, i. e. tensile strength f_{ct} and fracture energy G_F on the punching behaviour of flat slabs in terms of crack initiation and propagation as well as punching load. In contrast, the numerical investigations indicate a relatively small influence of the pure compressive strength (assuming constant tensile properties). However, this observation shall not lead to the conclusion that the punching failure was a purely tensile failure and a simplified modelling of the compressive concrete properties, e. g. elasto-plastic, was sufficient. The state of stresses and strains in the compression zone, first of all the confining effect of the tangential strains in the vicinity of the column also have to be modelled correctly. In other words, the material model should also be able to simulate the tensile splitting as well as shear failure of concrete in compression.

The comparison of existing mechanical models for punching with the test data bank in chapter 7 shows that the development of mechanical models based on experimental data only has limitations. Therefore, a realistic FE-model can be an important tool for the development or improvement of mechanical models for the punching failure since the evaluation of punching tests only may lead to several uncertainties. The main problems with the performing and the evaluation of punching tests can be summarised as follows: (1) The totality of the existing punching tests exhibits an enormous bandwidth of test setups and test results, (2) within comparable test programmes only a few parameters can be varied, (3) the tested slab thickness and column dimensions are relatively small (size effect), (4) important material properties of the concrete like fracture energy and tensile strength cannot directly be controlled and analysed, (5) the formation and propagation of internal cracks cannot directly be observed and (6) punching tests are expensive.

These problems can be overcome by combining experimental results with the information obtained in numerical simulations. The primary tasks of the numerical investigations should be the visualisation of the experimentally invisible fracture process inside the slab as well as the analysis of important geometrical and material parameters such as concrete tensile proper-

ties and slab thickness. Moreover, a numerical analysis might be helpful to design the setup or geometry of punching tests as well as details of test specimen, e. g. the boundary conditions or the distribution of shear reinforcement.

Though the far-reaching development of numerical methods and material models for concrete, it must be kept in mind that finite element simulations are not yet able to fully replace the information from existing or new punching tests. Therefore, a further development of the FEM for the analysis of reinforced concrete structures is in progress.

6 Databank on punching tests

Markus Staller and Rüdiger Beutel

6.1 Establishing a test databank

Since the beginning of the 20th Century a great number of experimental investigations of the punching problem with different aims are performed. The series of experiments are presented in the following for punching tests on reinforced concrete slabs with and without shear reinforcement in the rotationally symmetrical case.

The experimental study of the carrying and the deformation behavior of a flat slab system in its complete form is connected with a considerable expenditure. Therefore, most experimental investigations reduce the complete slab system to a system approximating the real structural behaviour as shown in Fig. 6.1-1.

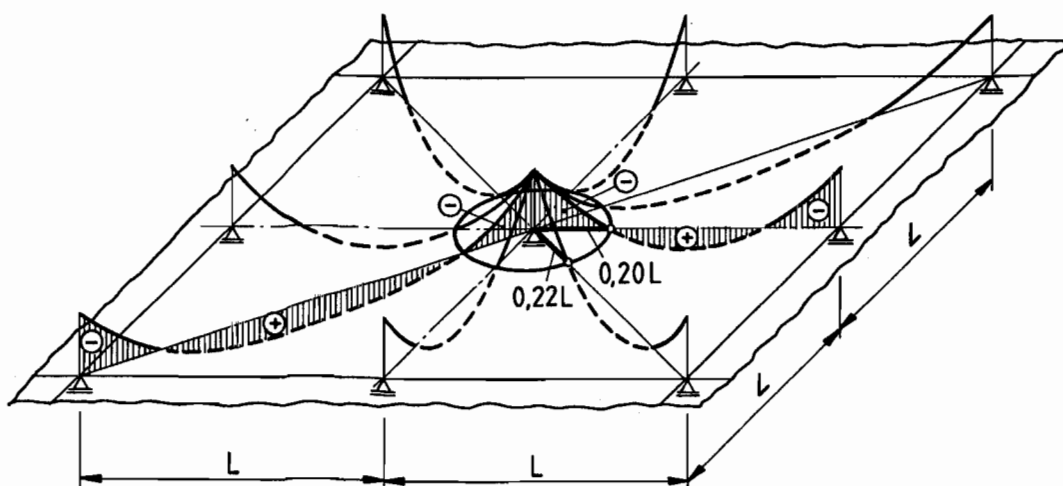


Fig. 6.1-1: Distribution of the radial moments of point supported flat slab under dead load on an internal column

From the great number of the experiments available in literature these experimental studies which describe the problem have to be extracted. If a study supplies no results in this specific respect, however, it can quite have been successful concerning the objective pursued in this series. The experiments occur chronologically, beginning with the newest experiments. Where several different series of experiments were performed at a place they are treated together for clarity.

The test data has to be critically investigated, not only for quality and grade, but also for description of test procedure and for test documentation accuracy. In this contribution the most important subjects of this comprehensive databank which is a part of the appendix in this bulletin are presented.

A full and clear list of all essential series of experiments with the experimental data indicated by the researchers can be taken from the appendices I and II of this Bulletin. These databanks contain more than 400 punching tests of centrally loaded slabs from 1956 to present with concrete with normal density aggregates.

For purpose of a better clarity, the "Collected data" and the "Input data for calculation" were separated. Thus the data of the individual experiments which were indicated by the researcher were included exclusively in the databank "Collected data" in appendices I-1

and II-1. For the limitation of the volume, a data sheet only includes the required columns for each series of experiments with individual denotation and footnotes. The quality of the testing procedure and especially of the concurrent derivation of the material parameters can in general be read off at the column number. Underlined values announce a direct measurement of the respective material characteristics (see Fig. 6.1-2).

Experimental investigations of Hallgren 1996

	geometry of slab and column				reinforcement				material properties											failure	
serial no. slab no.	total diameter of circular slab diameter of circle along the load was distributed total height of slab diameter of circular column stub ¹				effective depth	diameter of reinforcing steel ²	layout of reinforcement (x-direction)	layout of reinforcement (y-direction)	concrete compressive cylinder strength	concrete compressive cube strength	tensile splitting strength of concrete	tensile flexural strength of concrete ³	modulus of elasticity of concrete ⁴	maximum aggregate size	fracture energy ⁵	yield stress of reinforcing steel	tensile strength reinforcing steel	modulus of elasticity of reinforcing steel	failure load	type of failure ⁶	
notation unit	c	c ₁	h	c ₂	d	d _s	s _x	s _y	f _{c,cyl}	f _{c,cube}	f _{ct,sp}	f _{ct,fl}	E _c	d _a	G _I	f _y	f _t	E _s	P _u	-	
	mm	mm	mm	mm	mm	mm	mm	mm	MPa	MPa	MPa	MPa	GPa	m	N/m	MPa	MPa	GPa	MN	-	
1-1 HSC 0	2540	2400	240	250	200	16	120	30	-	112.9	4.5	-	51.8	18	-	643	807	-	0,965	P	
1-2 HSC 1	2540	2400	245	250	200	16	120	130	91.3	106.8	6.9	10.0	42.9	18	179	627	750	-	1,021	P	
... ..	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	
1-6 ⁷ N/HSC 8	2540	2400	242	250	198	16	120	130	94.9	100.1	6.9	10.2	43.5	18	-	631	754	213	0,944	P	
1-7 HSC 9	2540	2400	239	250	202	16	300	300	84.1	86.7	6.7	9.1	39.0	18	195	634	761	231	0,565	FP	

¹ monolithic

² anchored by steel plates (50/50/20 mm)

³ measured at time of test; four-point bending test on plain beams 100/150/800 mm

⁴ measured at time of test; cylinder $\varnothing=150$ mm, $h=300$ mm

⁵ three-point bending test on notched beams 100/100/800 mm acc. to RILEM 1985

⁶ FP = Flexural Punching, P = Brittle Punching

⁷ values for HSC (NSC in brackets)

Fig. 6.1-2: Part of databank I "Collected data", which includes exclusively the collected and recorded data given by the researchers

The table layout of the databanks "Input data for calculation" in appendices I-2 and II-2 of this bulletin is brought to a homogeneous form for all experiments with homogeneous columns. The data can be used directly for the evaluation code rules, mechanical and empirical models. Missing information is marked with "-" (see Fig. 6.1-3).

One of the efforts of these databank is, that not different values for the concrete compressive or tensile strengths are handed over to the evaluation and to the comparisons with calculated values for ultimate loads, but only the uniaxial strength values. In all those cases where different control specimens were tested, large discrepancies could occur between the different values for the concrete compressive strength or concrete tensile strength.

Databank II - Input Data for Calculation

running number	notations			geometry			reinforcement								material properties						failure				
	serial no.	slab no.	author	boundary conditions diameter along the load was distrib. / span of quad. slab total height of slab	shape of column diameter of circular / side length of quadratic col. stub	effective depth diameter of reinforcing steel diameter of compression reinforcing steel reinforcement ratio (inner section) reinforcement ratio (outer section) compression reinforcement ratio (inner section) compression reinforcement ratio (outer section)	compressive uniaxial strength tensile uniaxial strength modulus of elasticity maximum aggregate size	yield stress tensile strength modulus of elasticity	failure load failure load over flexure load (plasticity) lower bound of concrete strength																
no.	not.	n	A	BC	c_1/a_1	h	SC	c_2/a_2	d	d_s	d_{s2}	$\rho_{l,l}$	$\rho_{l,s}$	$\rho_{l2,l}$	$\rho_{l2,s}$	f_{1c}	f_{1cl}	E_l	d_s	f_y	f_t	E_s	P_u	P_u/P_{flex}	f_{1c-12}
unit	unit	-	-	-	mm	mm	-	mm	mm	mm	mm	%	%	-	-	MPa	MPa	GPa	mm	MPa	MPa	GPa	MN	-	MPa
1	1-1	HSC 0	Hallgren	k	2400	240	k	250	200	16	-	0,8	0,8	-	-	89,2	4,1	51,8	18	643	807	-	0,965	0,71	77,2
2	1-2	HSC 1	Hallgren	k	2400	245	k	250	200	16	-	0,8	0,8	-	-	86,7	6,2	42,9	18	627	750	-	1,021	0,77	74,7
...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
6	1-6	N/HSC8	Hallgren	k	2400	242	k	250	198	16	-	0,8	0,8	-	-	90,2	6,2	43,5	18	631	754	213	0,944	0,73	78,2
7	1-7	HSC 9	Hallgren	k	2400	239	k	250	202	16	-	0,3	0,3	-	-	79,9	6,0	39,0	18	634	761	231	0,565	1,01	67,9
...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...

Fig. 6.1-3: Part of databank "Input data for calculation", which includes exclusively the collected and recorded data given by the researchers

6.2 Explanations to the databank on slabs without shear reinforcement

A survey of definitions and general notations used in the databank is given in Fig. 6.2-1 which illustrates the most important geometrical dimensions of quadratic and circular slabs.

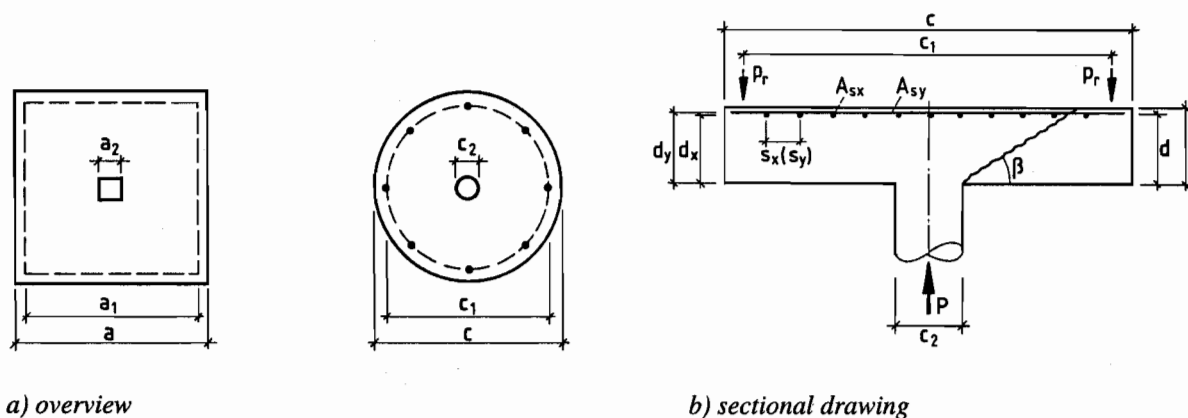


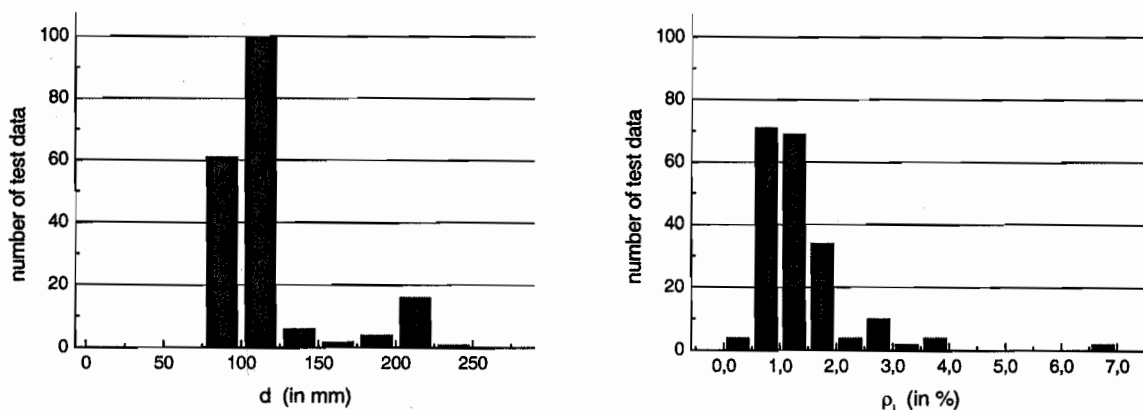
Fig. 6.2-1: Definition of notations.

Table 6.2-1 shows a clear list of the selected experimental investigations of rc slabs without shear reinforcement with the most important parameters in a chronological order beginning with the newest investigation.

Table 6.2-1: List of selected experimental studies

Author	Year of publication	Geometry of slab			Material	
		form	span (in mm)	depth (in mm)	strength (in MPa)	ratio of reinforcement
Hallgren	1996	circular	2400	240	84 - 109	0,3 - 1,2 %
Tomaszewicz	1993	quadratic	1100 - 2500	120 - 320	64 - 119	1,5 - 2,6 %
Ramdane	1993	circular	1372	125	34 - 123	0,6 - 1,3 %
Marzouk / Hussein	1991	quadratic	1500	90 - 150	30 - 80	0,4 - 2,1 %
Lovrovich / McLean	1990	circular	200 - 1200	100	40	1,7 %
Tolf	1988	circular	1190 / 2380	120 / 240	19 - 30	0,4 - 0,8 %
Regan	1986	quadratic	900 - 2745	80 - 250	12 - 53	0,8 - 2,4 %
Petcu / Stanculescu	1974	quadratic	2400	140 - 245	21 - 33	0,4 - 1,1 %
Base	1966	circular	1370	140	14 - 41	1,0 - 1,9 %
Manterola	1966	quadratic	3000	125	24 - 40	0,5 - 1,1 %
Yitzhaki	1966	circular	1170 / 1600	102	12 - 27	0,5 - 2,0 %
Moe	1961	quadratic	1780	152	20 - 35	1,1 - 2,6 %
Kinnunen / Nylander	1960	circular	1710	149 - 158	31 - 39	0,5 - 2,1 %
Base	1959	quadratic	560	76	26 - 30	0,7 - 2,8 %
Elstner / Hognestad	1956	quadratic	1780	152	13 - 51	0,5 - 6,9 %

Figure 6.2-2 demonstrates clearly the structure of the data available in literature on punching tests without shear reinforcement. Especially the distribution of the investigated effective depth of the slabs is very poor (Fig. 6.2-2 a)). Only less than 20 tests concern a slab thickness within a range of practical interest. A very similar arrangement of test data shows Fig. 6.2-2 b) with geometrical ratios of tensile reinforcement in a range around one percent. Exclusively the concrete compressive strength show a wide scatter of tested concrete strength. The result of Figure 6.2-2 is an urgent need for further experimental investigations especially of thicker slabs over $d = 250$ mm. The slabs with a total thickness of $h = 730$ mm (see data-bank II-1 a) tested by Tolf [Kinnunen/Nylander/Tolf (1980)] are not considered for further evaluation because of their (not symmetrical) boundary conditions.



a) effective depth

b) geometrical ratio of tensile reinforcement

Fig. 6.2-2: Histograms showing the structure of the data of tests without shear reinforcement

6.3 Explanations to the databank on slabs with shear reinforcement

The databank II for punching tests with shear reinforcement is divided in two parts. Part II-1a (Fig. 6.3-1 a)) contains the structure and material behaviour of the specimen and is similar to the databank I-1 without shear reinforcement. Part II-1b (Fig. 6.3-1 b)) describes additionally the shear reinforcement regarding type, quantity and arrangement.

Databank II contains 150 shear reinforced punching tests. The shear reinforcement types were limited to these types, which are established in the majority of the codes. Therefore, only stirrups (ST), shear ladders (SL), bent-up bars (SB) and hooks (HO) were considered. For shear reinforced flat slabs three different failure modes can occur (Fig. 6.3-2):

1. at the column face (Pc)
2. within the shear reinforced area (Ps and Pbs)
3. outside the shear reinforcement (Po)

For the first time this databank distinguished these failure types and therefore the arrangement of the shear reinforcement was extensive included. The quantity of the shear reinforcement was subdivided into five sections. The reinforcement in each section is characterised by the inclination α , the distance to the column face l_s and by the sum of the reinforcement amount A_{sw} (Fig. 6.3-2).

Hallgren, Kinnunen 1990

no.		geometry of slab and column					flexural reinforcement					concrete								flexural reinforcement	failure
serial no.	specimen no.	constraint conditions	diameter of the slab	diameter between the load or bearing points	total slab heights	column diameter	mean effective depth	Ø tension reinforcement	clear spacing of the tension reinforcement in x and y-direction	Ø compression reinforcement	clear spacing of the tension reinforcement in x and y-direction	compressive cylinder strength	compressive cube strength	tensile splitting strength	tensile flexural strength	modulus of elasticity	maximum aggregate size	fracture energie	yield strength	failure load	type of failure
notation	-	c2	c1	h	c	d	d _s	e _x /e _y	d _s	e _x '/e _y '	f _{c,cyl}	f _{c,cub}	f _{ct,s}	f _{ct,fl}	E _c	d _g	G _f	f _y	P _u	-	
units	-	mm	mm	mm	mm	mm	mm	mm/mm	mm	mm/mm	MPa	MPa	MPa	MPa	GPa	mm	N/m	MPa	MN	-	
14-1	HSC3S	k	2540	2400	240	250	200	16	160/ 160			92,4	97,5	6,5	7,7	37,4	18	158	632	1,329	P
14-2	HSC7S	k	2540	2400	240	250	200	16	210/ 210			85,0	99,4	7,4	10,6	39,4	18	156	630	1,106	FP

cylinder geometry $\phi/h = 150/300$ mm, cube side length 150 mm, geometry of the concrete tensile strength test specimen $l_b/h = 800/150/100$

a) Databank II-1a: Collected data of tests with shear reinforcement

no.		shape and distribution		row one			row two			row three			row four			row five			material properties			failure
serial no.	specimen no.	type of shear reinforcement	shear reinforcement distribution	distance to the column face	shear reinforcement cross section area	diameter/ inclination	distance to the column face	shear reinforcement cross section area	diameter/ inclination	distance to the column face	shear reinforcement cross section area	diameter/ inclination	distance to the column face	shear reinforcement cross section area	diameter/ inclination	distance to the column face	shear reinforcement cross section area	diameter/ inclination	yield strength	tensile strength	modulus of elasticity	place of the mayor shear crack
notation		-	-	lsw, 1	Asw,1	ds /?	lsw,2	Asw,2	ds /?	lsw,3	Asw,3	ds /?	lsw,4	Asw,4	ds /?	lsw,5	Asw,5	ds /?	fy	ft	Es	-
unit		-	-	mm	mm ²	mm/ o	mm	mm ²	mm/ o	mm	mm ²	mm/ o	mm	mm ²	mm/ o	mm	mm ²	mm/ o	MPa	MPa	GPa	-
14-1	HSC3S	SB	HA	90	4021,3	16/ 45													503			Pc
14-2	HSC7S	SB	HA	90	3217	16/ 45													501			Pc

b) Databank II-1b : Additional data to specify the shear reinforcement

Fig. 6.3-1: Example for the Databank II-1a "Collected data" and II-1b "Additional data for the shear reinforcement"

The databank II-2 "Input data for calculation" is similar to the databank I-2 (section 6.2) but it contains some information additionally (see databank II-1b). Therefore, individually explanations are not required.

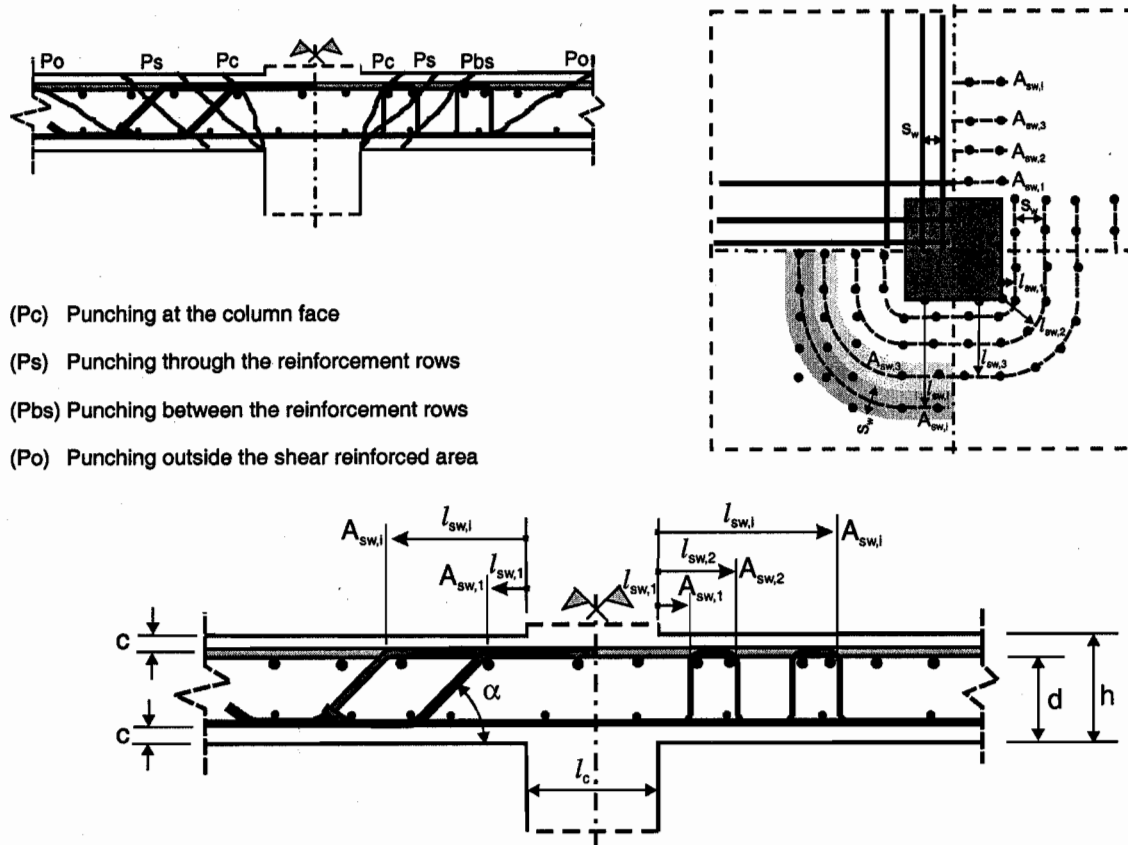


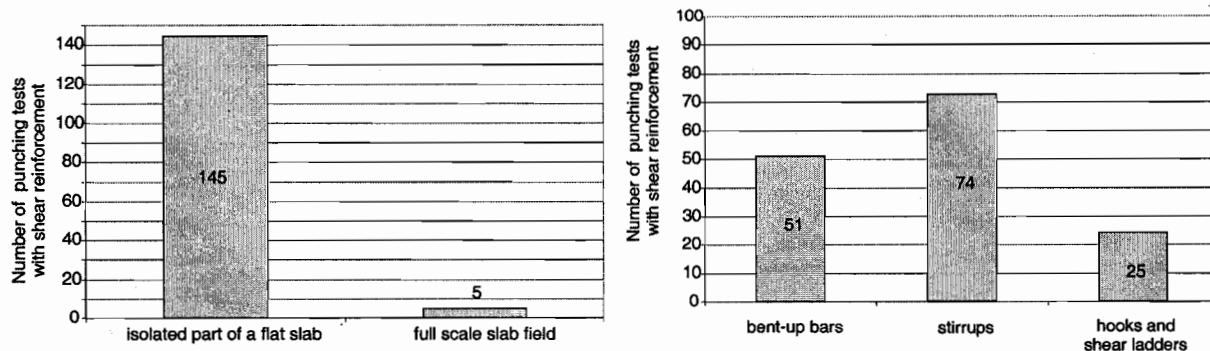
Fig. 6.3-2: Definitions according the punching failure types and the geometry of the distribution of the shear reinforcement

In Chapter 9 the databank II-2 will be used as an neutral basis to judge the code formulas of Chapter 2. Therefore, it is necessary to be clear on the structure of the data of the compiled punching tests. In Figure 6.3-3 all 150 tests are subdivided in different classes. The following items can be observed:

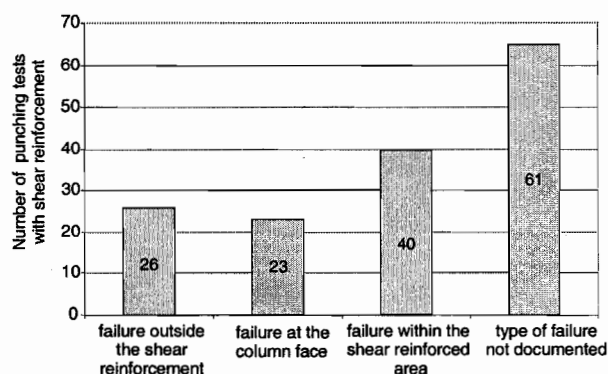
- 96,7 % of 150 tests were carried out by isolated slab column connections (Fig 6.3-3 a)). This shows that only five tests consider the influence of internal membrane forces in cracked flat slabs.
- 49% of the tests included stirrups and 34% bent-up bars. Hooks and shear ladders were included by 17% of the tests (Fig 6.3-3 b)).
- 40% of the evaluated papers and reports do not give any information about the failure mode. This concerns in first way the older tests which were documented without any saw-cut or cut out of the punching cone. The other tests indicated that 30% failed outside the shear reinforced area, 25% failed at the column face and 45% within the shear reinforced area (Fig 6.3-3 c)).
- 90% of all punching tests show a total depth of the slab less than 250 mm (Fig 6.3-3 d)). Normal total flat slabs depth in practise result in a value between 250 and 350 mm. Footings and foundation plates are built with a total thickness greater than 500 mm. This

comparison show that only thin plates were tested and therefore the following tendencies have to be taken into account:

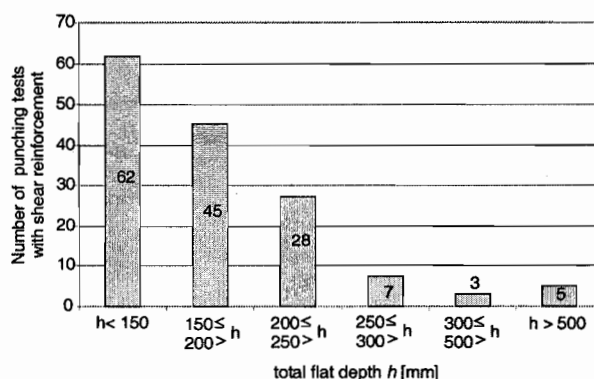
- a) A beneficial size effect, if the failure is due to crushing of the concrete.
- b) The anchorage of stirrups and shear ladders in the compression zone was underestimated because the height of the compression zone is in the range of the concrete cover.
- 70% of the tests show a concrete compression strength used in practice. Only nine specimens were tested with high strength concrete (Fig 6.3-3 e)).
- 78% of the tests include a flexural reinforcement ratio between 0,5% and 1,5% (Fig. 6.3-3 e)). This describes the flexural reinforcement ratio values in ordinary flat slabs.



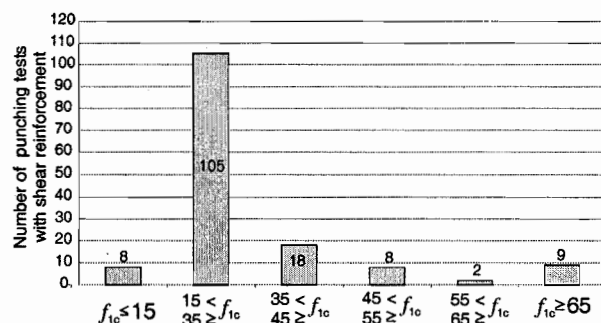
a) type of the structure



b) type of the shear reinforcement

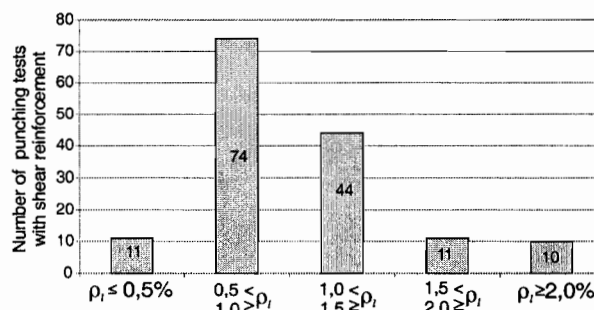


c) failure type



e) concrete compressive strength f_{lc}

d) total depth of the specimens



f) flexural reinforcement ratio ρ_l in the closeness of the column

Fig. 6.3-3: Structure of the data of the punching tests in databank II-1 and II-2

This structure of the data of the databank shows that punching tests of shear reinforced slabs including a thick depth or a full scale fields are rare. Therefore, it is recommended to complete the experimental studies in the area. It is well known that these tests are very expensive in costs and labour. Therefore, numerical studies could be an alternative solution to improve the knowledge (see Chapter 5).

6.4 Definitions and Conversion Factors of Concrete Properties

The definitions and conversion factors in this section are in general based on Model Code 90 (1993) and CEB Bulletin N° 228 "High Performance Concrete: Recommended Extensions to Model Code 90" (1995). The units used in this paper are: MN/m², N/mm², MPa

6.4.1 Concrete compressive strength

The concrete compressive strength should be taken as the prism strength f_{1c} . For the determination of f_{1c} the following priorities are defined:

1. The values tested on prisms should be taken, unless these could be regarded as not reliable, e.g. due to a small number of control specimens tested.
2. The value f_{1c} derived from $f_{c,cyl}$ should be taken, unless the values derived from cubes could be regarded as more reliable, e.g. due to a higher number of control specimens tested.
3. The value f_{1c} derived from cubes are taken if no cylinder tests are available, because of the high scatter of this kind of test.

The conversion of different concrete compressive strength into the uniaxial prism strength f_{1c} determined at a prism 100/100/500 mm can be calculated with the following equations:

$$f_{1c} = 0,79 f_{c,cube} \quad (6.4-1)$$

$$f_{1c} = 0,95 f_{c,cyl} \quad (6.4-2)$$

Thereby $f_{c,cube}$ is the uniaxial compressive strength of cubes 150/150/150 mm and $f_{c,cyl}$ is the uniaxial compressive strength of cylinders ($\varnothing = 150$ mm, $h = 300$ mm).

The conversion of different concrete compressive strength is defined as follows:

- The cube strength $f_{c,cube}$ is the uniaxial compressive strength of cubes 150/150/150 mm. Conversion of different sizes of cubes:

$$f_{c,cube} = 1,05 f_{c,cube,200} \quad \text{and} \quad f_{c,cube} = 0,90 f_{c,cube,100} \quad (6.4-3)$$

- The cylinder strength $f_{c,cyl}$ is the compressive strength of cylinders ($\varnothing = 150$ mm, $h = 300$ mm) stored in water at $20^\circ \pm 2^\circ$ C and tested at the age of 28 days in acc. with ISO 1920, ISO 2736/2 and ISO 4012. Conversion of different sizes of cylinders:

$$f_{c,cyl} = 1,05 f_{c,cyl,100/300} \quad (\text{i.e. cylinder } \varnothing = 100 \text{ mm, } h = 300 \text{ mm}) \quad (6.4-4)$$

6.4.2 Concrete tensile strength

The concrete tensile strength should be taken as the uniaxial tensile strength f_{1ct} derived from splitting test or flexural test. Therefore the following priorities are defined:

1. The value for f_{1ct} derived from the splitting test should prevail.
2. If no splitting tests were carried out, the f_{1ct} derived from flexural test should be taken.
3. The value f_{1ct} derived from axial tensile tests are taken if no other tests are available.

The splitting tensile strength $f_{ct,sp}$ is determined at cylinders ($\varnothing = 150$ mm, $h = 300$ mm) acc. to ISO 4108. If conversion factors are not available f_{1ct} may be estimated from the mean splitting tensile strength $f_{ct,sp}$ according to eq. (2.1-5), [CEB-FIP MC 90 (1993)]:

$$f_{1ct} = 0,9 f_{ct,sp} \quad (6.4-5)$$

The axial tensile strength f_{1ct} may be estimated from the mean flexural tensile strength $f_{ct,fl}$ according to eq. (2.1-6), [CEB-FIP MC 90 (1993)]:

$$f_{1ct} = f_{ct,fl} \frac{1,5 (h/h_0)^{0,7}}{1 + 1,5 (h/h_0)^{0,7}} \quad (6.4-6)$$

where: h = depth of beam (in mm)
 h_0 = 100 mm

6.4.3 Calculation of tensile strength out of compressive strength

For some verifications in design or for an estimate of other concrete properties it is necessary to refer to a mean value of tensile strength f_{1ct} associated with a specified characteristic compressive strength. In this case f_{1ct} may be estimated from eq. (2.1-4), [CEB-FIP MC 90 (1993)]:

$$f_{1ct} = f_{ctko,m} \left(\frac{f_{c,cyl} - \Delta f}{f_{cko}} \right)^{2/3} \quad (\text{in MPa}) \leq C 50 \text{ acc. to MC 90} \quad (6.4-7)$$

where: $f_{ctko,m} = 1,40$ MPa
 $f_{cko} = 10$ MPa
 $\Delta f = 8$ MPa

For concrete grades beyond C 50 ($f_{c,cyl} = 50$ MPa + Δf) the equation should be replaced by the following equation given by Remmel (1994):

$$f_{1ct} = 2,12 \cdot \ln \left(1 + \frac{f_{c,cyl}}{10} \right) \quad (\text{in MPa}) > C 50 \text{ acc. to MC 90} \quad (6.4-8)$$

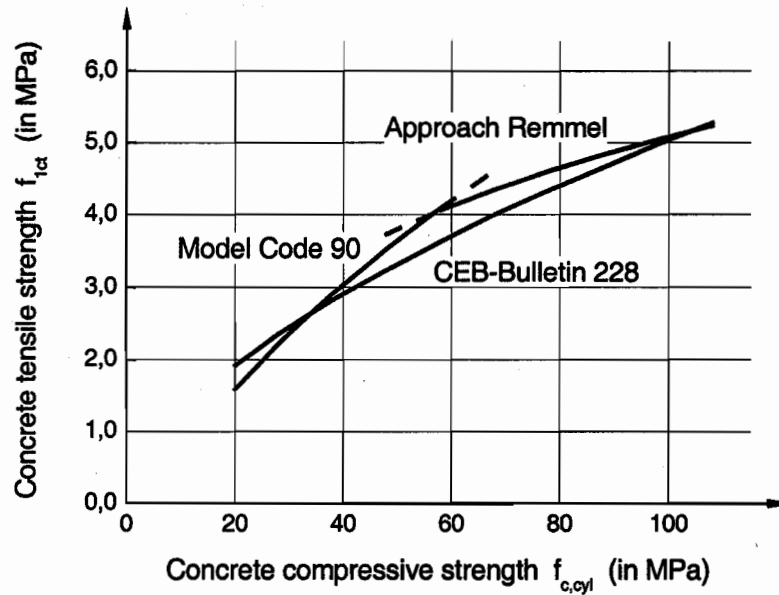


Fig. 6.4-1: Different approaches for determination of the uniaxial concrete tensile strength of concrete

An alternative regulation given by Reineck (1990 a) for concrete grades beyond C 50 leads to the same values (derivation less than 1%) for the concrete tensile strength:

$$f_{1ct} = 1,115 \cdot (f_{c,cyl} - \Delta f)^{1/3} \quad (\text{in MPa}) > C 50 \text{ acc. to MC 90} \quad (6.4-9)$$

6.4.4 Modulus of Elasticity

The modulus of elasticity E_c is defined as the tangent modulus of elasticity of the origin of the stress-strain diagram at a concrete age of 28 days acc. to eq. (2.1-16), [CEB-FIP MC 90 (1993)] (Note: in MC 90 E_c is called E_{ci}):

$$E_c = E_{c0} (f_{c,cyl} / f_{cm0})^{1/3} \quad (\text{in MPa}) \quad (6.4-10)$$

$$\begin{aligned} \text{where: } f_{cm0} &= 10 \text{ MPa} \\ E_{c0} &= 2,15 \cdot 10^4 \text{ MPa} \end{aligned}$$

Equation (2.1-16), [CEB-FIP MC 90 (1993)] is strictly valid up to concrete grade C 50. For concrete grades beyond this limit, the equation overestimate the modulus of elasticity of concrete and should be replaced by (eq. (2.1-15b), [CEB 228 (1995)]):

$$E_c = E_{c0} (f_{c,cyl} / f_{cm0})^{0,3} \quad (\text{in MPa}) \quad (6.4-11)$$

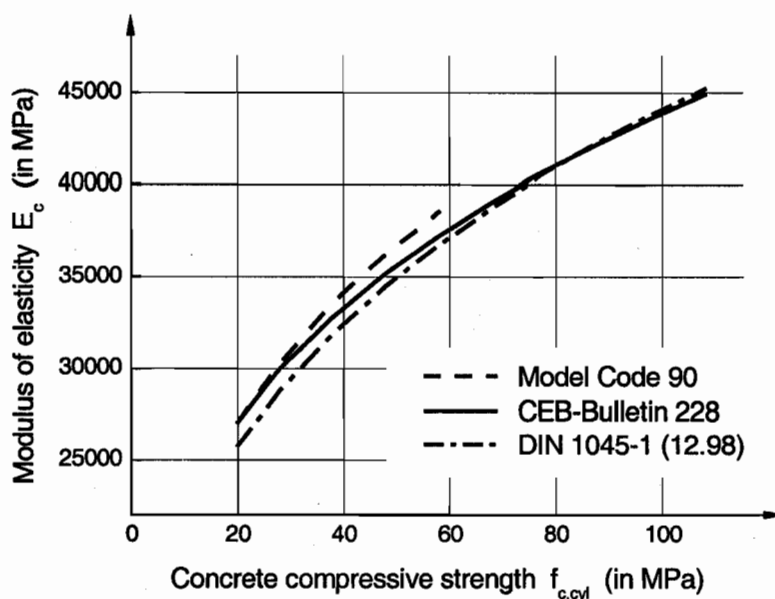


Fig. 6.4-2: Different approaches for determination of the modulus of elasticity of concrete

6.4.5 Fracture Energy

The fracture energy of concrete G_f is the energy required to propagate a tensile crack of unit area. According to the RILEM Draft Recommendation TC50-FMC (1994) the fracture energy G_f is determined on notched specimens loaded in flexure (eq. (2.1-7, [CEB-FIP MC 90 (1993)])):

$$G_f = G_{f0} \left(f_{c,cyl} / f_{cm0} \right)^{0.7} \text{ (in N/m)} \quad (6.4-12)$$

where: $f_{cm0} = 10 \text{ MPa}$

G_{f0} is the base value of fracture energy. It depends on the maximum aggregate size d_{max} as given in the following table:

d_{max} [mm]	G_{F0} [N/m]
8	25
16	30
32	38

Fracture energies for concrete strength over $f_{c,cyl} \approx 90 \text{ MPa}$ are lacking. The dependence of fracture energy on other parameters than aggregate size should be studied for all high strength concretes.

7 Comparison between mechanical models and test results of slabs without shear reinforcement

Markus Staller and Henrik Vocke

In this Chapter the ultimate punching loads predicted by selected mechanical models presented in Chapter 4 are compared with the results of punching tests without shear reinforcement collected in the databank (see Chapter 6). The analysis is carried out using a reviewed data base of 200 punching tests out of 250 tests collected from literature. The selection criteria for the reviewed data base are also presented in Chapter 6.2.

Table 7-1 shows a comparison of the experimental punching load to the calculated punching load $V_{\text{exp}}/V_{\text{cal}}$ in terms of mean value m , standard deviation s and coefficient of variation v . Table 7-1 is divided into three parts:

- In the first part, the comparison is carried out for the entire reviewed data base of 200 punching tests.
- In the second part, the comparison is carried out separately for tests where the maximum aggregate size d_a and the diameter of the tensile flexural reinforcement d_s were known and not known, respectively. This distinction is necessary for models where these parameters are used for the calculation of the punching load, namely the models of Men  trety, Hallgren and Staller. Where necessary, the maximum aggregate size was assumed to $d_a = 20$ mm and the diameter of the tensile flexural reinforcement was taken to $d_s = 10$ mm.
- In the third part, the comparison is carried out separately for normal strength concrete ($f_{lc} < 50$ MPa) and high strength concrete ($f_{lc} \geq 50$ MPa). This distinction is necessary for mechanical models which were not aimed for high strength concrete, namely the models of Kinnunen, Moe, Braestrup and Georgopoulos.

Para- meters	number of tests	$V_{\text{exp}}/V_{\text{cal}}$	Kinnunen	Moe	Braestrup	Georgo- poulos	Men��trety	Hallgren	Staller
all	200	m	1,174	1,057	0,864	1,167	1,294	1,030	0,939
		s	0,304	0,226	0,403	0,315	0,401	0,243	0,176
		v	0,259	0,214	0,466	0,270	0,310	0,236	0,188
d_a and d_s known	140	m	not applicable				1,213	1,049	0,942
		s					0,285	0,222	0,164
		v					0,235	0,212	0,174
d_a and d_s assumed	60	m					1,483	0,986	0,933
		s					0,548	0,282	0,204
		v					0,370	0,286	0,219
$f_{lc} < 50$	156	m	1,198	1,061	0,950	1,091	distinction not necessary		
		s	0,290	0,241	0,388	0,257			
		v	0,242	0,227	0,409	0,235			
$f_{lc} \geq 50$	44	m	1,087	1,043	0,455	1,433			
		s	0,337	0,167	0,114	0,359			
		v	0,310	0,160	0,252	0,251			
m = mean value, s = standard deviation, v = coefficient of variation									

Table 7-1: Summary of the statistical parameters from the comparison between mechanical models and punching test results

In addition, in figures 7-1 to 7-7 the values V_{exp}/V_{cal} are plotted for each mechanical model as a function of the uniaxial compressive concrete strength (prism strength) f_{1c} , the flexural reinforcement ratio ρ_l and the effective depth of the slab d . The figures are drawn for the entire reviewed data base of 200 punching tests. The following distinctions are contained in the figures:

- For the models of Kinnunen, Moe, Braestrup and Georgopoulos, different symbols were used for normal strength concrete ($f_{1c} < 50$ MPa) and high strength concrete ($f_{1c} \geq 50$ MPa), respectively (figures 7-1 to 7-4).
- For the models of Menétrey, Hallgren and Staller, different symbols were used for tests where the maximum aggregate size d_a and the diameter of the tensile flexural reinforcement d_s were known and not known, respectively (figures 7-4 to 7-7).

The approximately horizontal trendlines in fig. 7-7 demonstrate that the model of Staller (2000) leads to a good estimation of the three parameters f_{1c} , ρ_l and d . Table 7-1 shows that this model leads to the lowest coefficient of variation $v = 0,188$ (entire reviewed data base) of the value V_{exp}/V_{calc} . This can be explained among other things by the fact that the model by Staller is an empirical model which was derived from a similar data base [Staller (2000)]. As a result, the model of Staller leads to a coefficient of variation v which is comparable to the values of current empirical design code formulas (see table 8-2 in section 8.1.4).

In Staller's model, the geometrical reinforcement ratio ρ_l is taken into account in terms of the mechanical reinforcement ratio $\omega^{1/3} = (\rho_l \cdot f_y / f_{1c})^{1/3}$. In contrast to other models, this approach leads to a very good prediction of the influence of the geometrical reinforcement ratio ρ_l (figure 7-1b). According to table 7-1, the model is not very sensitive to unknown or assumed values of the maximum aggregate size d_a .

In fig. 7-6 it can be seen that the Modified Mechanical Model by Hallgren (1996) also gives a good prediction of the experimental results and that the main parameters are monitored correctly. Moreover, the coefficient of variation $v = 0,236$ for V_{exp}/V_{calc} is relatively low. Compared to that, the model of Kinnunen, Nylander (1960, 1961) that was the basis for Hallgren's model leads to a coefficient of variation $v = 0,259$ for V_{exp}/V_{calc} which is slightly higher than with the model of Hallgren. This is first of all due to the fact that Hallgren introduced into his model a size effect law based on nonlinear fracture mechanics. Therefore, the trendline for the effective depth in figure 7-6a is approximately horizontal while the corresponding trendline for Kinnunen/Nylanders's model (figure 7-1a) is inclined leading to unsafe values for higher effective depths. However, fig. 7-1 shows that the original model by Kinnunen/Nylander leads to reasonable results, especially when higher concrete strengths are not considered.

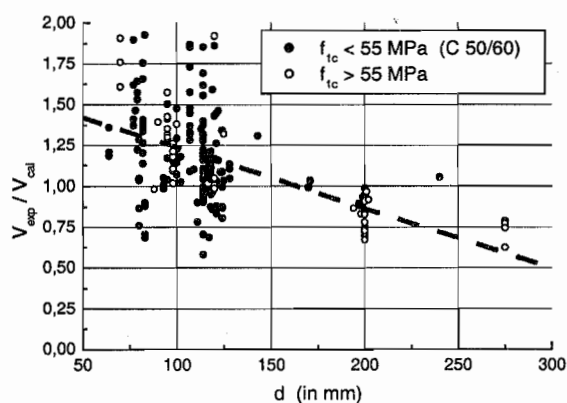
According to fig. 7-5 the model of Menétrey (1994) slightly underestimates the punching capacity, but on the other hand correctly predicts the influence of the three main parameters. Table 7-1 shows that coefficient of variation for V_{exp}/V_{calc} is clearly influenced by unknown or assumed values of the maximum aggregate size d_a and the diameter of the tensile flexural reinforcement d_s . For the tests where all parameters are known, the coefficient reaches a relatively low value of $v = 0,235$. The results with the model of Menétrey which is based on the tensile strength of concrete could be more accurate if the tensile strength was given for all experimental results. However, Menétrey's assumption of a constant tensile stress along the punching crack means a simplification. Finally, it must be pointed out that the model of Menétrey is the only model to also account for shear reinforcement and prestressing.

The model of Moe (1961) leads to surprisingly good results in terms of both mean value and coefficient of variation (table 7-1). It is remarkable that the model of Moe is able to realistically predict the influence of high strength concrete as well as the size effect (fig. 7-2) even though the model dates back to 1961. This is even more astonishing knowing that the model of Moe was primarily derived on empirical basis. However, Moe's basic approach in eq. (3.2-3) where he assumes an interaction between the flexural and shear capacity seems to be the key to his realistic prediction. In addition, he already used the mechanical reinforcement ratio $\omega = \rho_l f_y / f_c$ instead the geometrical reinforcement ratio ρ_l what improved the predictions for high strength concrete and high reinforcement ratios.

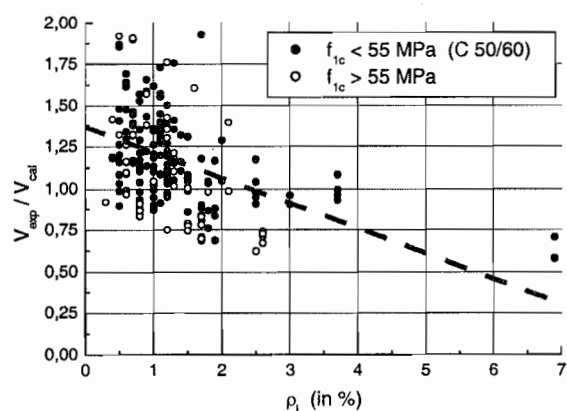
The plasticity model of Braestrup, Nielsen et al. (1976) generally overestimates the experimental punching strength and shows the highest coefficient of variation for V_{exp}/V_{cal} (table 7-1). Since the influence of the reinforcement ratio is ignored in the model, the prediction of its influence fails (fig. 7-3b). Moreover, the comparison with the test results in fig. 7-3c shows that the model underestimates the punching capacity with lower concrete strength and overestimates it with higher concrete strength. This is probably due to the linear influence of the concrete compressive strength f_c in the model.

The results obtained with the model of Georgopoulos (1987) show relatively low coefficients of variation v but strongly depend on the concrete compressive strength. Table 7-1 indicates for normal strength concrete a mean value $m = 1,091$ for V_{exp}/V_{cal} , while for high strength concrete a mean value $m = 1,433$ is calculated. This is probably due to the fact that with the model of Georgopoulos high concrete strengths lead to very steep inclinations α of the assumed punching cone (see section 4.3) and therefore to relatively low vertical components of the tensile stresses in the punching crack. However, fig. 7-4 demonstrates that with the model of Georgopoulos the size effect as well as the influence of the reinforcement ratio are monitored correctly.

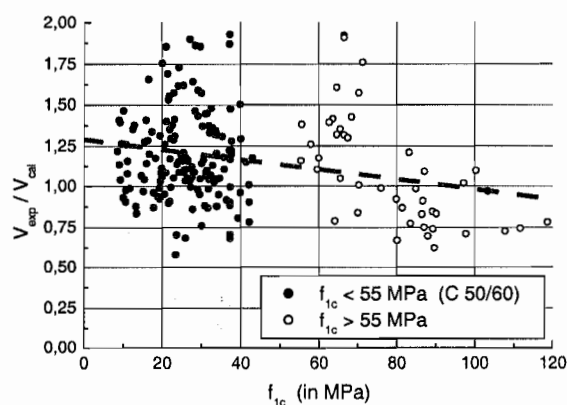
Model of Kinnunen / Nylander



a) effective depth



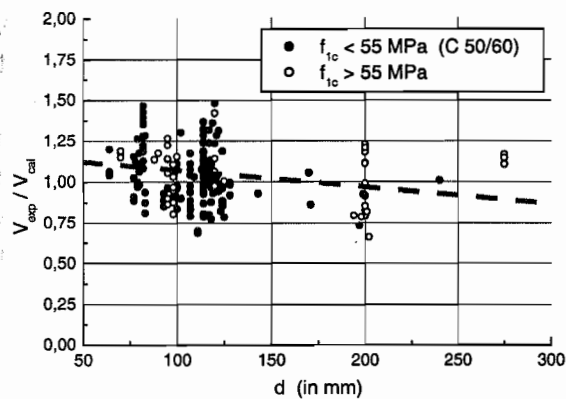
b) geometrical ratio of tensile reinforcement



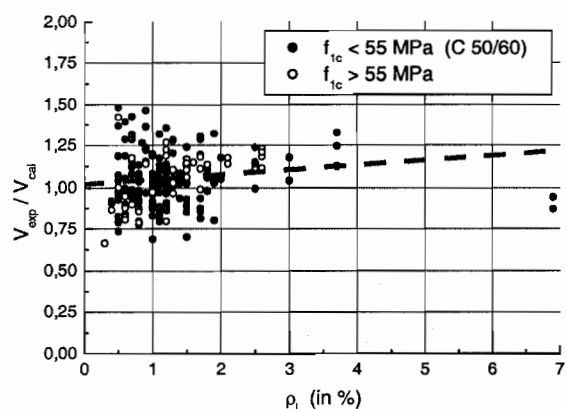
c) concrete compressive strength

Fig. 7-1: Comparison of Model Kinnunen / Nylander and experimental data

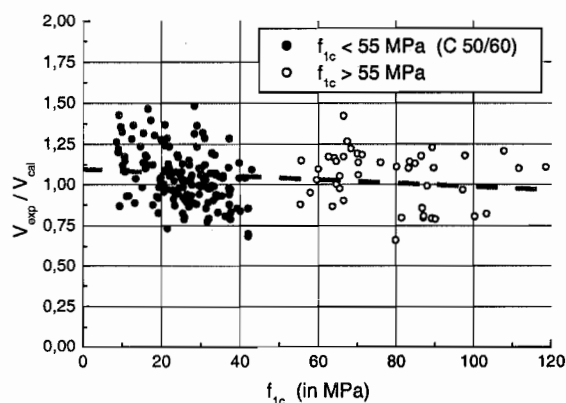
Model of Moe



a) effective depth



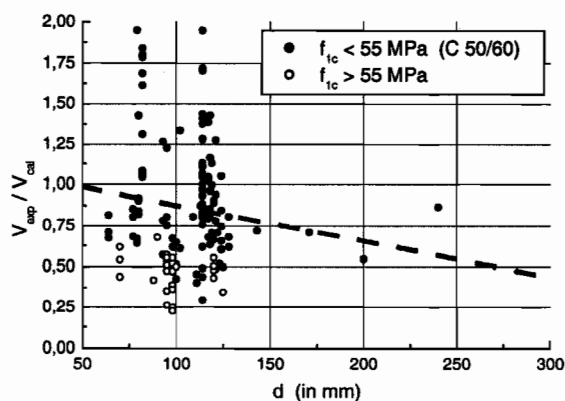
b) geometrical ratio of tensile reinforcement



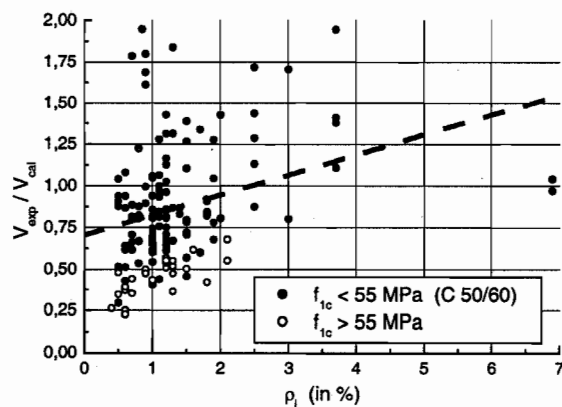
c) concrete compressive strength

Fig. 7-2: Comparison of Model Moe and experimental data

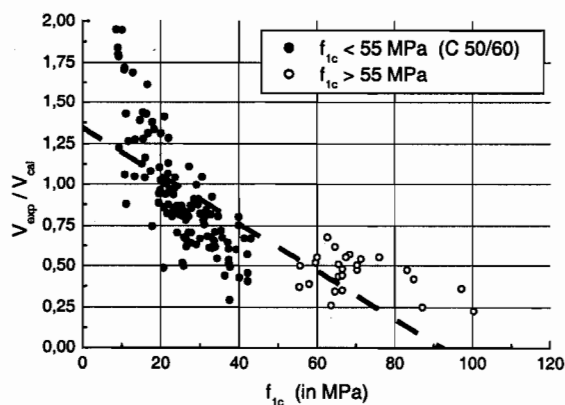
Model of Braestrup/Nielsen et al.



a) effective depth



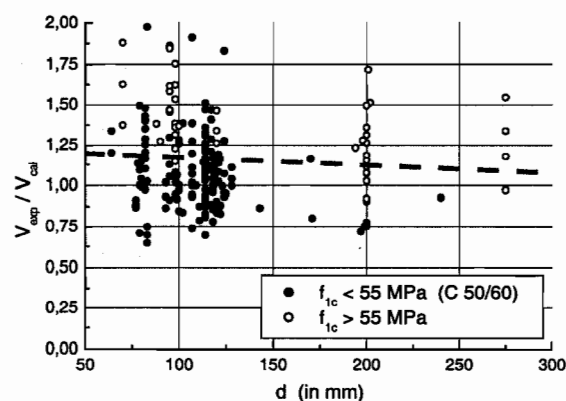
b) geometrical ratio of tensile reinforcement



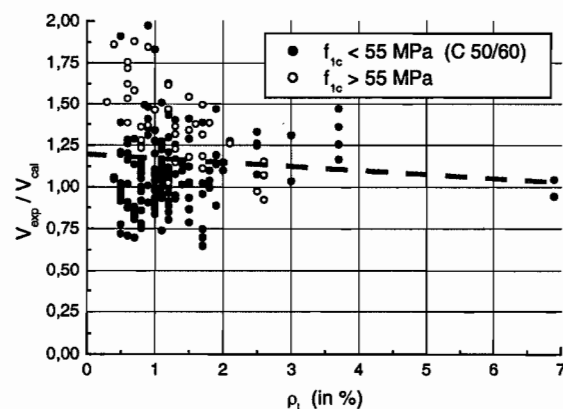
c) concrete compressive strength

Fig. 7-3: Comparison of Model Braestrup/Nielsen et al. and experimental data

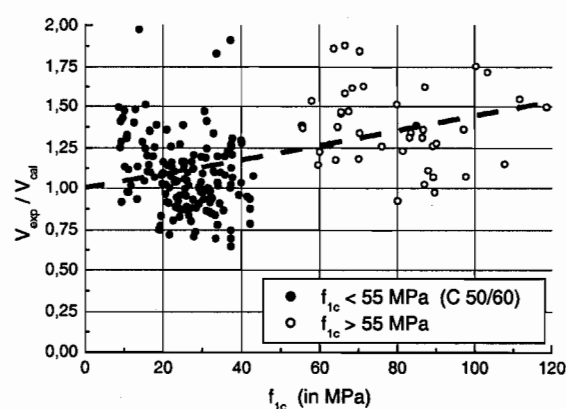
Model of Georgopoulos



a) effective depth



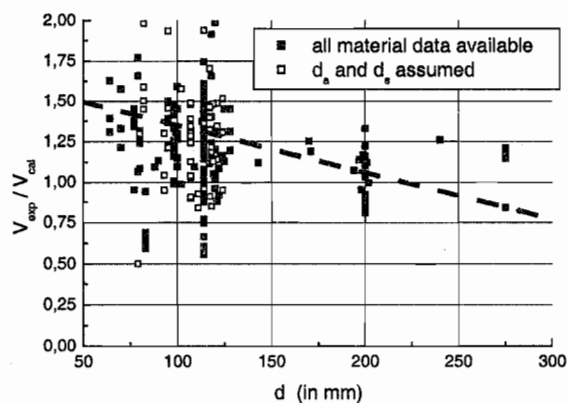
b) geometrical ratio of tensile reinforcement



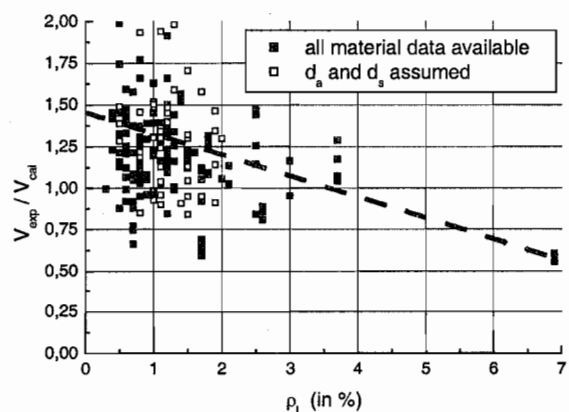
c) concrete compressive strength

Fig. 7-4: Comparison of Model Georgopoulos and experimental data

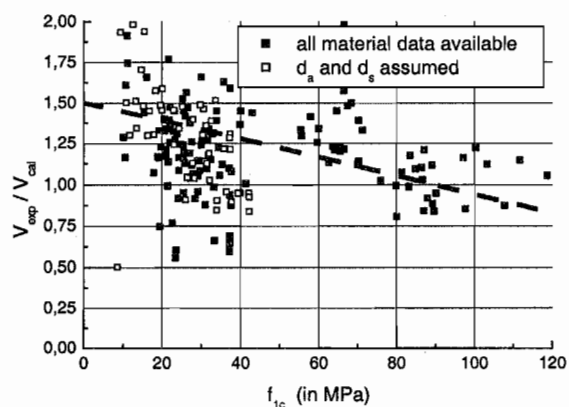
Model of Men  trety



a) effective depth



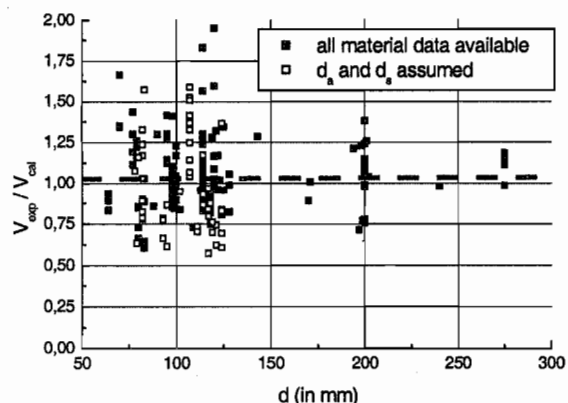
b) geometrical ratio of tensile reinforcement



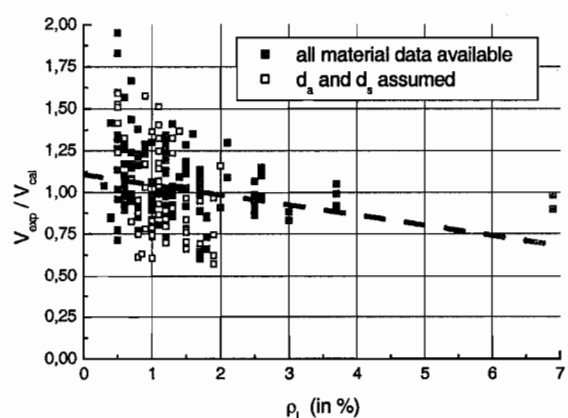
c) concrete compressive strength

Fig. 7-5: Comparison of Model Men  trety and experimental data

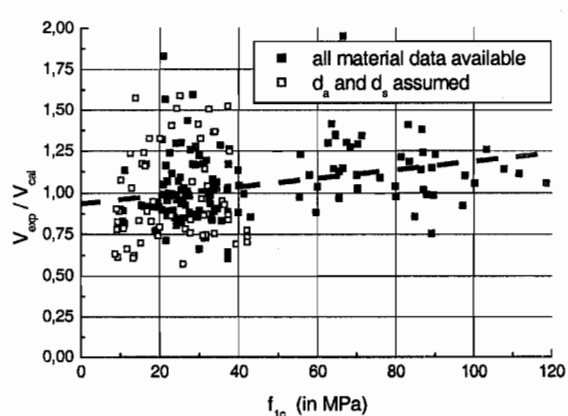
Modified Mechanical Model of Hallgren



a) effective depth



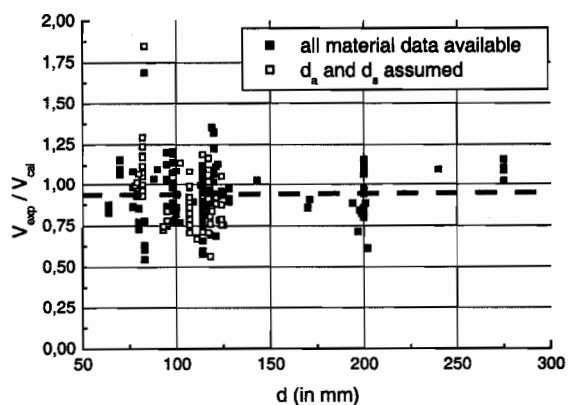
b) geometrical ratio of tensile reinforcement



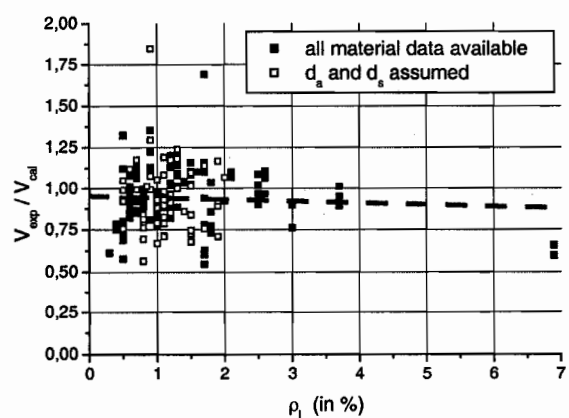
c) concrete compressive strength

Fig. 7-6: Comparison of Model Hallgren and experimental data

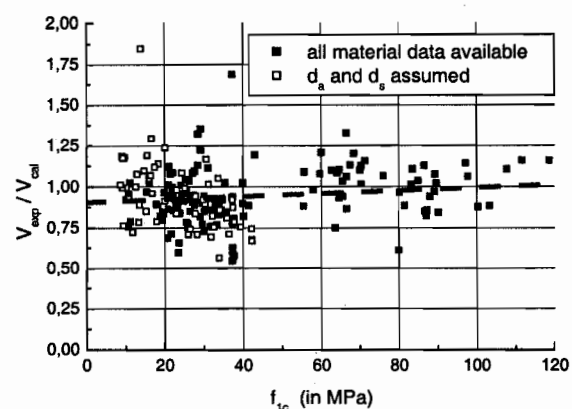
Empirical Model of Staller



a) effective depth



b) geometrical ratio of tensile reinforcement



c) concrete compressive strength

Fig. 7-7: Comparison of empirical Model Staller and experimental data

8 Comparisons between code rules and tests for flat slabs without shear reinforcement

Rüdiger Beutel, Markus Staller

8.1 Introduction

The punching databank without shear reinforcement “I-2” contains 205 punching tests. The comparison between these tests and selected codes (Chapter 2) is based on the ratio of the observed failure load V_{Test} and the punching resistance $V_{Ru,code}$ at ultimate limit state. This ratio is calculated for each test and statistically evaluated. Additionally, all individual results are archived in the databank “III”. Finally, all individual results $V_{Test} / V_{Ru,code}$ are printed versus the main punching parameters (d , ρ_b , f_c) to give some information about the trend of the approximation between tests and codes.

For this statistical comparison firstly a clear specification of the punching failure in the test is necessary. Therefore, in the following a bending failure is checked arithmetically and bond failures at end supports or individual influences are checked by evaluating the reports. Secondly, the quotient $V_{Test} / V_{Ru,code}$ has to be comparable. This comparability is evident if all punching parameters are considered by the codes. Unfortunately the shear slenderness is not considered in all codes. Therefore, the statistical evaluations were limited to slender specimen.

The numerical calculation of the quotient $V_{Test} / V_{Ru,code}$ is based on the following principles:

- (1) The code equations are solved with mean values of the material strength. Characteristic values are not taken into account because any material over-strength is considered in a mechanically correct way, and secondly the differences between site- and laboratory-conditions are approximately considered. Therefore the safety factor $\xi = V_{Test} / V_{u,Code}$ tends to be conservative.
- (2) The concrete strength is considered as a short time strength. A reduction due to the time dependent behaviour of concrete was not taken into account as the loading time period was not documented for most of the tests. Therefore, the safety factor tends to be conservative, too.
- (3) An upper limit of the flexural reinforcement ratio was not taken into account although some codes give a limit for it. This was necessary because a lot of tests contained a high flexural reinforcement ratio to avoid a flexural failure (Chapter 6), and because it is well known, that the flexural reinforcement ratio influences the punching shear capacity.
- (4) An upper limit for the size-effect was considered for an effective depth less than 200 mm if this parameter is required. Therefore, all evaluations for codes which do not consider a size effect law tend to be progressive.

In section 8.5 a procedure is proposed permitting the evaluation of experimental data from various test sites with respect to the usability for the analysis and description of mechanical or empirical failure-models and code rules. For this purpose statistical adaptability tests for normal and lognormal distribution functions and tests for outliers are presented as well as variance- and mean-value tests. According to the applied methods, the data of objectively selected experimental values are rendered at disposal for further evaluation. A focused assessment of quality and reliability of semi-empirical design equations in codes, i.e. DIN 1045-1 or Model Code 90 with reliable and economic viability is carried out by evaluating test data of the data bank “I-2” given in the appendix of this bulletin.

8.2 Compilation of a representative and comparable data base

The test series show a large spectrum of investigated parameters. To achieve a representative and comparable set of test data the following failure types, geometrical and mechanical influences have to be considered. Data not fulfilling the required terms have to be disregarded for further evaluation.

(a) Flexural failures

The experimental study of Kuang and Morley (1992) shows that an arithmetical classification of a pure flexural failure is only possible if significant membrane forces are not existing. Most of all isolated flat slab specimens were tested by a set-up which excluded external membrane forces (Chapter 10.1). Internal membrane forces because of an arch shear transfer mechanism or direct strut could not be fully excluded. Therefore, a pure flexural failure was characterised by a ratio $V_{Test} / V_{u,flex}$ greater than 1,15 ($V_{Test} / V_{u,flex} > 1,15$). $V_{u,flex}$ represents the punching shear capacity of a plasticity approach. A flexural punching failure was defined by $0,95 < V_{Test} / V_{u,flex} < 1,15$ and a brittle punching failure by $V_{Test} / V_{u,flex} \leq 0,95$.

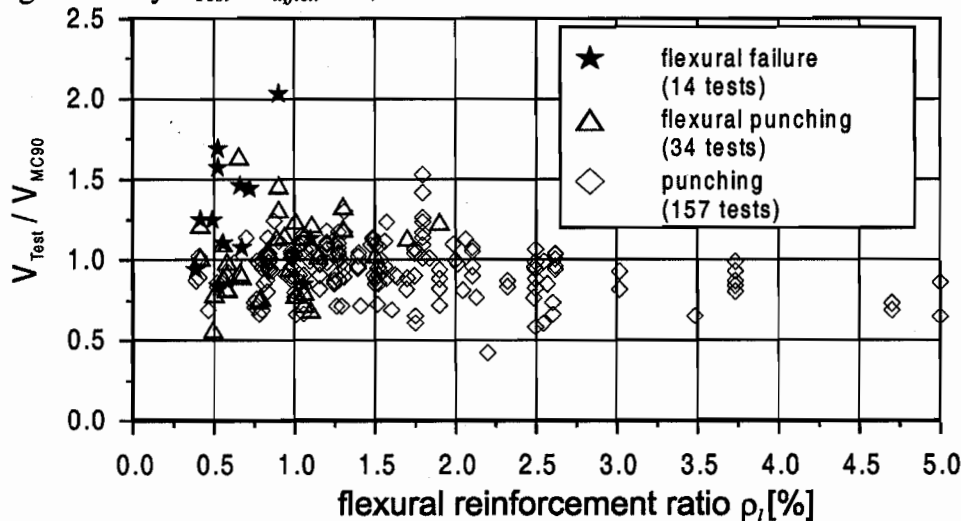


Figure 8-1: Compilation of a flexural, a flexural punching and a brittle punching failure according to the punching shear capacity of Model Code 90 - $V_{Test} / V_{Ru,MC 90}$ versus the flexural reinforcement ratio ρ_f

From Figure 8-1 it can be observed that pure flexural failures (*) tend to higher quotients than flexure or brittle punching failures. Therefore, 14 tests which failed due to pure flexure were excluded from further evaluations.

(b) Bond failure at end supports

Bond failures at end supports were only indicated by the reports. Arithmetical checks were not installed because in many cases the description of the test set-up or of the flexural reinforcement strains was not sufficient. Therefore, tests which failed due to a bond failure at end supports were not considered in further evaluations (databank "I-2").

(c) Influence of the shear slenderness

The databank contains tests with a shear slenderness (a/d) between 1 and 14. The slenderness a/d considers the moment-shear force relationship. This ratio depends on the radial distance from the column face to the bearing points, a and the effective depth, d .

From uniaxial shear tests it is well known that the shear resisting capacity is influenced by the shear slenderness. Therefore, 13 specimens which were tested by a shear slenderness less than four were excluded from further evaluations, too.

(d) Individual influences

The evaluation of the 25 tests by Elstner and Hognestand (1956) leads to a systematic low mean value of the punching shear capacity according to all considered codes. This could be due to the concrete technology in 1956 and due to the fact that the flexural reinforcement was less effectively ribbed. Four tests of Ladner/ Schaeidt/ Gut (1977) with a real slab system result in a systematically high punching shear capacity. All these 29 tests were excluded, too.

Summary

The check of the 205 punching tests without shear reinforcement resulted in a representative and comparable data base of $205 - 14 - 13 - 29 = 149$ tests without shear reinforcement. The number of the collected tests was reduced to about 73 % of the collected data.

8.3 Statistical parameters and required safety level

Table 8.3-1 shows the definitions of all considered statistical parameters. An analytical approximation of the test failure loads by the codes would be characterised by a safety ratio $\xi = 1,0$ and a standard deviation $\sigma = 0$. But these values are statistically unattainable. Therefore, a good approximation of the tests by the codes is characterised by a small value of σ and v .

Table 8.3-1: Compiling and definition of the statistic parameters

Parameter	Explanation
n/n_0	Ratio of the number of tests with equal failure types n to the number of all tests of the representative database n_0 .
$\xi_i = \frac{V_{Test}}{V_{u,calc}}; \xi_m = \frac{1}{n} \cdot \sum \xi_i$	Individual and mean value of safety ratio punching without shear reinforcement: $V_{u,calc} = V_c$
$\sigma = \sqrt{\frac{1}{n-1} \cdot \sum (\xi_i - \xi_m)^2}$	Standard deviation
$v = \frac{\sigma}{\xi_m}$	Variation coefficient
$\xi_{5\%} = \xi_m - 1,645 \cdot \sigma$	5% fractile- characteristic value of the safety ratio according to Eurocode 1

A general statement regarding the required safety level of different design code equations is not possible, because a uniform definition is not available. Therefore, this Chapter takes into account the required resistance safety level of Eurocode 1, Annex D for all codes. Eurocode 1 enables the following two ways to calculate the required resistance safety level:

Method (1): Assessment via the characteristic value:

This method demands the characteristic value R_k on the 5% fractile based on a statistical normal distribution. The partial material safety factor γ_m can be calculated by equation(8-1).

$$R_d = \frac{R_k}{\gamma_m} \quad (8-1)$$

Method (2): Direct assessment of the design value:

This method calculates the design value by a probabilistic procedure. Therefore, the criteria for a safe design resistance R_d is evaluated by a failure probability. The partial safety factor concept requires a separation of the reliability effects for actions and resistance. This behaviour could be considered by the weight factors α_R and α_S .

The results of Method (1) are presented in Chapter 8.4 and the results of Method (2) in Chapter 8.5. If the design strength is judged only by laboratory tests than special attention has to be paid to the differences between the laboratory and the building site conditions (see Chapter 8.3).

8.4 Comparisons between code rules and tests based on mean values of the material strength

In this Chapter 149 laboratory punching tests without shear reinforcement are compared with the ultimate punching shear capacities of BS 8110-97, ACI 318-95, Eurocode 2- Part 1, Model Code 90, DIN 1045 and the new German design code DIN 1045-1. The considered design equations and partial material safety factors of each code were mentioned in Chapter 2.

Code design capacities R_d depend on characteristic values of the material strength. A characteristic material strength is a reference strength (e.g. Model Code 90: $f_{ck} = f_{cm}-8$) to capture the unavoidable scattering of the material behaviour and placing circumstances on site. This influences must be considered if safety levels of the codes are judged by tests, too. Therefore, all comparisons in this chapter are based on mean values of the concrete compressive strength and yield strength of the reinforcement. This means, that all parameters in the design formulas which are based on characteristic values are replaced by parameters based on mean values. The evaluation of the resistance based on mean values has the advantage that a material overstrength is considered mechanically correctly but generally the calculated safety factor ξ_m tends to be conservative. Finally, the safety level $\xi_{5\%}$ (Table 8.3-1, line 5) contains the scattering of the punching strength which is only based on the well defined laboratory conditions. Influences on site, which reduce the resistant capacity are not calculated. Therefore, this conservative approach based on mean values considers approximately the neglected influences on site.

Some codes e.g. Eurocode 2 or DIN 1045(88) restrict the lower and the upper value of the flexural reinforcement ratio. The upper limit of this codes is fixed by 1,5 % to guarantee a semi-ductile punching behaviour. In contrast to this code restriction, a lot of tests were reinforced with a flexural reinforcement ratio between 1,5 % and 2,5 %. If this discrepancy is neglected the safety analysis tends to be progressive because a high flexural reinforcement ratio increases the punching strength.

The comparisons between tests and each code were subdivided into the following steps:

- (a) The application limits of each code were considered. E.g. DIN 1045(88) is not applicable for high strength concrete. Therefore, the comparisons for DIN 1045 neglected all tests with a concrete compressive cylinder strength higher than 55 MPa.
- (b) The application rules of each code were neglected. Therefore, the comparisons considered all 149 comparable tests.
- (c) Finally, the necessity of the application rules could be identified by the statistical parameters σ and v . If step (b) increases this statistical parameters, the application rules are confirmed. Otherwise, these rules can be less restricted.

All statistical results of the comparisons between tests and codes are arranged in table 8.3-2. Row 2 shows the considered number of tests in contrast to the number of comparable tests. In the case of the German code DIN 1045(88) 65 of 149 tests must be neglected because a concrete strength $f_c > 50$ MPa and a flexural reinforcement ratio $\rho_l < 0,5\%$ are excluded by the application rules. DIN 1045(88) overestimates the influence of a concrete strength f_c greater than 50 MPa

(Figure 8.4-1(c)). Therefore, the design rules of DIN 1045(88) are limited to normal strength concrete. The 5% fractile $\xi_{5\%}$ of the safety factor results in 0,84 and this value is less than the required value of 1,0. The reason for this safety lack according to Eurocode 1 is found due to the approximation of the influence of the concrete strength. The influences of the effective depth and the flexural reinforcement ratio are considered to be acceptable (Figure 8.4-1(a) and (b)). The safety standard according to Eurocode 1 will be reached if the punching strength is reduced about 16%.

Table 8.3-2: Statistic parameters of the punching shear resistance without shear reinforcement for seven design codes

Code	n/n_0	ξ_m	σ	v	$\xi_{5\%}$
DIN 1045(88)	84/ 149	1,22	0,23	0,19	0,84
Model Code 90, FIP Recommendations	149/ 149	0,98	0,16	0,16	0,72
British Standard 8110-97	149/ 149	1,03	0,17	0,16	0,76
Eurocode 2, Part 1 (1992)	112/ 149	1,28	0,25	0,20	0,87
ACI 318-95 (1995)	149/ 149	1,29	0,27	0,21	0,85
DIN 1045-1 (2000)	149/ 149	1,38	0,23	0,17	1,00

Model Code 90, FIP-Recommendations and BS 8110 show the best approximation of the mean punching strength over all parameters (Figure 8.4-2(a) to (c) and 8.4-3 (a) to (c)) but the demanded 5% design value is not reached. This design value will be reached if the shear capacity is decreased about 27% for Model Code 90 or FIP-Recommendations and about 24% in the case of BS 8110. The limitation of BS 8110 regarding the concrete strength to 40 MPa is not necessary.

The evaluation for Eurocode 2, Part 1 considers 112 tests because 37 high strength concrete tests were excluded by the application rules. This limitation of the concrete strength is equal to DIN 1045(88). It has to be pointed out that in the case of Eurocode 2 less tests could be neglected because the design equation is more robust than the one of DIN 1045(88). From Figure 8.4-4 (b) it can be observed that Eurocode 2 underestimates the influence of a high flexural reinforcement. Additionally, it is shown that the influence of a high concrete compressive strength is overestimated (Fig. 8.4-4). Therefore, the recommendations by Kordina (1994) are confirmed (Chapter 2). The safety standard of Eurocode 1 will be reached if the concrete shear strength is reduced about 13%.

ACI 318-95 neglects the influence of the flexural reinforcement ratio. Therefore the punching capacity tends to be conservative for a flexural reinforcement ratio greater than 1,5 % (Fig. 8.4-5 (a)). The wide scattering of ratio ξ founds well to the ignored flexural reinforcement ratio. This leads to the highest standard deviation and variation coefficient. Therefore the 5% fractile does not reach the demanded safety level according to Eurocode 1. It is recommended to consider the flexural reinforcement ratio. The demanded safety level of Eurocode 1 will be reached if the punching strength of ACI-318-95 is reduced about 25%.

Finally, the new German design code DIN 1045-1 is presented. This punching design equations were calibrated on the presented punching data bank. Therefore, it is not surprising that the 5% fractile is equal to the required value of 1,0. The design equations of Model Code 90 and DIN 1045-1 are approximately equal (compare Equation 8.3-2 with the design formula in Figure 2-3) because Model Code 90 shows the best approximation of the tests for mean values. In contrast to Model Code 90 the 5% fractile of DIN 1045-1 reaches the required value because the punching shear capacity was reduced by a smaller control perimeter. The established distance between the control perimeter and the column face is fixed to 1,5 d (comparable to Eurocode 2). This value is 25% smaller than the required distance of Model Code 90 (2,0 d).

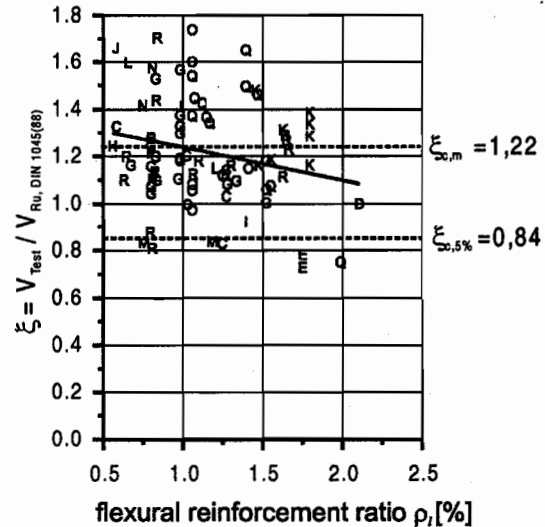
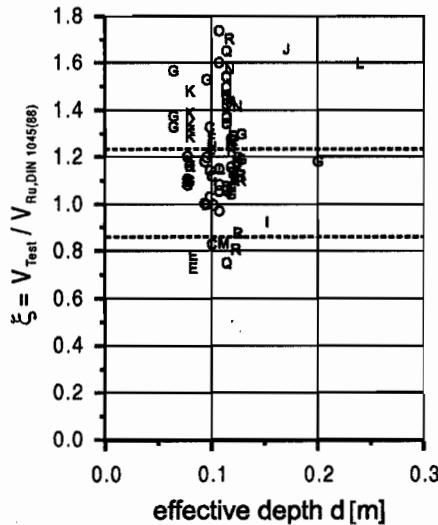
$$V_{Ru,DIN1045-1} = \gamma_m \cdot v_{Rd,ct} \cdot u_{krit} \quad [\text{MN}] \quad (8.3-1)$$

$$v_{Rd,ct} = 0,12 \cdot (100 \cdot \rho_l \cdot f_{ck})^{1/3} \cdot \kappa \cdot d \quad [\text{MN/m}] \quad (8.3-2)$$

where: $\gamma_m = 1,5$, $\kappa = 1 + \sqrt{\frac{200}{d}} \leq 2,0$, d in mm
for circular columns: $u_{krit} = c_2 + 3 \cdot d \cdot \pi$ [m]

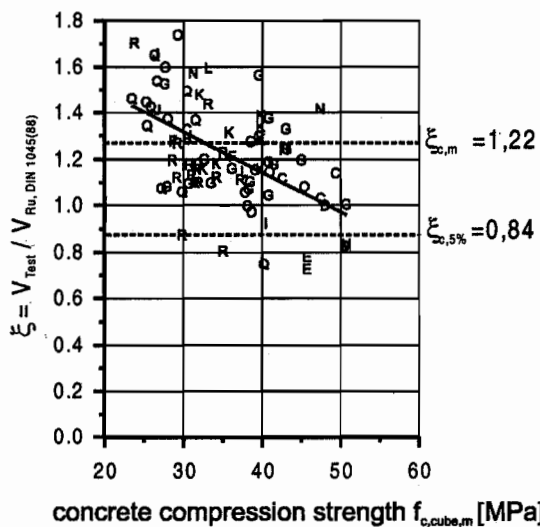
8.4.1 Plots of the comparisons between punching tests and codes

8.4.1.1 DIN 1045 (88)



a) $V_{Test} / V_{Ru,DIN 1045(88)}$ versus effective depth

b) $V_{Test} / V_{Ru,DIN 1045(88)}$ versus flexural reinforcement ratio



c) $V_{Test} / V_{Ru,DIN 1045(88)}$ versus concrete compression cube strength

number of considered tests: 84

— linear regression

C	Ramdane	1993
D	Marzouk/ Hussein	1991
E	Lovrovich/ McLean	1990
F	Tolf	1988
G	Regan	1986
H	Swamy/ Ali	1982
I	ETH-Zürich	1977/ 1979
J	Schaefers	1978
K	Ladner/ Schaeidt/ Gut	1977
L	Ladner et al.	1970/ 1973
M	Corley/ Hawkins	1968
N	Base	1966
O	Manterola	1966
Q	Moe	1961
R	Kinnunen/ Nylander	1960

Figure 8.4-1: Comparisons between 84 punching tests without shear reinforcement and the punching shear capacity of DIN 1045(88)

8.4.1.2 Model Code 90 and FIP Recommendation

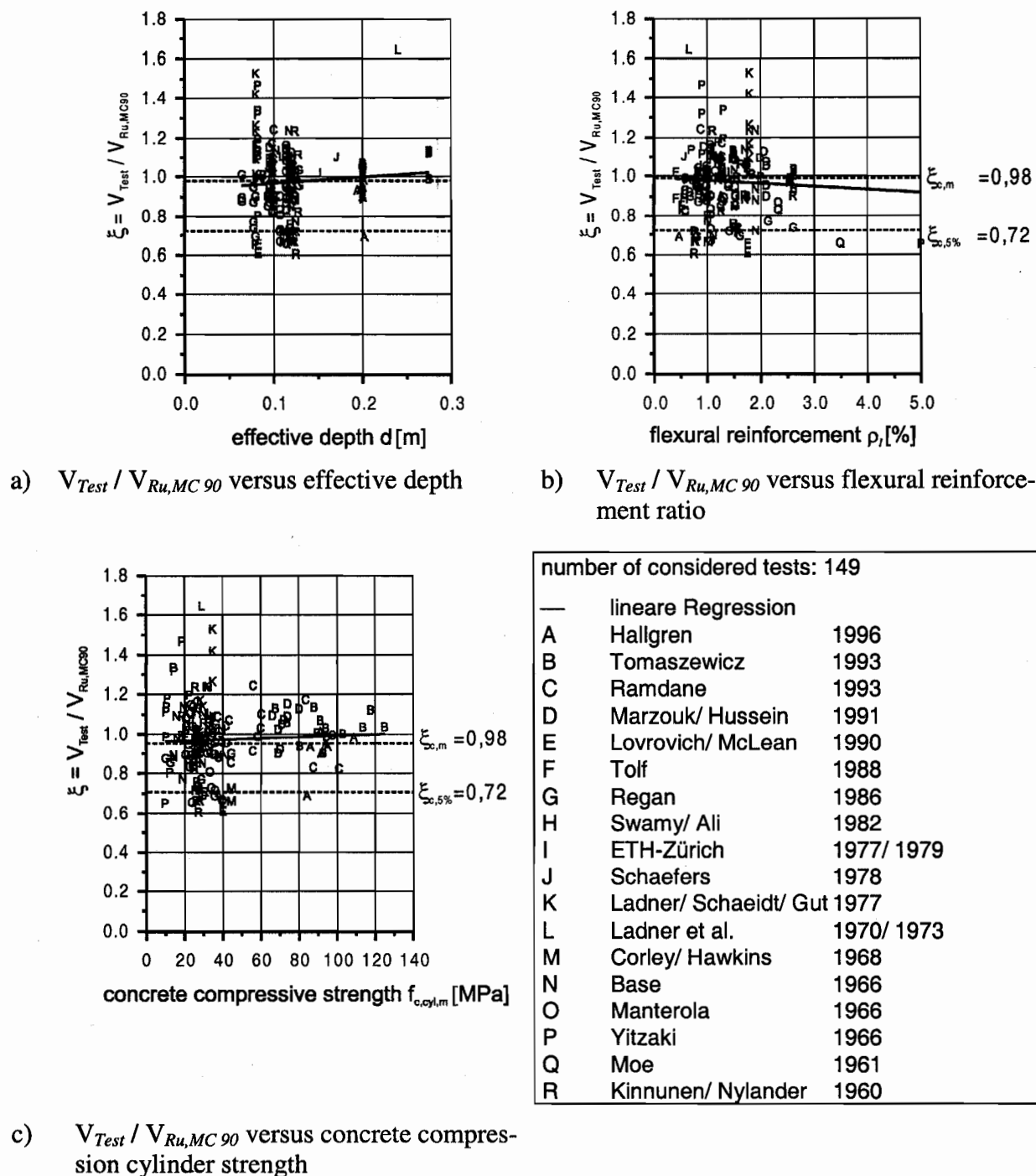
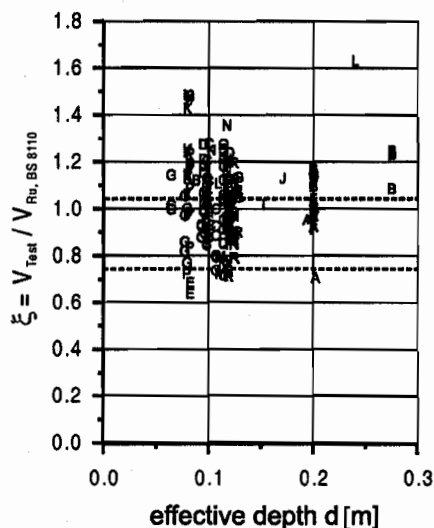
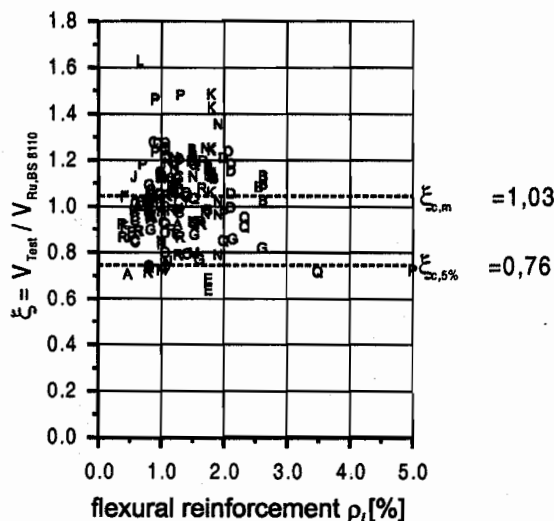


Figure 8.4-2: Comparisons between 149 punching tests without shear reinforcement and the punching shear capacity of CEB/ FIP Model Code 90 and FIP Recommendations

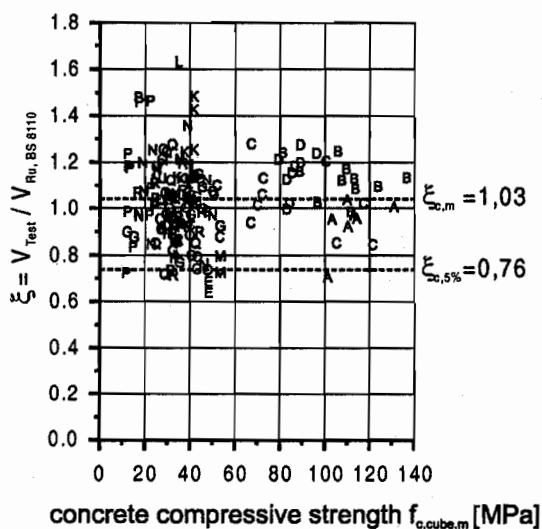
8.4.1.3 British Standard BS 8110-97



a) $V_{Test} / V_{Ru, BS 8110-97}$ versus effective depth



b) $V_{Test} / V_{Ru, BS 8110-97}$ versus flexural reinforcement ratio



c) $V_{Test} / V_{Ru, BS 8110-97}$ versus concrete compression cube strength

number of considered tests: 149

—	linear Regression	
A	Hallgren	1996
B	Tomaszewicz	1993
C	Ramdane	1993
D	Marzouk/ Hussein	1991
E	Lovrovich/ McLean	1990
F	Tolf	1988
G	Regan	1986
H	Swamy/ Ali	1982
I	ETH-Zürich	1977/ 1979
J	Schaefer	1978
K	Ladner/ Schaeidt/ Gut	1977
L	Ladner et al.	1970/ 1973
M	Corley/ Hawkins	1968
N	Base	1966
O	Manterola	1966
P	Yitzaki	1966
Q	Moe	1961
R	Kinnunen/ Nylander	1960

Figure 8.4-3: Comparisons between 149 punching tests without shear reinforcement and the punching shear capacity of BS 8110-97

8.4.1.4 Eurocode 2, Part 1

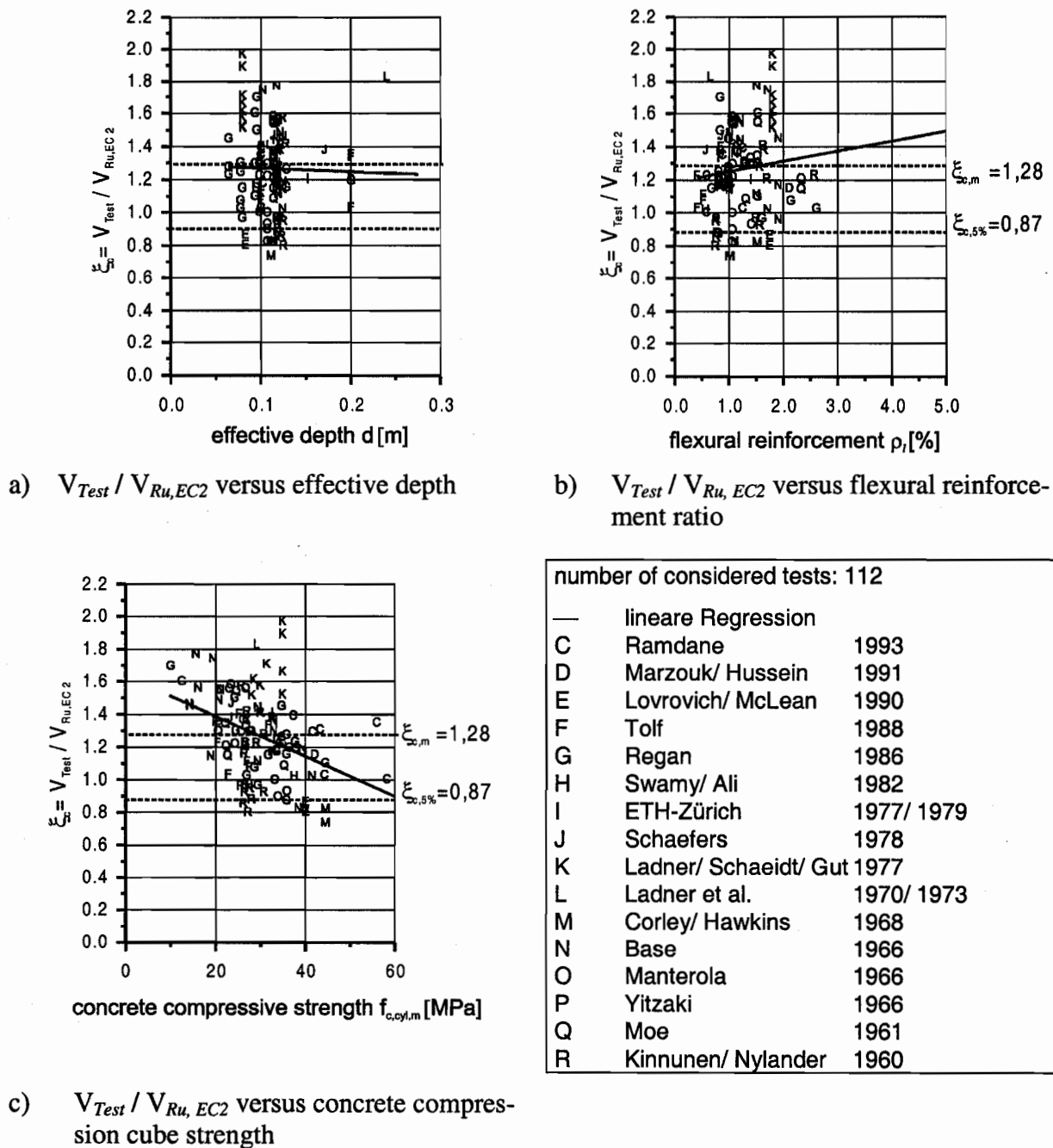
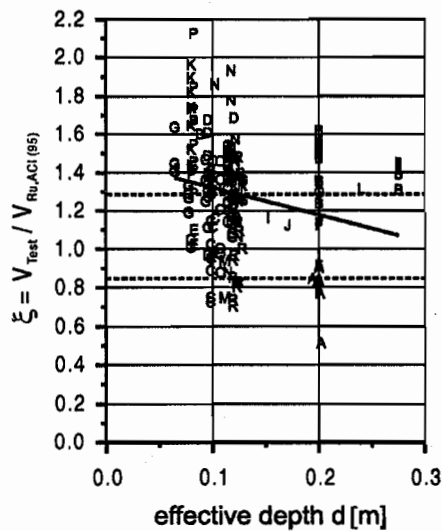
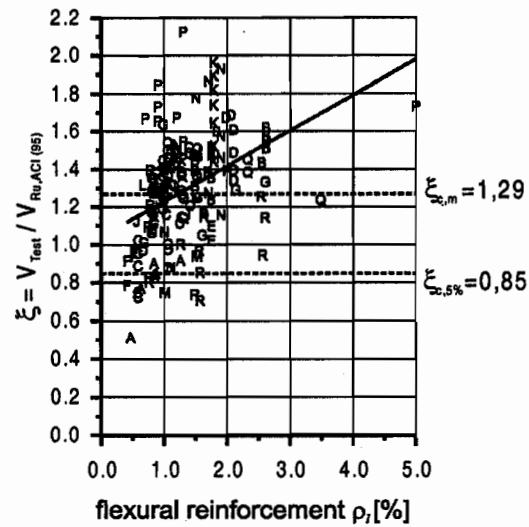


Figure 8.4-4: Comparisons between 112 punching tests without shear reinforcement and the punching shear capacity of Eurocode 2

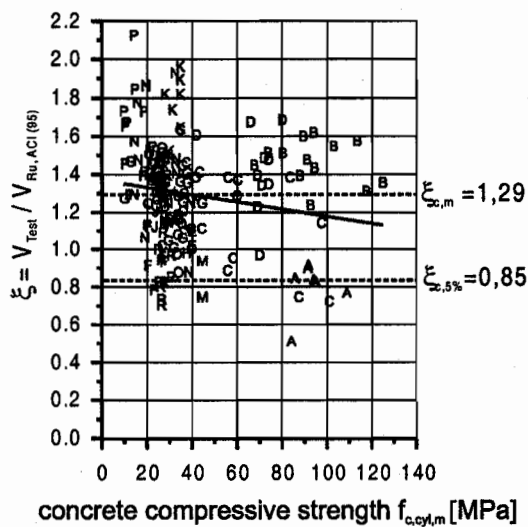
8.4.1.5 ACI 318-95



a) $V_{Test} / V_{Ru,ACI 318-95}$ versus effective depth



b) $V_{Test} / V_{Ru,ACI 318-95}$ versus flexural reinforcement ratio



c) $V_{Test} / V_{Ru,ACI 318-95}$ versus concrete compression cube strength

number of considered tests: 149

— lineare Regression

A	Hallgren	1996
B	Tomaszewicz	1993
C	Ramdane	1993
D	Marzouk/ Hussein	1991
E	Lovrovich/ McLean	1990
F	Tolf	1988
G	Regan	1986
H	Swamy/ Ali	1982
I	ETH-Zürich	1977/ 1979
J	Schaefer	1978
K	Ladner/ Schaeidt/ Gut	1977
L	Ladner et al.	1970/ 1973
M	Corley/ Hawkins	1968
N	Base	1966
O	Manterola	1966
P	Yitzaki	1966
Q	Moe	1961
R	Kinnunen/ Nylander	1960

Figure 8.4-5: Comparisons between the 149 punching tests without shear reinforcement and the punching shear capacity of ACI 318-95

8.4.1.6 DIN 1045-1

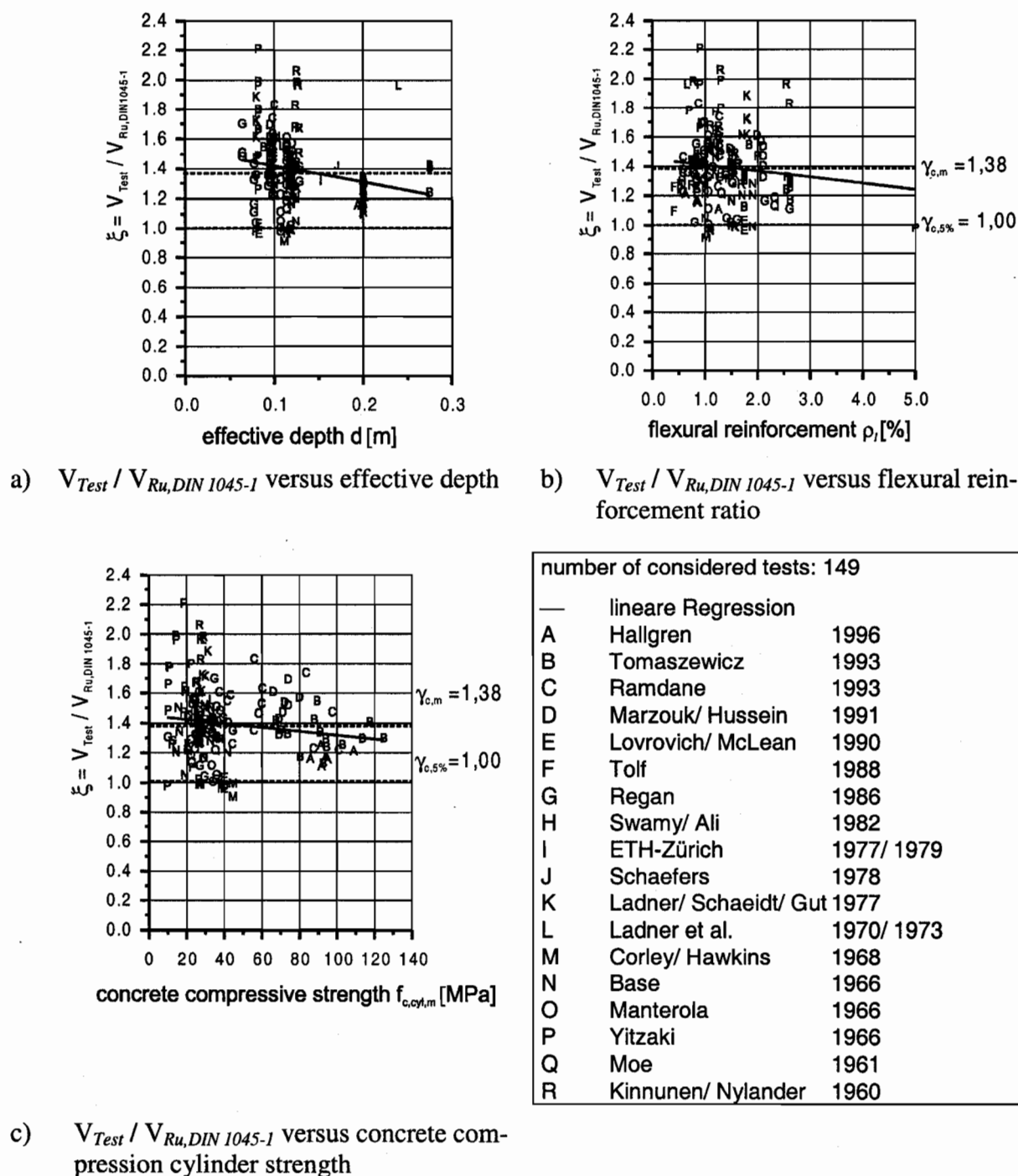


Figure 8.4-6: Comparisons between the 84 punching tests without shear reinforcement and the punching shear capacity of DIN 1045-1

8.5 Statistical Method of Evaluating Experimental Data

8.5.1 Statistical Evaluation Procedure

To ascertain the overall mean value or the overall standard deviation from the random samples at various test points, the steps shown on the flow chart (Fig. 8.5-1) must be carried out in the order indicated. Thereby the appropriate statistical test must be passed before the subsequent test can be applied. If a test fails, the cause must be eliminated and the procedure continues at an appropriate stage.

The statistical tests introduced below are basically applicable, even when given a low number of test results. However, it should be noted that the quality of the statements and the selection would be considerably improved, if the scope of random samples were greater.

All of the tests cited here [Benjamin/Cornel (1970), John (1979)] are objective decision-making aids for the engineer, which are prepared from statistical perspectives. The engineer can use the test results to decide if a common evaluation of the various test results is basically possible from a statistical point of view. After applying these statistical methods in connection with the engineering criteria (see chapter 6), one gets objectively experimental data for further evaluation.

8.5.2 Application of Statistical Methods

After calculating the mean value and standard deviation, one can verify the sample from a population as being normally distributed, by applying the *Goodness-of-fit* test according to *Kolmogorov-Smirnoff*. Measured values that clearly deviate from the other values in a series are found by using the freak-test method according to *Grubbs*. With the help of the Bartlett-test combined with the *F*-test, the different positions of mean values and standard deviations basically appearing in engineering test series can be identified. It can be ensured that the different samples belong to a common population and may then be unified to an overall sample.

This statistical evaluation requires several iterations necessary for sorting out freak-values and individual test units not meeting the imposed criteria. With this, the number of the test data permitted for evaluation possibly, substantially reduced.

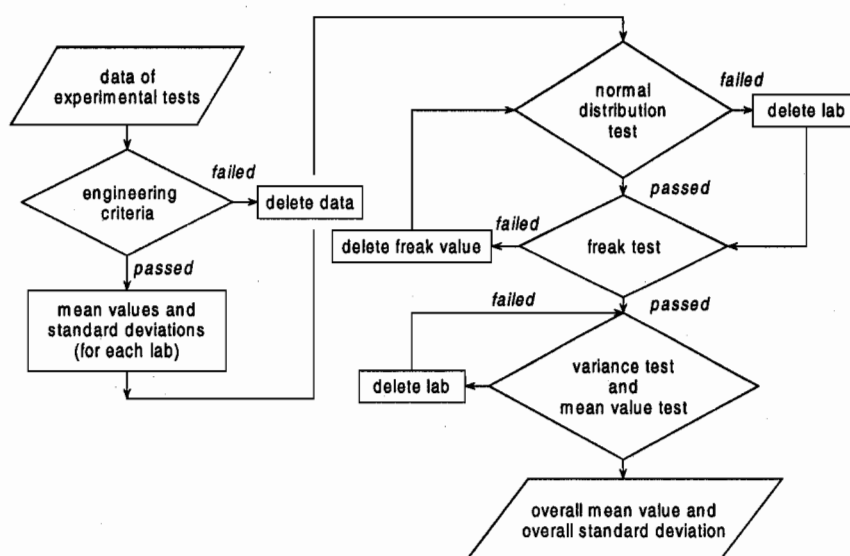


Fig. 8.5-1: Flow chart for the statistical evaluation procedure

8.5.3 Characteristic and design values

Code rules are mainly based on empirical reliability methods, supplemented with simplified first order reliability methods. The required reliability level, described as failure probability p_f or reliability index β , can be described with the help of theoretical reliability considerations, depending on the limit state to be observed.

The reliability index $\beta = 3.8$ is based on the entire life-span of the building according to Eurocode 1 [EC 1 (1995)] for the ultimate limit state. This corresponds to a failure probability $p_f = 7.2 \cdot 10^{-5}$ [Schneider (1994), Schuëller (1981)]. A Classification of a material as – either brittle or ductile – in different failure types is not intended.

The partial safety factor is derived with the help of the weighting factors α_R and α_S describing the ratio of scatters to the total scatters. These factors show the weight with which both the actions and the resistance side are involved in failure probability. In this contribution the weighting factors $\alpha_S = -0.7$ and $\alpha_R = 0.8$ are specified according to Eurocode 1.

8.5.4 Derivation of characteristic values and design values

Characteristic values and design values are determined as follows, by considering the scatter in experimental values based on the material's mean values that are measured experimentally. The equation used to determine the load bearing capacity of reinforced concrete slabs therefore is a function of the tensile strength f_{lct} of concrete.

A ratio v_2 is defined and used to figure out the design value derived from the calculated stress at failure v_{cal} , based on the actual failure stress v_{exp} that is experimentally measured. The equation for v_{cal} in this ratio can be chosen independently of the calculated value, which serves the statistical selection of experimental data, exclusively [Staller (2000)]. Therefore, evaluating arbitrary approaches and models or the exchange of mean values of material strengths, via their characteristic value is possible.

8.5.5 Determining the design value for code rules

The selection method that is introduced into the experimental data about the determination of characteristic and design values provides the possibility of determining optimised pre-factors for code rules, such as DIN 1045-1 [DIN 1045-1 (2000)], Model Code 90 [CEB-FIP MC 90 (1993)] or other empirically deduced product formulae, under theoretical reliability perspectives. This shall now be illustrated as an example for DIN 1045-1. In the process, it is assumed that the form of the equation and under the consideration of individual parameters (without longitudinal stress), such as the compressive strength of concrete, the ratio of reinforcement or effective depth conforms to the real facts with sufficient accuracy.

$$V_{Rd,ct} = c_d \kappa (100 \rho_l f_{ck})^{1/3} d u_{crit} \quad \text{with: } \kappa = 1 + \sqrt{\frac{200}{d}} \quad (d \text{ in mm}) \quad (8.5-1)$$

The searched pre-factor c_d for the design value of the punching load capacity in the ultimate limit state can be determined according to the eq. 8.5-2.

$$c_d = \frac{e^{-\alpha_R \beta \sqrt{\ln(v_2^2 + 1)}}}{\mu_{v_2} e^{-\frac{1}{2} \ln(v_2^2 + 1)}} \quad (8.5-2)$$

For the values in eq. 8.5-2, the values for the overall mean value and the variation coefficient based on the characteristic compressive strength of concrete are entered. The characteristic value $c_{k;0.05}$ can be determined by replacing $\alpha_R \beta$ by $k = 1,645$ in eq. 8.5-2:

$$c_{k;0.05} = \frac{e^{-k \sqrt{\ln(v_{v_2}^2 + 1)}}}{\mu_{v_2} e^{-\frac{1}{2} \ln(v_{v_2}^2 + 1)}} \quad (8.5-3)$$

Deriving the characteristic value and the design value of a pre-factor in a design equation is exclusively based on the approximation of a specified failure probability p_f or on the reliability index β , derived from the latter in connection with the weighting factor α_R .

For eq. 8.5-1, the pre-factor c_d is calculated by considering the database created in the appendix of this bulletin. When determining the characteristic and the design values, the variation of compressive strength of concrete are to be considered besides the scattering of experimental values. The design equation is a function of the characteristic compressive strength f_{ck} of concrete. Concrete compressive strengths $f_{c,exp}$ specified in the experimental series are generally the mean values acquired from one or several samples. Since determining the characteristic value of the concrete compressive strength using statistical methods is not possible due to missing or insufficient information, the assumption of a statistically secured standard deviation of 5 MPa. This leads to $\Delta f = 5 \text{ MPa} \cdot 1.645 = 8.22 \text{ MPa}$ which corresponds to the value $\Delta f = 8 \text{ MPa}$ given in Model Code 90 or in DIN 1045-1 approximately. Therefor the following evaluation is carried out with $f_{c,exp} - f_{ck} = 8.22 \text{ MPa}$ and the lower limit value $f_{c,exp} = f_{ck}$.

The choice of experimental data based on statistical points of view is made with the help of the empirical model developed in chapter 8.5.2 on the assumption of a lognormal distribution for the experimental data [Staller (2000)]. This does not allow any inference that only the values with parameters common with the investigated equation should be approved for evaluation.

In Table 8.5-1, the overall results of the statistical evaluation process and the theoretical reliability evaluation of experimental data for lognormal distribution based on the characteristic concrete compressive strength are presented.

A pre-factor $c_d = 0.14$ for the design value of is recommended, based on the results and experiences won within the scope of this work [Staller (2000)]. Through the limitation demanded in the code formula of DIN 1045-1 of the size-effect factor $\kappa \leq 2$ and the ratio of flexural reinforcement $\rho_1 \leq 2 \%$, eq. 8.5-1 contains additional safety elements. Regarding the construction practice sensible "designing" retaining these limitations is apparently reasonable.

Table 8.5-1: Evaluation of experimental data based on the characteristic concrete compressive strength for the lognormal distribution

	$f_{c,exp} - f_{ck} = 8.22 \text{ MPa}$	$f_{c,exp} = f_{ck}$
Number of experiments	93	93
Overall mean value	0.2465	0.2154
Standard deviation	0.0438	0.0333
Variation coefficient	0.1775	0.1546
Characteristic value	0.1816	0.1653
Design value	$0.1421 \approx 0.14$	$0.1334 \approx 0.13$

8.6 Conclusion

The statistical methods illustrated in this chapter are subdivided in a selection method and an evaluation method. The first one serves the objective choice of suitable problem description of experimental data from different testing sites, from a large quantity of non-arranged data. In the process statistical adaptability tests for certain probability distribution functions and tests for freak values are presented as well as variance- and mean value-tests. After applying engineering-based selection criteria, a unified database for further evaluation is made available in the selection process. Weighting of special experimental data is done neither during the selection nor during the following evaluation process.

The subsequently introduced statistical evaluation method according to "classical" approximations is used in determining the characteristic and design values, e.g. such as pre-factors, and design equations. This is exemplified in the code formulation for the punching shear capacity according to DIN 1045-1.

The comparisons of seven code formulas on an admissible load level in Chapter 2.5.1 carried out that significant differences concerning to the influence of the flexural reinforcement and the level of the punching shear capacity in the ultimate limit state. These differences can be judged now by the results of the comparisons between tests and codes.

- (1) From Table 8.3-2 (row 3) can be observed that all codes reach or exceed the mean ratio $\xi_m = V_{Test} / V_u = 1,0$. The approximation of the punching shear capacity is established well on average. But on the level of characteristic values only the new German design Code DIN 1045-1 reaches the required ratio $\xi_{5\%} = V_{Test} / V_u = 1,0$ according to Eurocode 1 and all other codes fall below. It must be concluded that the demanded safety level according to Eurocode 1 (Method (1) in Chapter 8.3 and 8.4) is not reached and the calculated punching shear capacities of all codes have to be reduced.
- (2) In contrast to Method (1) the example of DIN 1045-1 according to Method (2) of Eurocode 1 (Chapter 8.5) carried out that the punching shear capacity can be increased of approximately 8% to 16% ($0,13/0,12 = 1,08$ to $0,14/0,12 = 1,16$). Therefore, the scale of the punching capacities of the codes can be confirmed. But it has to be pointed out, that this example of Method (2) considers less tests and only the well defined laboratory boundary conditions. The influences of the concreting and placing of the reinforcement on site which could decrease the punching shear capacity are neglected. As well the increase of the punching shear capacity by membrane forces, which act in real flat slab systems are neglected, too. It is recommended to assess these influences and therefore the punching shear capacities of all mentioned codes (Chapter 2) tend to high shear capacities which do not reach the required safety level according to Eurocode 1.
- (3) The comparisons between Table 8.3-2, row 4 and Figures 8.4-1 to 8.4-6 indicate that a higher variation coefficient v is due to the considered influence of the flexural reinforcement ratio or the concrete compressive strength. Good approximations are given by Model Code 90, FIP-Recommendations, BS 8110 and DIN 1045-1.

In summary - it can be stipulated that with the application of the introduced methods, optimised and thus economical design values for strengths of building components can be determined objectively by applying theoretical reliability approximations. The theoretical investigations performed in this chapter point out that the decisive parameters in the design formulas of DIN 1045-1 and Model Code 90 lead to a good description of the load-bearing behaviour of point-supported slabs subjected to punching shear.

9 Comparisons between codes, models and tests for flat slabs with shear reinforcement

Rüdiger Beutel, Philippe Menétrey

9.1 INTRODUCTION

Flat slabs are more and more built in buildings with a high level of technical installations, or in buildings with flexible room arrangements during their life time such as offices. Often the main problem in practice is to provide the resistance against a punching failure at the flat slab-column connections.

The punching resistance can be increased by using a larger column diameter, a larger effective depth, more flexural reinforcement, by increasing the concrete compressive strength or by using shear reinforcement. Large columns are often rejected by the architect, a larger slab depth increases the dead load and the cost of the footings and columns. Increasing the flexural reinforcement ratio and the concrete strength is less effective and in many cases not practical. In this situation providing an effective shear reinforcement is the most economic solution.

Many shear systems were developed to provide against a punching failure (Fig. 9-1 and Fig. 9-2). The purpose of this chapter is to compare experimental punching failure loads of shear reinforced flat slabs with models and tests. Therefore, the evaluations are limited to shear systems which are established in all codes. This is the case for stirrups, closed links, hooks, bent-up bars and shear assemblies like shear ladders (Fig. 9-1). In addition, double headed studs, stud-rails and shear heads are established by some codes by approvals. But most of the experimental investigations for the latter shear systems (Fig. 9-2) touch the economic interests of companies and therefore they are not published and not considered in this chapter.

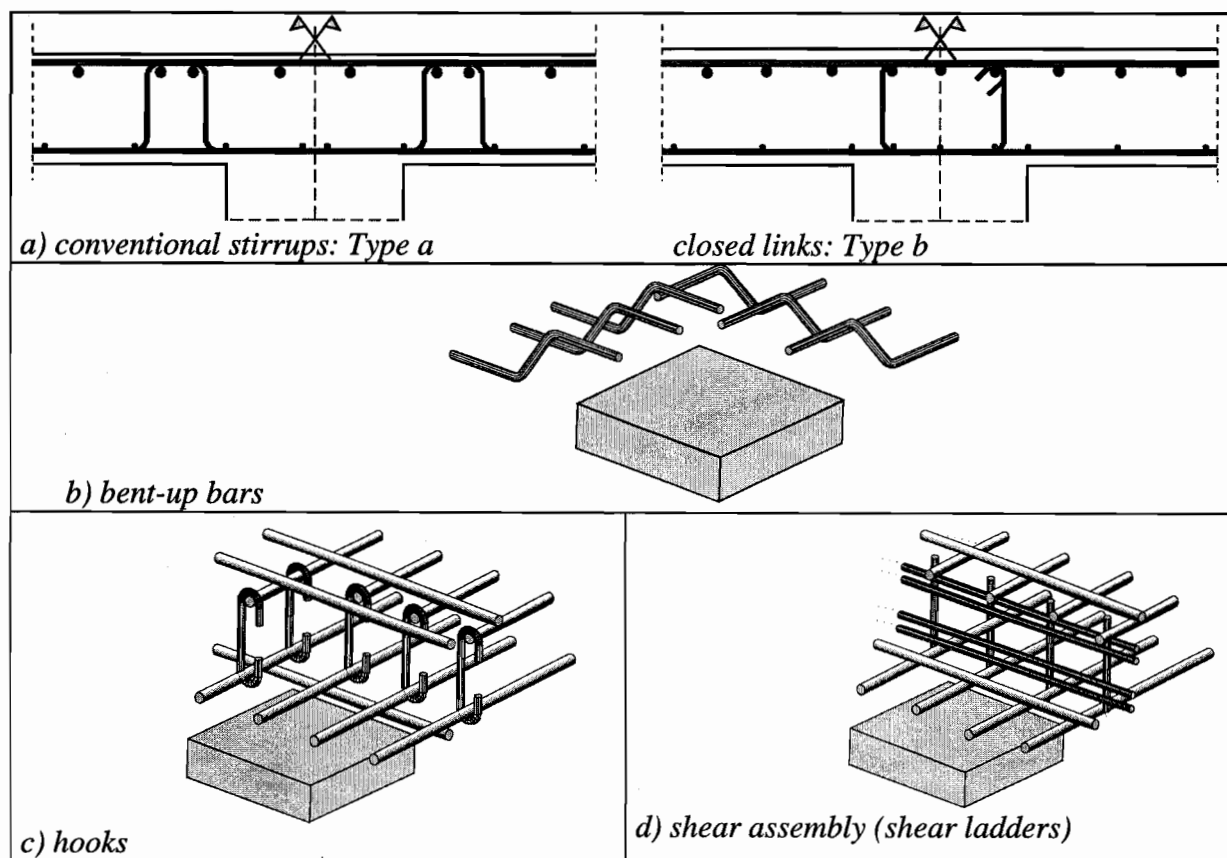


Fig. 9-1: Considered shear systems in the databank of punching tests with shear reinforcement

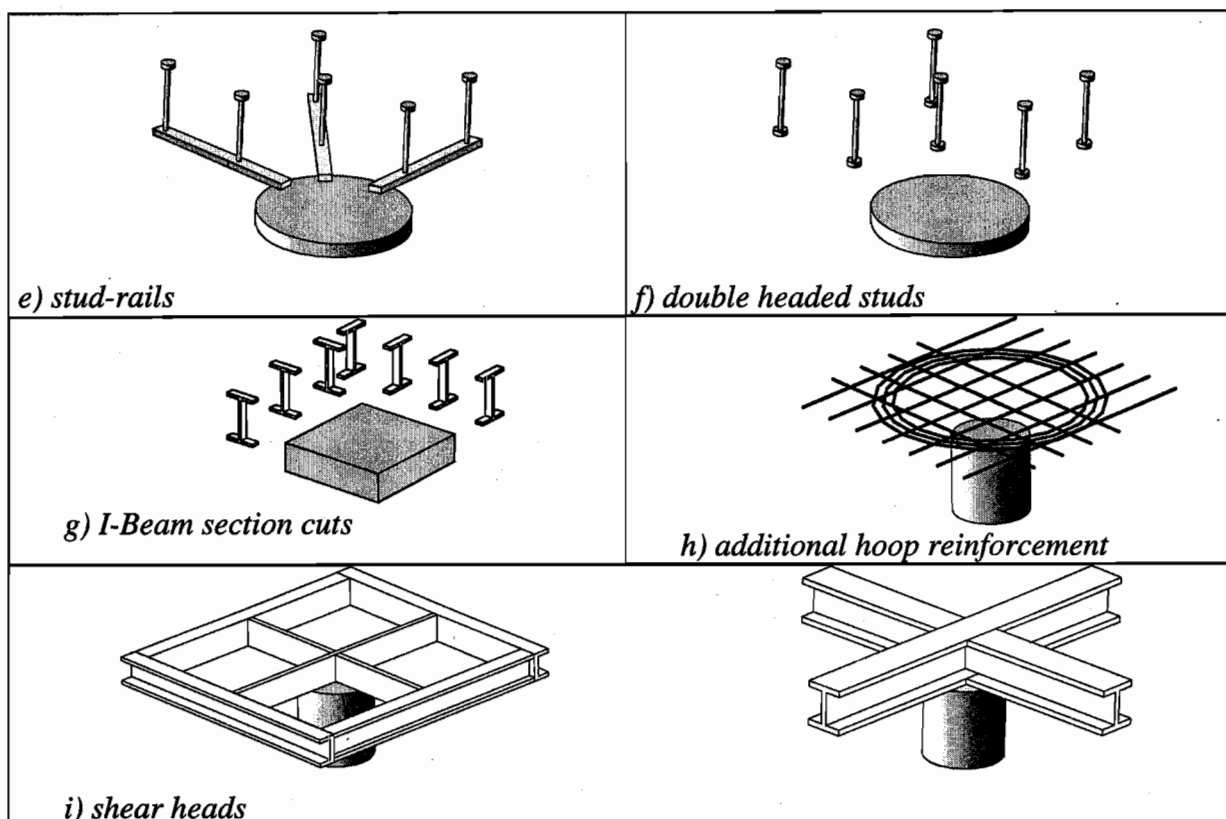


Fig. 9-2: Not considered shear systems in the databank of punching tests with shear reinforcement

9.2 Compilation of a representative data base

In the presented databanks “II-1a” and “II-1b” data on slabs with shear reinforcement is collected and they contain 150 tests. To guarantee a compilation of pure punching failures, all tests were arithmetically checked for flexural failures and for bond failures at end supports by the reports (similar to Chapter 8.2).

In contrast to a punching failure without shear reinforcement it can be observed from Figure 9-3 that the shear slenderness (a/d) influences the punching shear capacity only marginally and therefore no test was excluded by this criteria.

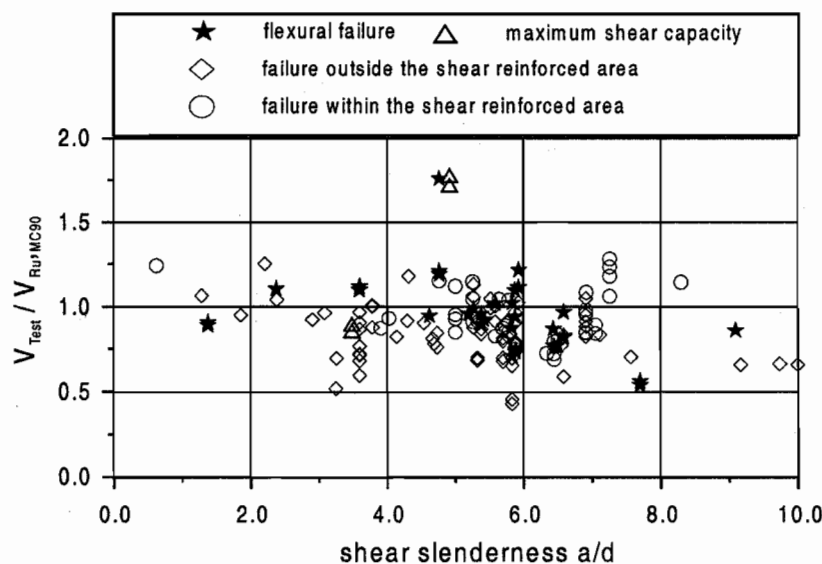


Fig. 9-3: $V_{Test} / V_{Ru,MC 90}$ versus the shear slenderness a/d

It has to be pointed out that an arithmetic classification of a flexural punching failure on the one hand and a punching failure with shear reinforcement on the other hand was not sufficient. From Fig. 9-4 it can be observed that the failure loads of flexural failures and failures at maximum punching shear capacity (crushing of the compression zone) are close together for low flexural reinforcement ratios. This behaviour indicates that the height of the compression zone in these tests was relatively small. Therefore the failure was due to a crushing of the compression zone which could be induced either by yielding of the flexural reinforcement or by a high shear stress. A classification according to a local yielding of the orthogonal flexural reinforcement is not sufficient either, as the maximum shear capacity can only be reached if the main part of the moments were carried in the tangential direction.

Characteristic differences can only be observed looking at the load deflection curve because of the brittle concrete crushing and the ductile flexural failure. In addition, mixed mode failures were observed and classified as a flexural punching failure in the databank "II". Due to the fact that a lot of papers do not present load-deflection curves and in many cases the tests reports were not available, flexural failures could not be sorted out consequently.

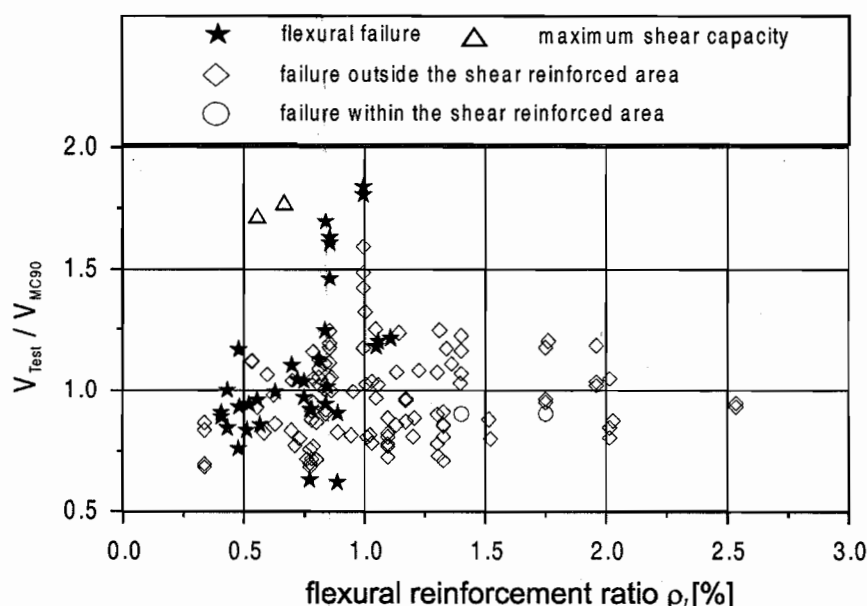


Fig. 9-4: $V_{Test} / V_{Ru,MC90}$ versus the flexural reinforcement ratio ρ_l for flexural, flexural punching and brittle punching failures

Finally, the quantity of the shear reinforcement was checked by the value of the minimum reinforcement according to Model Code 90. It was observed that all collected tests include more than the minimum shear reinforcement.

Therefore, all 150 collected tests are assumed for further evaluations.

9.3 Comparison between code rules and tests based on mean values of the material strength

All comparisons between tests and codes considered all punching failure modes of shear reinforced flat slabs arithmetically.

- concrete crushing at the column face (maximum shear capacity)
- failure within the shear reinforced area (failure of the shear reinforcement)
- failure outside of the shear reinforced area

The comparisons between tests and codes differentiate between these failure modes. The statistical results are shown in Table 9-1 to 9-3. The statistical parameters are equal to Chapter

8., Table 8.3-1. In addition, more information about the efficiency of the shear systems, the punching parameters (ρ_l , d , f_c) and the radius of the shear reinforced area (l_s) are given for selected codes in the Figures 9-5 to 9-7.

a) Maximum punching shear capacity

From Table 9-1 (row three) it can be observed that the mean ratio $\xi_m = V_{Test} / V_{u,Code}$ of all codes exceeds the value 1,0 but the 5% fractile does not reach the demanded safety level according to Eurocode 1. Secondly, the number of tests which failed on the level of the maximum shear capacity varies between 13 and 85.

Table 9-1 – Statistic parameters for the comparison between tests and the maximum punching shear capacity for six codes

Code	n/n_0	ξ_m	σ	v	$\xi_{5\%}$
DIN 1045(88)	81/ 93	1,29	0,26	0,20	0,86
Model Code 90	42/ 141	1,31	0,26	0,20	0,88
British Standard 8110-97	85/ 141	1,15	0,32	0,28	0,62
Eurocode 2, Part 1	37/ 141	1,26	0,31	0,25	0,74
ACI 318-95	13/ 141	1,27	0,30	0,24	0,78
DIN 1045-1 (2000)	39/ 141	1,34	0,19	0,14	1,02

n : number of tests which indicate arithmetically a punching failure at the level of the maximum shear capacity

n_0 : number of all considered tests

In addition, the following points can be observed:

- (1) DIN 1045(88): 87% of the tests failed on the maximum shear level (Table 9-1, row one). This high number of tests points out that the maximum shear capacity contains a limit according to a failure outside of the shear reinforced area. This can be confirmed by Figure 9-5(d) because it shows that the ratio $\xi_m = V_{Test} / V_{Ru, max DIN 1045 (88)}$ is increased with an increasing radius of the shear reinforced zone l_s . In addition, the following influences can be observed from Figure 9-5: (a) the ratio ξ_m increases with an increasing the effective depth, (b) the parameter μ_g shows no tendency and (c) the ratio ξ_m decreases with an increasing of the concrete compressive strength.
- (2) Model Code 90: In the case of Model Code 90 29% of the tests failed at the level of the maximum shear capacity because a punching failure outside of the shear reinforced area is considered. Only the parameters effective depth and shear reinforcement quantity show a tendency. From Figure 9-6(a) it can be observed that the punching shear capacity in the test is increased according to the calculated failure load with increasing the effective depth. This is due to the fact that the anchorage behaviour of the shear reinforcement is improved because one part of the stirrup forces will be anchored by bond stress and the anchor elements (mandrels or transverse bar) are less stressed. In Figure 9-6(d) it is shown that the maximum shear capacity will be improved if the quantity of the shear reinforcement is increased. This is due to the effects of an improved confinement of the concrete compression zone and an increased radius of the shear reinforced area.

(b) shear capacity within the shear reinforced area

Table 9-1 (row three) shows that the mean ratio $\xi_m = V_{Test} / V_{u,Code}$ of all codes exceeds the value 1,0 but the 5% fractile does not reach the demanded safety level according to Eurocode 1. Secondly, it is evaluated that according to DIN 1045(88), Eurocode 2, Model Code 90 and British Standard only approximately 10% of the tests failed within the shear reinforced

area. In contrast to these codes 86% of the tests according to ACI 318-95 failed within this area.

Table 9-2 – Statistic parameters for the punching resistance within the shear reinforced area

Code	n/n_0	ξ_m	σ	v	$\xi_{5\%}$
DIN 1045(88)	12/ 93	1,47	0,16	0,11	1,29
Model Code 90	16/ 141	1,30	0,29	0,23	0,82
British Standard 8110-97	22/ 141	1,08	0,25	0,23	0,67
Eurocode 2, Part 1	12/ 141	1,27	0,30	0,23	0,78
Model Code 90	16/ 141	1,30	0,29	0,23	0,82
British Standard 8110-97	22/ 141	1,08	0,25	0,23	0,67
ACI 318-95	122/ 141	1,73	0,57	0,33	0,80
DIN 1045-1 (2000)	71/ 141	1,71	0,41	0,24	1,16

n : number of tests which indicate arithmetically a punching failure within the shear reinforced area

n_0 : number of all considered tests

(c) Punching outside of the shear reinforced area

In Table 9-3 (row 2) it is shown that according to Eurocode 2 and Model Code 90 approximately 70% of the tests failed arithmetically outside of the shear reinforced zone. In contrast to Eurocode 2 and Model Code 90, ACI 318-95 carried out this type of failure for only 4% and British Standard for 24% of the tests. Therefore, this type of failure is judged very variously in the code formulas. In addition, it is shown that the mean ratio $\xi_m = V_{Test} / V_{u,Code}$ of all codes exceeds approximately the value 1,0 but the 5% fractile does not reach the demanded safety level according to Eurocode 1

Table 9-3 – Statistic parameters for the punching resistance outside of the shear reinforced zone

Code	n/n_0	ξ_m	σ	v	$\xi_{5\%}$
DIN 1045(88)	no rules				
Model Code 90	100/ 141	1,18	0,20	0,17	0,86
British Standard 8110-97	34/ 141	0,91	0,14	0,15	0,68
Eurocode 2, Part 1	104/ 141	1,26	0,31	0,25	0,74
ACI 318-95	6/ 141	(1,25)	(0,36)	(0,29)	(0,66)
DIN 1045-1 (2000)	31/ 141	1,24	0,14	0,11	1,02

n : number of tests which indicate arithmetically a punching failure outside of the shear reinforced zone

n_0 : number of all considered tests

Summary

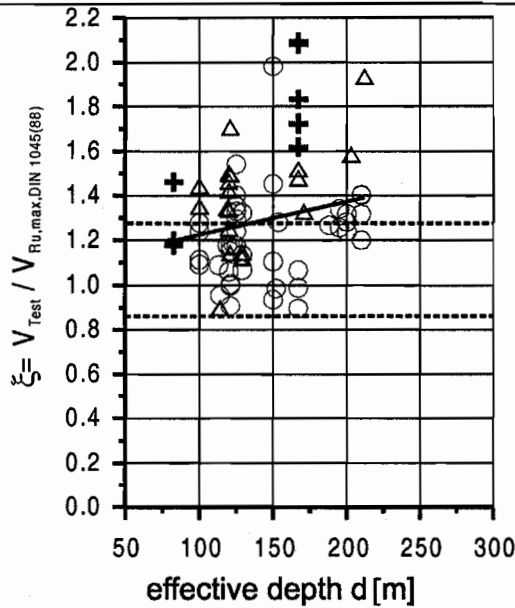
The statistical evaluation pointed out that the types of the punching failure are judged variously and the 5% fractile of all failure types does not reach the demanded safety level according to Eurocode 1. Therefore, the presented punching data bank was used to present one possibility of a improved code formula for all punching failure types. This recommended approach was developed for the new German design code DIN 1045-1 and therefore it will be presented in section 9.4.

DIN 1045 (88)

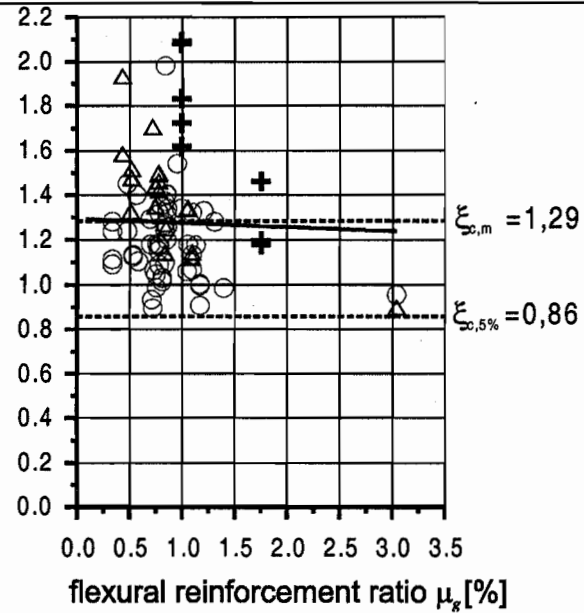
Maximum punching shear capacity

Legend: $\oplus$ hooks, $\triangle$ bent-up bars, $\circ$ stirrups and closed links

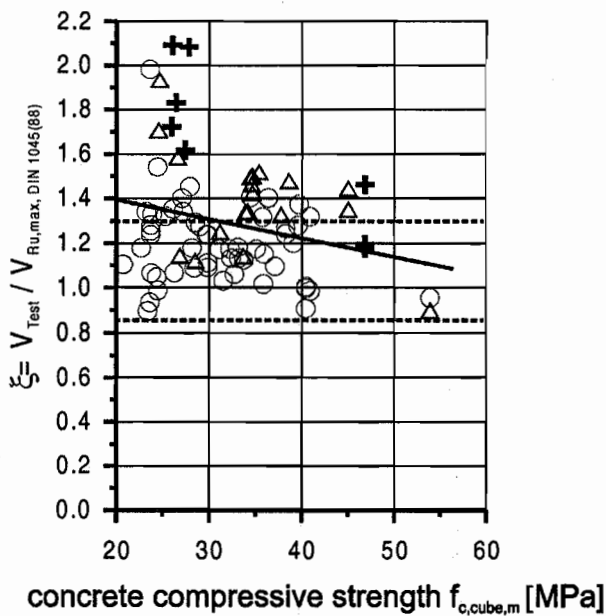
number of tests: 81



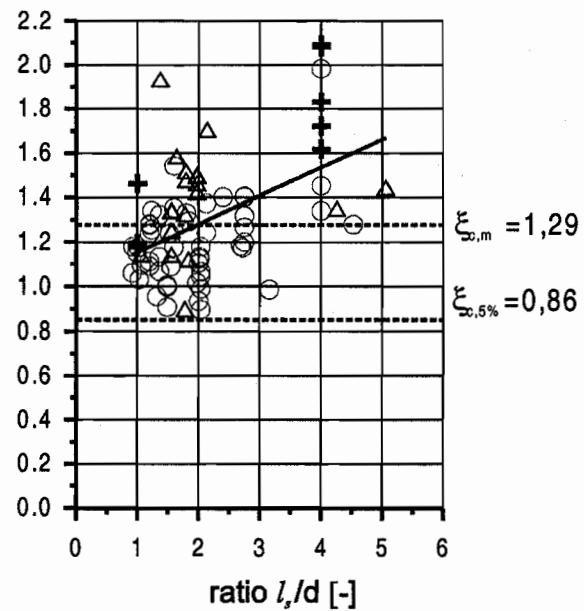
a) approximation of the effective depth



b) approximation of the flexural reinforcement



c) approximation of the concrete compressive strength



d) influence of the shear reinforced radius l_s in terms of d

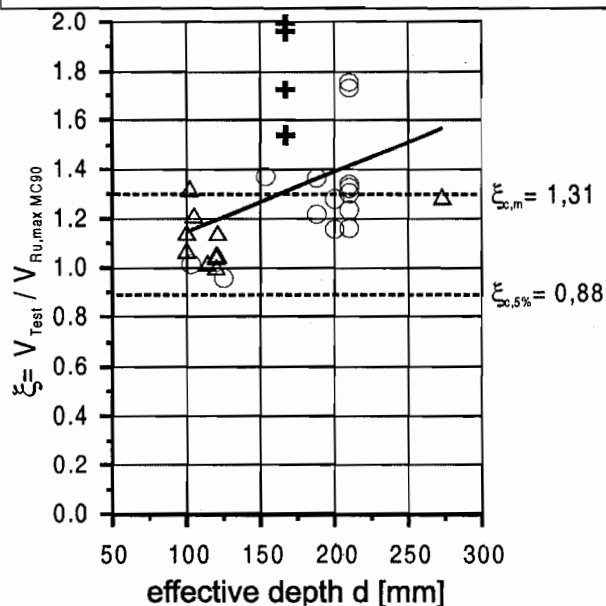
Fig. 9-5: Comparison between tests and DIN 1045(88) for all tests which failed on the level of the maximum shear capacity

CEB-FIP Model Code 90

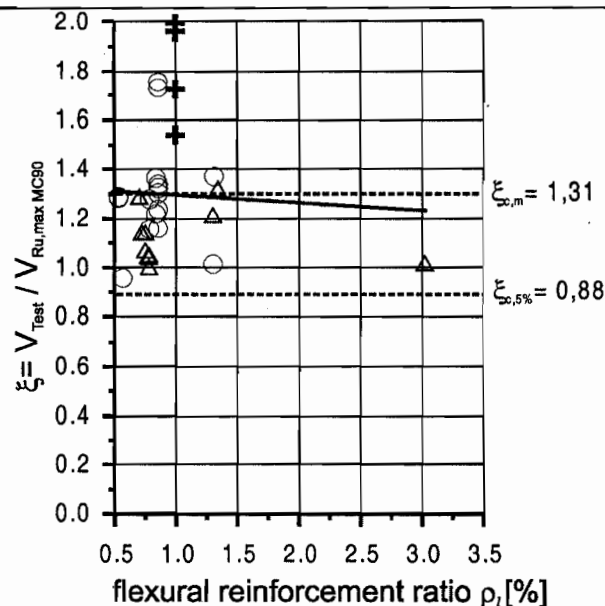
Maximum punching shear capacity

Legend: + hooks, Δ bent-up bars, $\circ$ stirrups and closed links

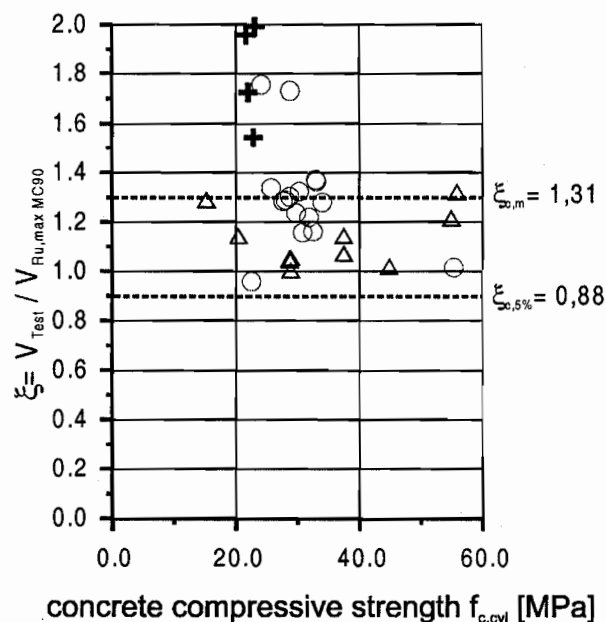
number of tests: 42



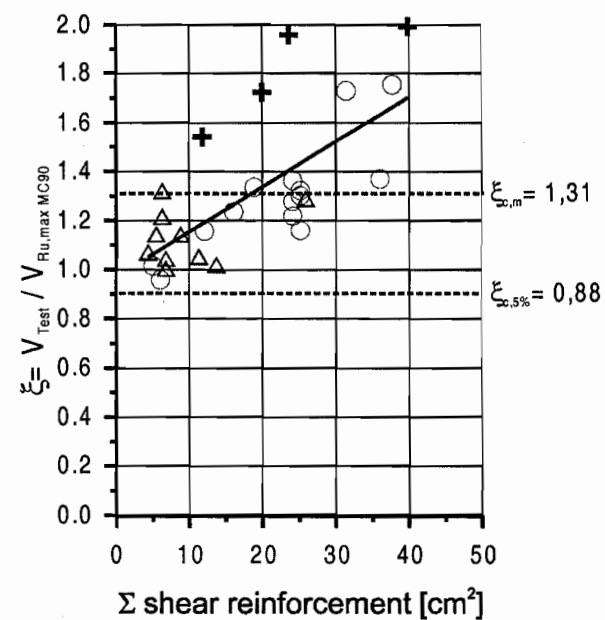
a) approximation of the effective depth



b) approximation of the flexural reinforcement



c) approximation of the concrete compressive strength



d) influence of the shear reinforcement quantity

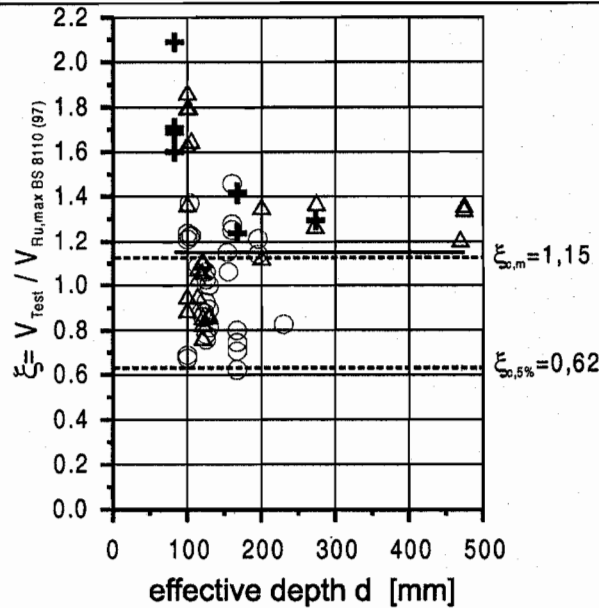
Fig. 9-6: Comparison between tests and Model Code 90 for all tests which failed on the level of the maximum shear capacity

British Standard 8110-97

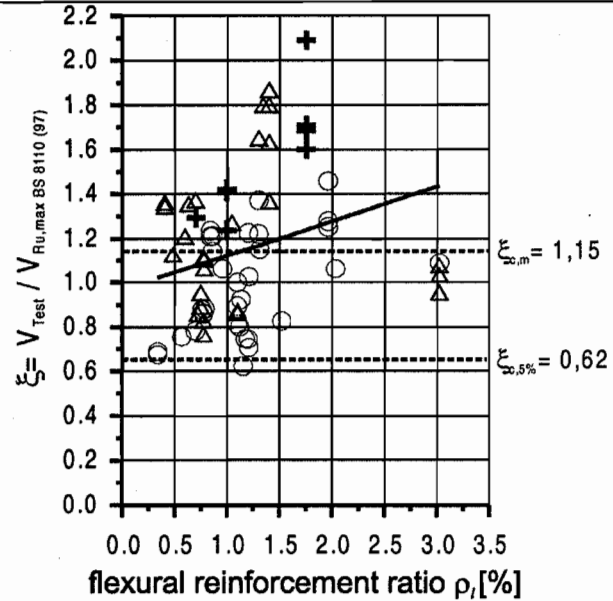
Maximum punching shear capacity

Legend: $\oplus$ hooks, $\triangle$ bent-up bars, $\circ$ stirrups and closed links

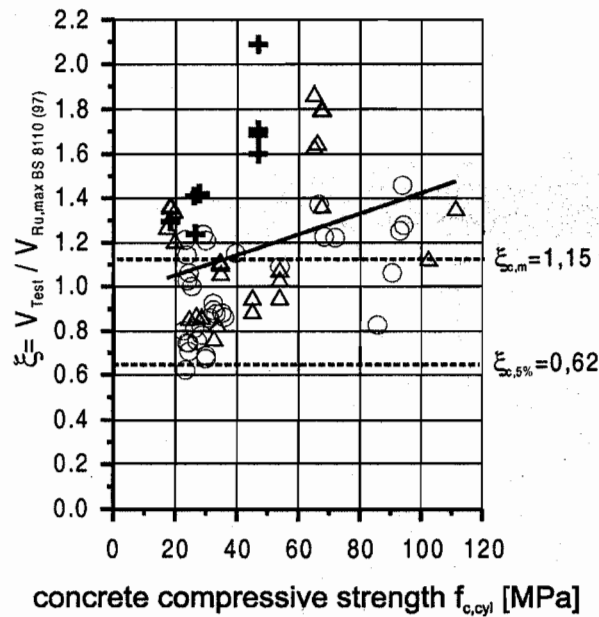
number of tests: 85



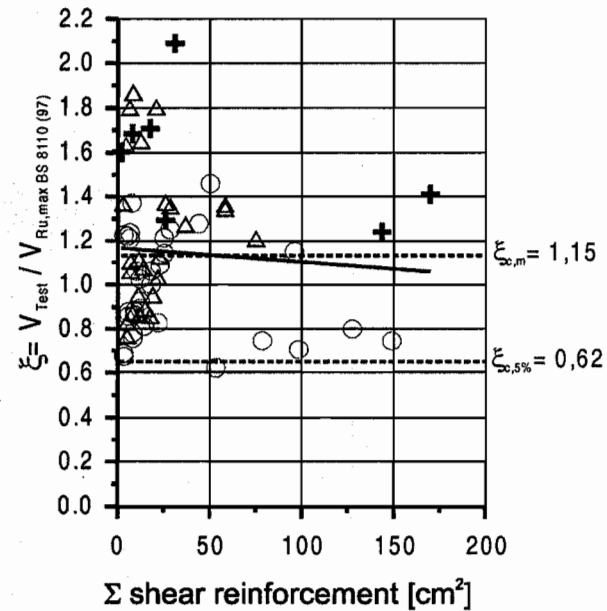
a) approximation of the effective depth



b) approximation of the flexural reinforcement



c) approximation of the concrete compressive strength



d) influence of the shear reinforcement quantity

Fig. 9-7: Comparison between tests and BS 8110 for all tests which failed on the level of the maximum shear capacity

9.4 Code Recommendations for Punching with shear reinforcement – DIN 1045-1 (2000)

9.4.1 Punching shear capacity without shear reinforcement

The punching design strength without shear reinforcement is based on the principle of Model Code 90. A Comparison between the punching strength formula of Model Code 90 and tests show that the design strength describes a mean value of the test strength. Eurocode 1 defines a characteristic value for the design strength. Therefore the control perimeter of Model Code 90 was decreased from $2.0 d$ to $1.5 d$. Special attention has to be paid to the effective flexural reinforcement ratio. It is well known from tests that the compression reinforcement does not influence the punching strength. So only the flexural reinforcement force which is balanced by the concrete strength can be considered calculating the punching strength.

$$V_{Rd,ct} = v_{Rd,ct} \cdot u_{krit} \quad [\text{MN}], \quad u_{krit} = l_c + 3 \cdot d \cdot \pi \quad [\text{m}] \quad (9-1)$$

$$v_{Rd,ct} = 0,12 \cdot (100 \cdot \rho_l \cdot f_{ck})^{1/3} \cdot \kappa \cdot d \quad [\text{MN/m}]; \quad \rho_l \leq 0,40 \cdot \frac{f_{cd}}{f_{yd}} \leq 0,02 \quad (9-2)$$

$V_{Rd,ct}$: punching shear resistance,

d : effective depth [m],

l_c : column diameter,

κ : size effect parameter,

$v_{Rd,ct}$: punching shear capacity perimeter

u_{krit} : control perimeter

ρ_l : flexural reinforcement ratio [-]

f_{ck} : characteristic compressive strength.

9.4.2 Punching with shear reinforcement

(a) Maximum punching shear capacity

The own tests and tests from literature confirm that a characteristic value of the maximum punching capacity of conventional stirrups and bent-up bars is given by equation (9-3). The failure at the maximum punching shear capacity is caused by concrete crushing at the column face. In order to simplify the code rules the control perimeter of the maximum shear capacity is identically defined as the control perimeter of the punching resistance without shear reinforcement. The dependence on the punching resistance without shear reinforcement has to be put down to the fact that in the first line the height of the compression zone and the compressive concrete strength are considered.

$$V_{Rd,max} = 1,7 \cdot V_{Rd,ct} \quad (9-3)$$

(b) Punching shear strength within the shear reinforced area

The shear capacity of each stirrup row is given by a constant concrete contribution (v_{crd}) and by the shear strength of the stirrups. As mentioned in Chapter 10.3 the concrete contribution can be specified by the punching resistance without shear reinforcement. Taking into account the stirrup anchorage slip in thin flat slabs, the design strength of the shear reinforcement is fixed to 70% of the yield strength. This strength is directly linked to a required slab thickness of 200 mm. If the slab thickness increases the anchorage behaviour of the stirrups will be improved and leads to an increasing design value of the stirrups (equation 9-6).

$$V_{Rd,sy,i} = v_{Rd,sy} \cdot u_i \quad [\text{MN}] \quad (9-4)$$

$$v_{Rd,sy} = v_{crd} + \frac{\kappa_s \cdot A_{sw,i} \cdot f_{yd}}{u_i} \quad [\text{MN/m}] \quad (9-5)$$

$$\text{stirrups and bent-up bars: } \kappa_s = 0,7 + 0,3 \frac{d - 400}{400} \begin{matrix} \geq 0,7 \\ \leq 1,0 \end{matrix}, d \text{ in mm} \quad (9-6)$$

$$\text{interior columns: } \Delta A_{sw} = 2 \cdot \pi \cdot s_w \cdot \frac{v_{crd}}{\kappa_s \cdot f_{yd}} \quad (9-7)$$

$V_{Rd, sy, i}$: Shear capacity in every stirrup row i ;

$v_{Rd, sy}$: shear capacity in every stirrup row per meter;

v_{crd} : Concrete contribution of the shear capacity for stirrups and bent-up bars $v_{crd} = v_{Rd, ct}$;

$A_{sw, i}$: Sum of the stirrup cross section area in each row;

f_{yd} : design yield strength;

κ_s : effectiveness factor of the shear reinforcement;

u_i : perimeter of each stirrup row; s_w : spacing of the stirrups in the radial direction;

l_w : radial distance between the column face and the last stirrup row.

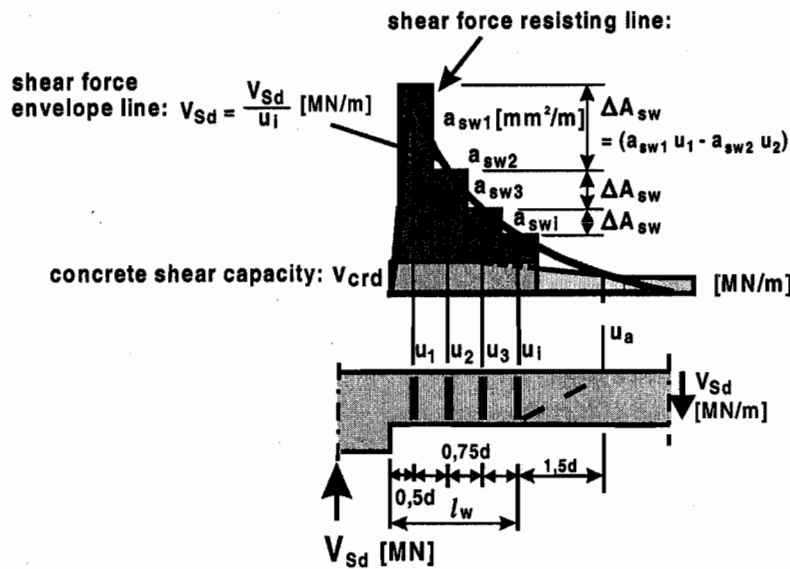


Fig. 9-8: Diagram of the acting and resisting shear force

The best way to explain the design procedure is to compare the acting shear force envelope line to the shear force resisting line (Fig. 9-8). Therefore in each stirrup row i the acting and resisting shear force has to be calculated. The design of the stirrups leads to a staggered shear force resisting line approximating the acting shear force envelope line.

(c) Punching outside of the shear reinforced area

The distance between the last stirrup row and the control perimeter outside the shear reinforced area can be assumed to $1.5 d$. This value is exactly equal to the distance between the column face and the control perimeter for the punching capacity without shear reinforcement. The design value of the punching resistance outside the shear reinforced area decreases from the punching resistance without shear reinforcement to the shear capacity of continuous supported slabs which is 20% lower than the punching resistance (formula 9-10).

$$V_{Rd, cta} = v_{Rd, cta} \cdot u_a \quad [\text{MN}]; u_a: \text{exterior control perimeter} \quad (9-8)$$

$$v_{Rd, cta} = \kappa_a \cdot v_{Rd, ct} \quad [\text{MN/m}] \quad (9-9)$$

$$\kappa_a = 1 - \frac{0,167 \cdot l_w}{3,5 \cdot d} \geq 0,83 \quad (9-10)$$

9.4.3 DIN 1045-1

Maximum punching shear capacity

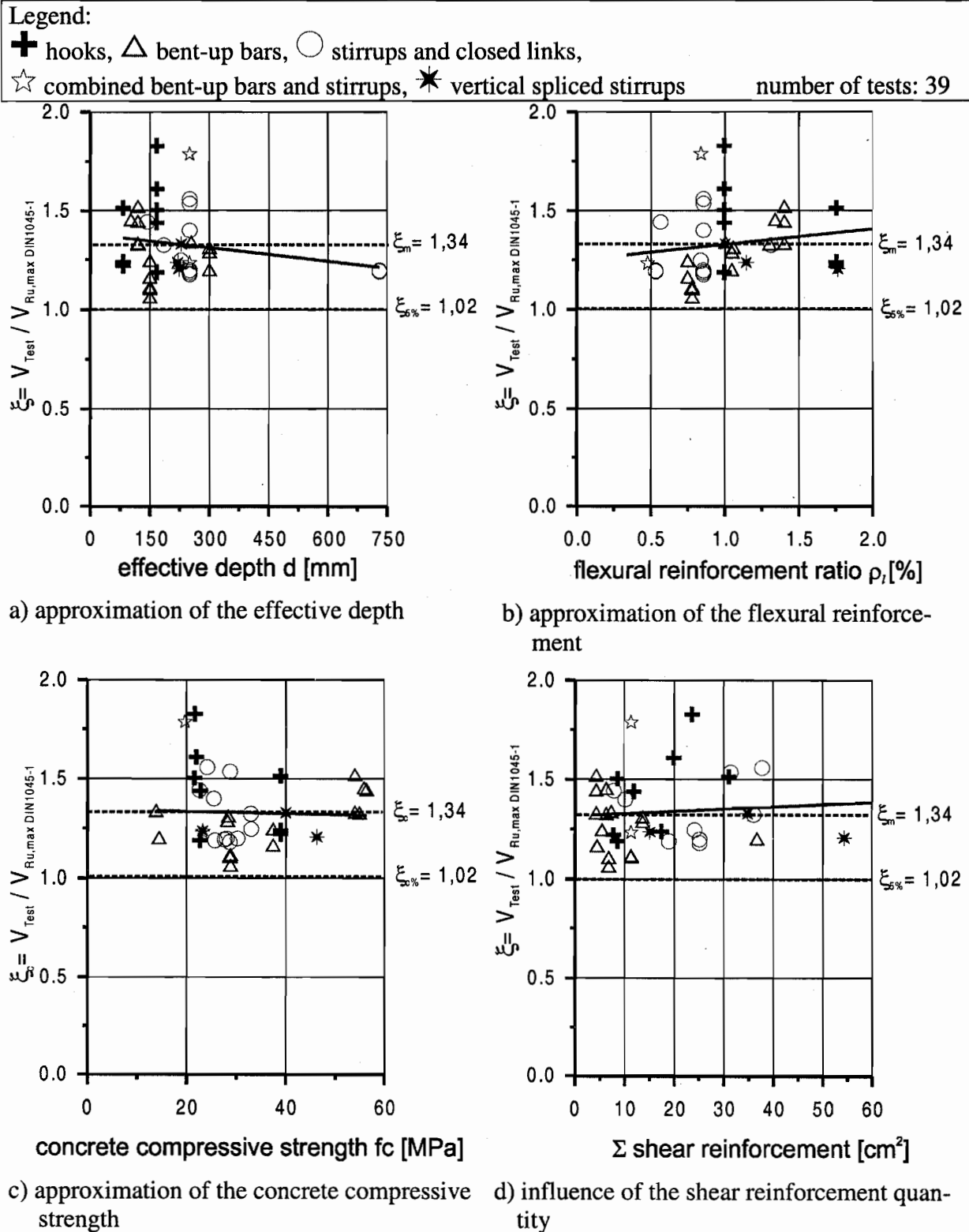


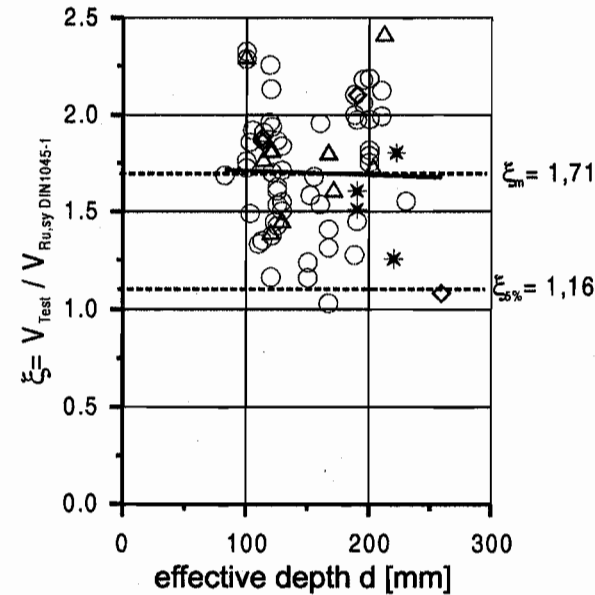
Fig. 9-9: Comparison between tests and Model Code 90 for all tests which failed on the level of the maximum shear capacity

Punching shear capacity within the shear reinforced area

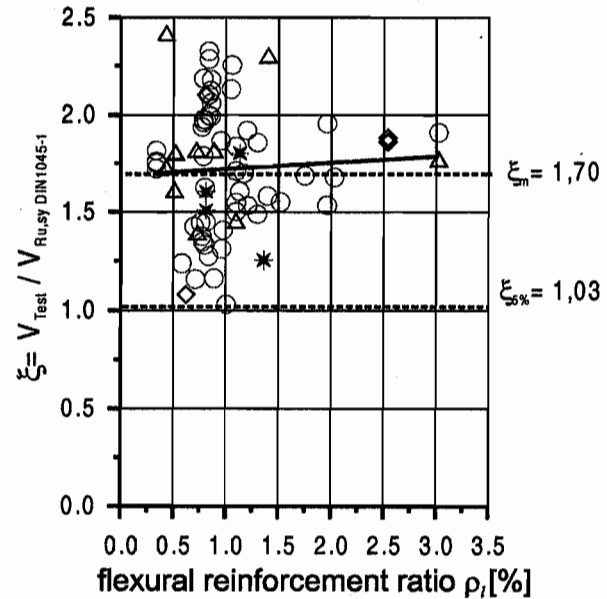
Legend:

+ hooks, Δ bent-up bars, $\circ$ stirrups and closed links,
 $\star$ combined bent-up bars and stirrups, $\star$ vertical spliced stirrups

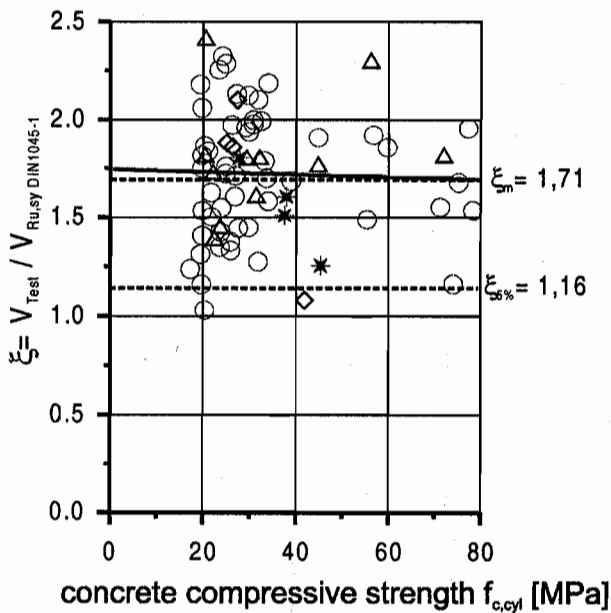
number of tests: 71



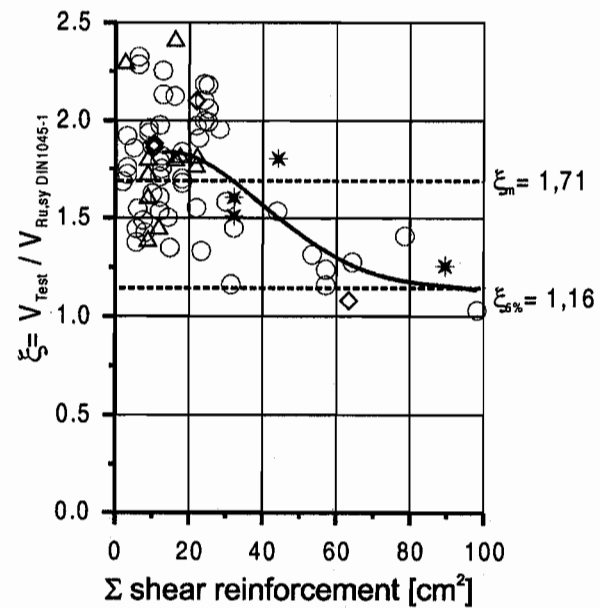
a) approximation of the effective depth



b) approximation of the flexural reinforcement



c) approximation of the concrete compressive strength



d) influence of the shear reinforcement quantity

Fig. 9-10: Comparison between tests and Model Code 90 for all tests which failed within the shear reinforced zone

Punching shear capacity outside the shear reinforced zone

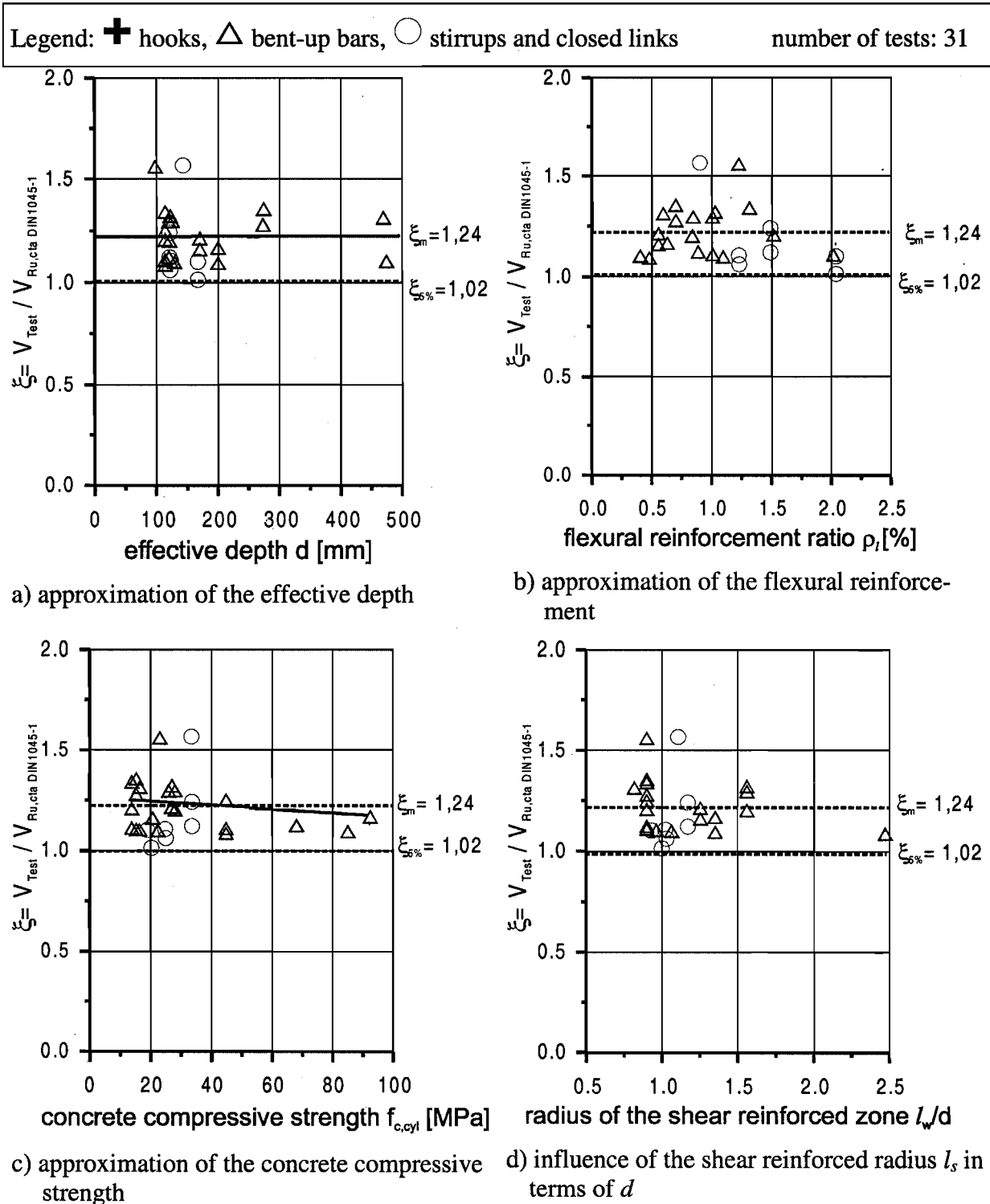


Fig. 9-11: Comparison between tests and Model Code 90 for all tests which failed on the level of the maximum shear capacity

9.5 Brief review on mechanical models for slabs with shear reinforcement

The rigid sector model of Kinnunen and Nylander (1960) was modified. Special attention on the behaviour of the shear reinforcement was investigated by Narasimhan (1971), Nilsson (1983), Shehata (1990) and Gomes (1991).

Models with shear reinforcement based on the plasticity theory were developed by Brändli (1984), Braestrup (1979) and Marti (1990).

Strut and tie models in the punching area with shear reinforcement were developed by Andrä (1981) and Kanellopoulos (1986). Models by using inclined concrete cone shells were investigated by Polonyi and Bollinger (1983) and Göricke (1999). In addition, only the presented model by Menétrey (1994) considers shear reinforcement (Chapter 4).

Comparison between a mechanical model and tests results of slabs with shear reinforcement

The purpose of this chapter is to compare the experimental punching load of concrete slabs with shear reinforcement with the one predicted analytically. The database used in the previous chapter to compare the code predictions will be considered.

From the analytical models described in Chapter 4, only the one by Menétrey (1994) allows to predict the punching load of a slab with shear reinforcement, so that the prediction with this model will be compared to the database.

Different failure mechanisms

Shear reinforcements such as studs, stirrups, bent-bars or bolts are used in order to increase the failure load and to reduce the sudden decrease of the load carrying capacity. The computation of the failure load in such slabs should differentiate the following failure mechanisms (7.2-1 for a slab with studs).

- 1) A failure mechanism for which the punching crack is located in between the column face and the first row of shear reinforcement.
- 2) A failure mechanism for which the punching crack is initiated outside the last row of shear reinforcement.
- 3) A failure mechanism for which the punching crack crosses the shear reinforcement.

Comparison with the database

The predictions of the model are compared with the experimental database. It has to be mentioned, that the computation of the punching strength of a slab with shear reinforcement is much more tedious than the one for a slab without shear reinforcement. This is not only due to the different failure mechanisms which have to be considered, but is also due to the fact that by varying the punching crack inclination, is the contribution of the shear reinforcement modified.

Due to the amount of tests and required data, some simplifications were made:

- 1) The punching crack inclination was assumed to be always 30° .
- 2) Some data like the aggregate size and the diameter of the flexural reinforcement were missing for various tests. In order to pursue with a database with many tests, the data were completed and the aggregate size was assumed to be $d_a=20$ mm and the diameter of the flexural reinforcement was assumed to be $\phi_s=10$ mm.
- 3) No distinctions were made between circular and square slabs or columns (treated as circular ones).

- 4) The differences between stirrups and studs as presented in sec. 4.7.5 is not considered, because the concrete fracture energy necessary to express this difference was almost never reported experimentally. Therefore, eq. 4.7-17 was considered, were the full strength of the shear reinforcement is activated.

The prediction of the punching load in three sections was performed as illustrated in Fig. 4.7-2. The predicted failure load and the experimental one are shown in Fig. 9-12 for the three mechanisms. It can be seen that the dispersion of the prediction is large.

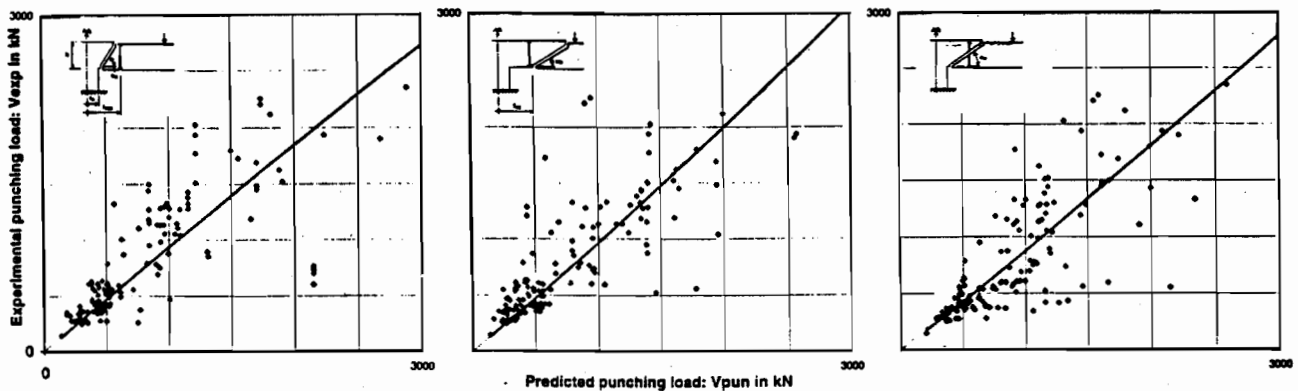


Fig. 9-12 Failure load in slab with shear reinforcement for the three mechanisms

However, the punching load prediction should consider the minimum value of the ones obtained with the three mechanisms and this value is plotted in Fig.9-13. It can be seen that the dispersion is mainly reduced. Furthermore, the prediction is not diffuse on the overall area, but is restricted to the area on the safety side. These justify that the failure load should be computed for various mechanisms, and the minimum value should be considered.

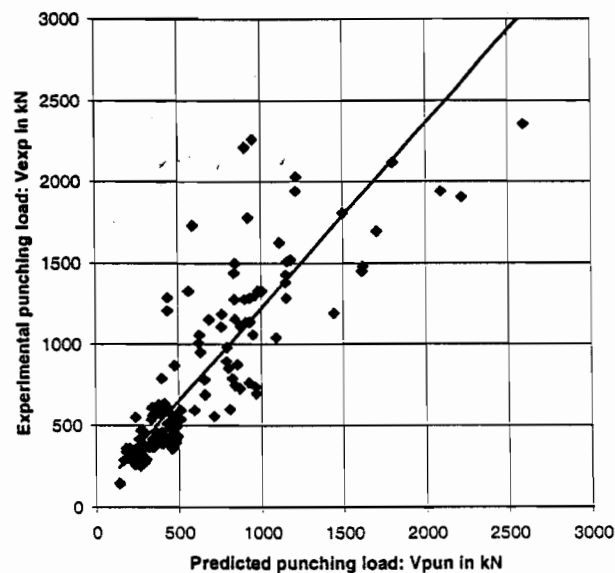


Fig. 9-13 Punching load prediction in slab with shear reinforcement (minimum value)

10 Experimental investigations

10.1 Relation between structure and test set-up

Alaa Sherif

10.1.1 Introduction

The problem of punching shear strength of reinforced concrete flat slabs is a topic that has been covered extensively by many researchers. Due to the complexity of the problem, any analytical model developed to describe the punching behaviour of flat slabs has to rely to some extent on experimental results. Since testing of real structures is not feasible, specimens are tested to study the behaviour of slab-column connections in real structures. In most of these tests little attention is given to the effects of the test set-up, such as the boundary conditions, on the behaviour of the connection tested. Concerns also rise if the test specimens represent the true behaviour of a slab-column connection. In this study the different test set-ups for testing slab-column connections are critically reviewed. The effect of the test set-up on the punching behaviour of slab-column connections is investigated.

10.1.2 Test set-ups

There are mainly two types of set-ups which have been used for testing interior slab-column connections. These are single column tests and slab-column subsystems.

10.1.2.1 Single column tests

Most of the tests on slab-column connections are single column tests often referred to as isolated slab-column connections. The test specimen consists of a column or a column stub surrounded by a portion of the slab. The dimensions of the slab portion are chosen to model the points of contra flexure or zone of negative moments of the real slab as shown in Fig. 10.1-1. The main advantage of single column tests is that they are relatively inexpensive and permit testing of full scale specimens. Thus, the failure shear stresses are not influenced by the size effect which causes a continuous reduction of shear strength with increasing depth of the slab. However, their disadvantages centre around concerns that they do not realistically model the behaviour of a slab-column connection in a real structure. The effects of boundary conditions, confinement and in-plane forces (membrane forces) are largely ignored, besides the fact that load redistribution is not possible.

Alexander (1986) distinguishes between two types of single column tests: 1. the slab is supported along its boundary, and the loads are applied to the column (see Fig. 10.1-2a), and 2. the slab is supported by the column and the loads are applied at a fixed distance from the column (Fig. 10.1-2b). These two testing methods are not equivalent. The first method allows the slab some freedom in determining its own force distribution along the boundary.

(a) Effect of boundary conditions

The effects of the boundary conditions and force distribution along the boundary of the slab specimen on its punching shear strength can be studied using test results by Elstner and Hognestad (1956). The failure load F_u normalized with respect to $\sqrt{f'_c}$, where f'_c is the concrete cylinder compressive strength, are listed in Table 10.1-1 for square slabs with the following support conditions: all four edges are simply supported, two opposite edges are simply supported,

and the four corners are simply supported. Although the shear stress distributions around the column and at $d/2$ from the column face (d is the effective shear depth of the slab) obtained from an elastic finite element analysis are almost identical for the three cases (see Fig. 10.1-3), the test results indicate a significant reduction in shear strength if the edges are not continuously supported as illustrated in Table 10.1-1. For a slab supported along its four sides, boundary forces develop which seem to enhance the shear strength.

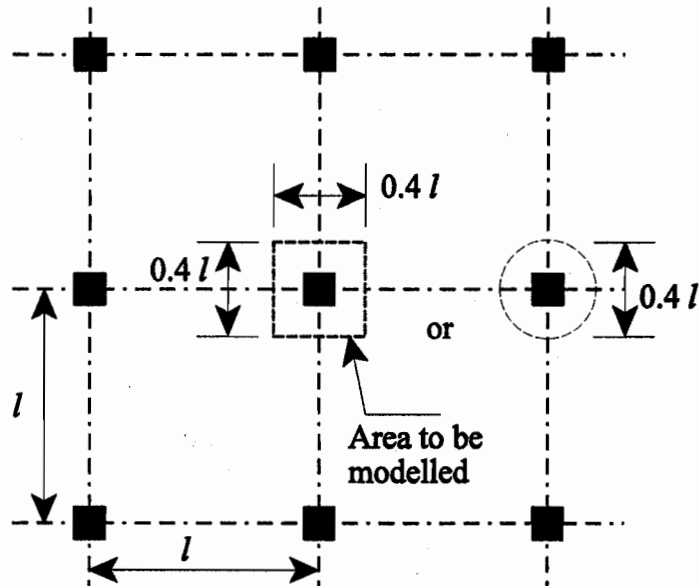
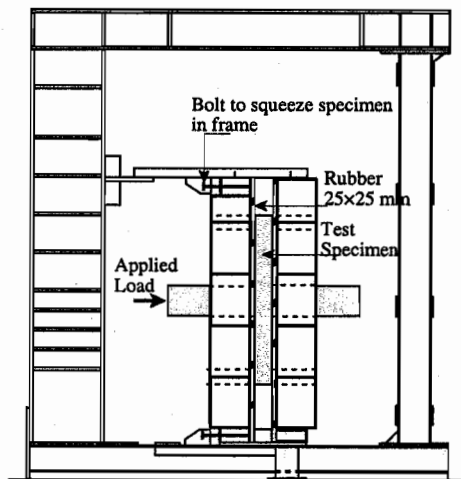
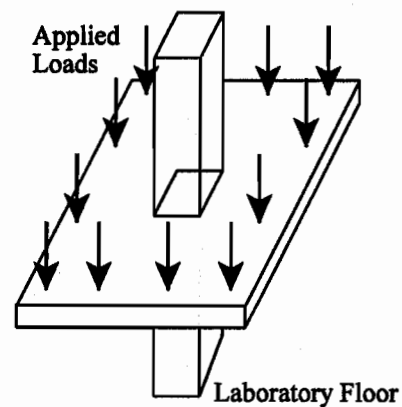


Fig. 10.1-1: Size of slab specimen used in isolated slab-column tests



(a) Slab supported along boundary
(University of Calgary)



(b) Slab supported at column
(Hawkins et al 1989)

Fig. 10.1-2: Test set-ups for a single column specimen

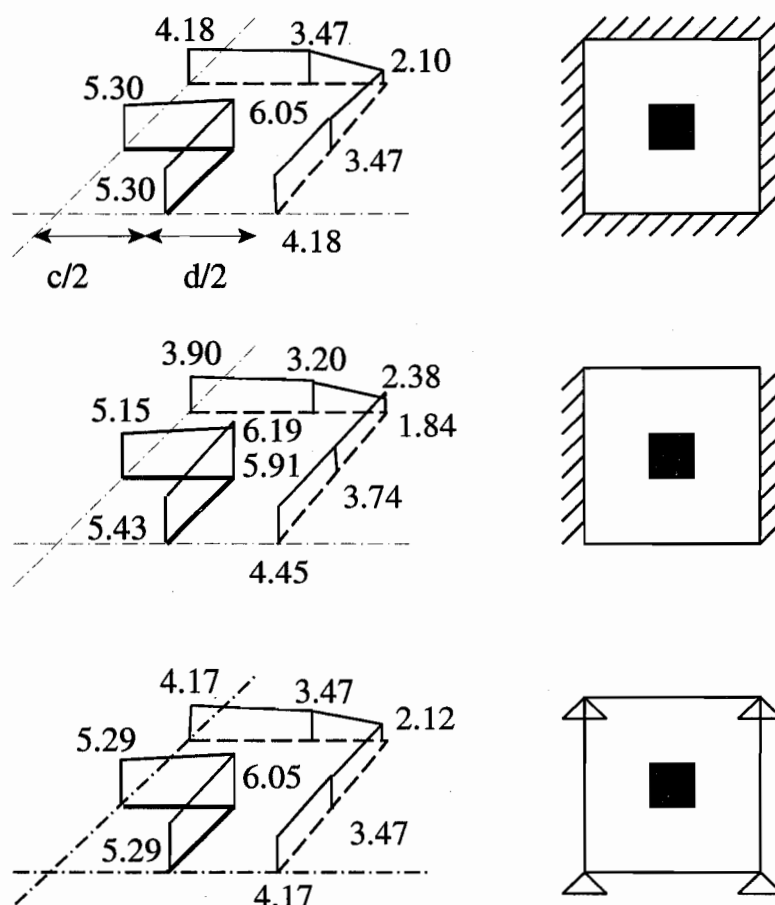


Fig. 10.1-3: Finite element shear stress distribution (MPa) due to 1 MN shear force for slabs tested by Elstner and Hognestad (1956)

Researcher	Test	ρ_l %	d mm	c mm	$F_u / \sqrt{f'_c}$	Note
Elstner & Hognestad (1956)	average of A-2a, A-2b, A-2c, A-7b	2.47	114.3	254	89.2	4 edges simply supported
	A-7	2.47	114.3	254	75.8	2 opposite edges simply supported
	A-7a	2.47	114.3	254	53.9	4 corners simply supported
Alexander & Simmonds (1992)	P19RB	0.53	126.0	200	64.1	rotations of edges & corners restrained
	P19RE	0.53	126.0	200	57.5	rotations of edges restrained
	P19RC	0.53	126.0	200	52.6	rotations of corners restrained

Note c = column width

Table 10.1-1: Effect of boundary conditions on the failure load F_u

However, it should be noted that the effect of the moment to shear ratio on the punching shear

strength is also included in these tests. The moment to shear ratio for the slab supported on its four corners is higher than for the one simply supported along its four edges. Therefore, it is expected that the later exhibit a higher punching shear strength.

The punching strength is also influenced by the shear distance a_v from the loaded area to the support (see Fig. 10.1-4). Regan and Braestrup (1985) studied the effect of the ratio a_v/d known as the shear span ratio, on the shear strength. Although the data in this area is limited, it could be concluded that the shear strength rises quite sharply when a_v/d is less than about 1.5 but is relatively constant for larger values of the ratio. For very short shear spans a_v , the support interferes significantly with the failure surface. According to Gardner (1990), for punching shear failure to occur, at least three slab thicknesses are necessary between the punch and the slab support.

To assess the effect of rotational boundary restraint, Alexander and Simmonds (1992) used a

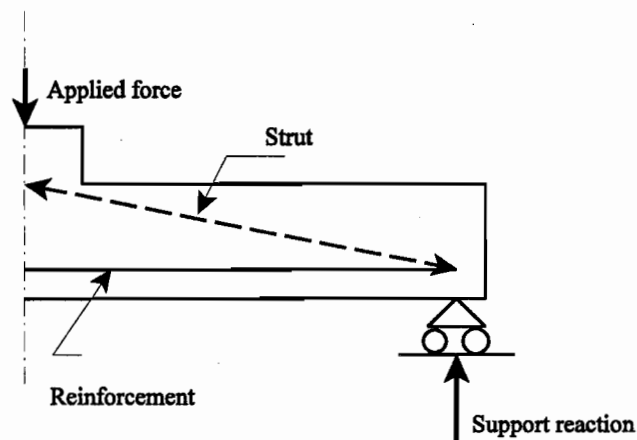


Fig. 10.1-4: Concept of compressive strut increasing the resistance to applied shear force

test set-up in which the slab had purely rotational boundary restraint. This was achieved by using roller supports and a system allowing the application of positive moments at the edge and corners of the slab. Three types of boundary conditions were investigated: 1. rotations of edges and corners are restrained, 2. rotations of edges are restrained, and 3. rotations of corners are restrained. The results are listed in Table 10.1-1. Alexander and Simmonds (1992) concluded that the application of rotational restraint at the slab boundary without any in-plane restraint increases the shear capacity of the connection.

(b) Effect of compressive membrane action

The phenomenon of compressive membrane action is generally considered to be a secondary effect which occurs after cracking of the concrete or yielding of the reinforcement. It has been found to result in substantial enhancements in the ultimate load capacity of continuous slab systems and beam-slab systems such as bridge decks. Series of full panel tests by Long and Bond (1967) showed those full panel specimens gave much greater strengths than their conventional counterparts which are extending to the nominal line of contra flexure in a flat plate structure. Masterson and Long (1974) developed a rational method of predicting the punching strength of interior slab-column connections which allows for the effect of compressive membrane action. In this method the portion of the slab inside the nominal line of contra flexure is considered to be laterally restrained by the surrounding zone of the slab and therefore compressive membrane action is induced as the slab deflected (see Fig. 10.1-5). Masterson and Long calculated the membrane moment which occurs when the restraining zone of the slab cracks due to the expansive forces from the slab inside the line of contra flexure. This compressive membrane moment is then included in their flexure criterion of failure.

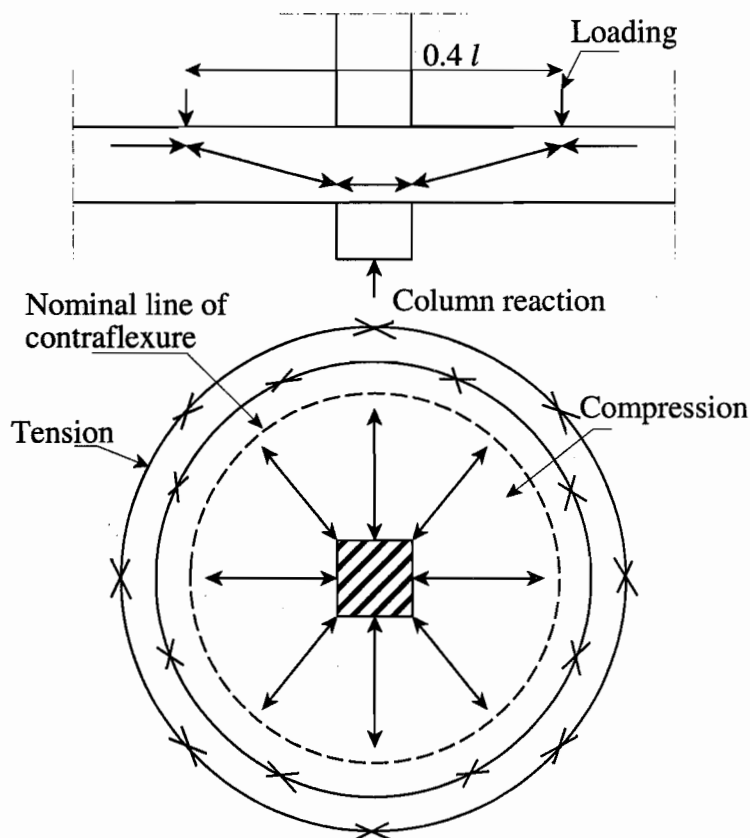


Fig. 10.1-5: Concept of compressive membrane action at interior slab-column connection

To verify the concept of enhanced punching strength due to compressive membrane action Rankin and Long (1987) conducted an extensive experimental program of 17 tests. The type of model ranged from the conventional specimen representing the area of a slab surrounded by the nominal line of contra flexure ($0.4l$) to the full panel specimen representing the area of the slab from midspan to midspan between columns (see Fig. 10.1-6). Both types of model were simply supported at the nominal line of contra flexure and had therefore statically equivalent loading and support arrangements.

The $\frac{1}{4}$ scale model sizes were taken to range from 640 mm ($0.4l$) to 1600 mm (l) with a column size of 100 mm and overall slab depth range of 45.5 to 64 mm. Parameters studied for each span were the reinforcement ratio and span depth ratios. The results indicated a definite increase in the ultimate load of the interior slab-column connections as the size of the specimen increased from the nominal line of contra flexure ($0.4l$) to the full span (l). The smallest increase in strength was approximately 30% for the more heavily reinforced full panel specimen ($\rho_f=1.1\%$) and up to 50% for the more lightly reinforced full panel specimen ($\rho_f=0.5\%$). Similar strength enhancements were found for low ($l/h=25$) and high ($l/h=35$) span to depth ratios respectively.

Rankin and Long proposed an analytical model to include the effect of compressive membrane action in the shear strength of interior columns. They also assumed that the portion of the slab inside the nominal line of contra flexure is restrained against lateral expansion by the surrounding

slab. However, the treatment of the compressive membrane action differs from Masterson and Long in that the arching moment is first calculated on the assumption of rigid lateral restraint and then the effect of partial restraint is allowed for by taking a certain proportion of this arching moment to contribute to the flexural capacity of the slab.

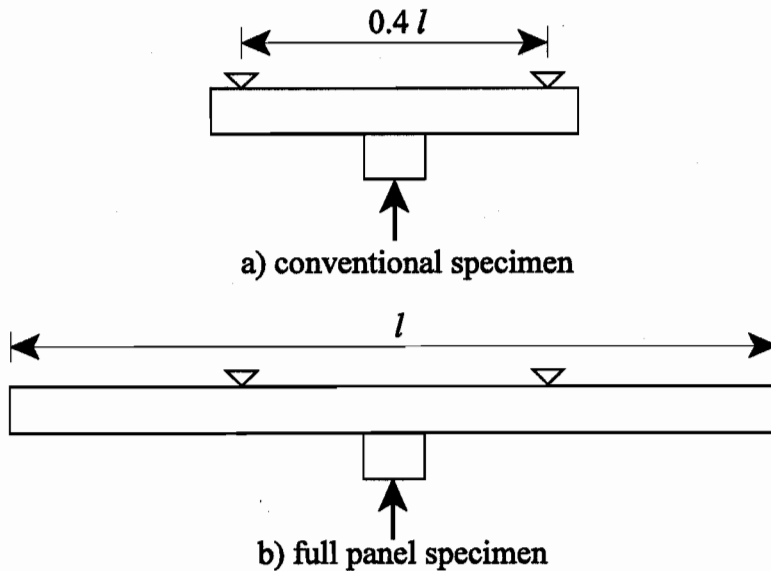


Fig. 10.1-6: Loading and support conditions for test specimens by Rankin and Long (1987)

Although these tests prove the enhancement of the shear strength in full panels, it is believed that the strengths achieved in these tests are too high. Using the ACI critical section ($d/2$ from column face) the stresses for the full panel tests ($d = 40.5\text{mm}$) are as follows: $0.495\sqrt{f'_c}$ for $\rho_l = 0.517\%$, $0.521\sqrt{f'_c}$ for $\rho_l = 0.802\%$, and $0.692\sqrt{f'_c}$ for $\rho_l = 1.107\%$. The effect of the boundary forces created along the simply supported perimeter ($0.4l$) which also increase the shear strength is included in these tests. In a real slab these boundary forces are missing so that the strength of a real slab is expected to be lower than the results of Rankin and Long. In addition, in a real slab restrained shrinkage introduces tensile stresses which reduce the shear strength.

Interesting in this context is the approach by Nölting (1984), which for a slab with depth d predicts an increase in the shear strength with an increase in its span length. This may be attributed to the compressive membrane action. However, this increase is till a certain value of the span, after that an increase in the span results in a decrease in the shear strength.

10.1.2.2 Slab subsystems

Long (1978) tested the slab subsystem shown in Fig. 10.1-7. Although it is believed that such a system is more realistic than single column tests, two aspects are of concern in this model. First the simply supported edge identified in Fig. 10.1-7 creates boundary forces with unknown effect on the punching shear resistance of the slab. Secondly, the thickness of the slab of only 63.5 mm is small so that the shear failure stresses are influenced by the size effect. This effect is difficult to separate from the other parameters influencing the punching shear strength. Sherif (1996) used a test set-up to test full scale 150 mm thick, 5.0 m span continuous reinforced concrete flat slabs. The test set-up facilitates the testing of continuous slabs by providing realistic boundary conditions along lines of zero shear of part of a slab centred about an exterior column and the adjacent interior column (see Fig. 10.1-8a). Boundary frames allowing vertical deflections but no

rotation about axes parallel to the slab edges provide these edge conditions (Fig. 10.1-8b). The columns are hinged at the top and the bottom of their midheights, i.e., 1.5 m. The loading system consisting of a series of beams equally distributes the actuator load to 16 points in the full span and 8 points in the half span. According to a finite element analysis this can be considered as a uniformly distributed load. A schematic of the test set-up is shown in Fig. 10.1-9. Based on the results of the tested slabs, Sherif (1996) concluded that the punching shear strength of interior slab-column connections is the same for isolated slab-column and full slab tests. Sherif and Dilger (2000) modified the test set-up and used two full spans as shown in Fig. 10.1-10 to test a high strength concrete flat slab.

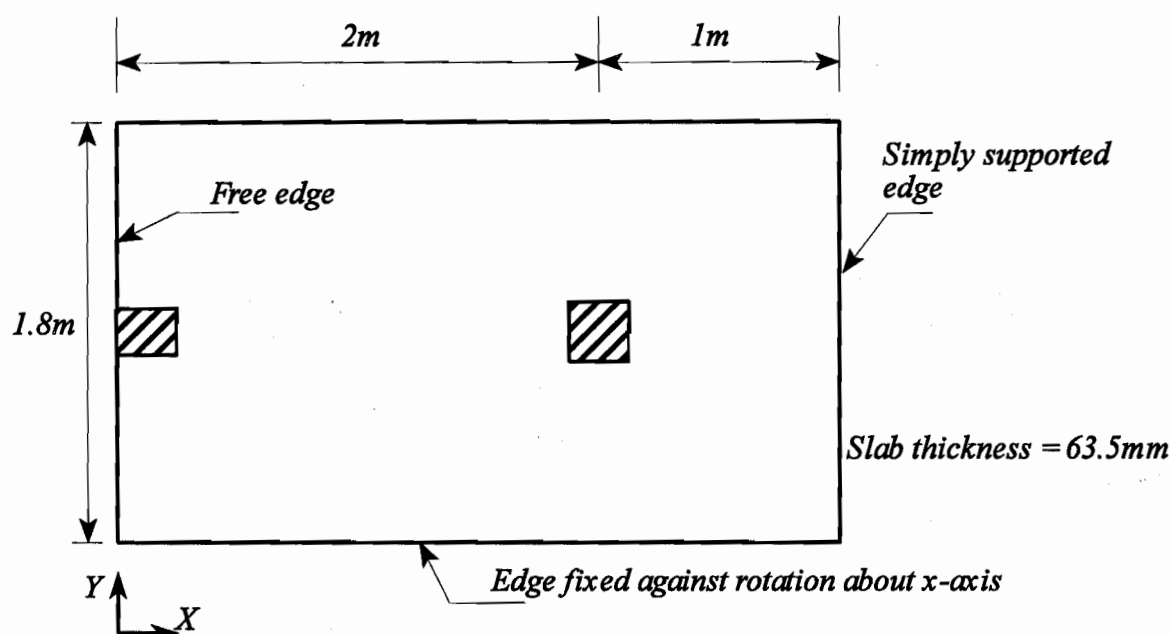
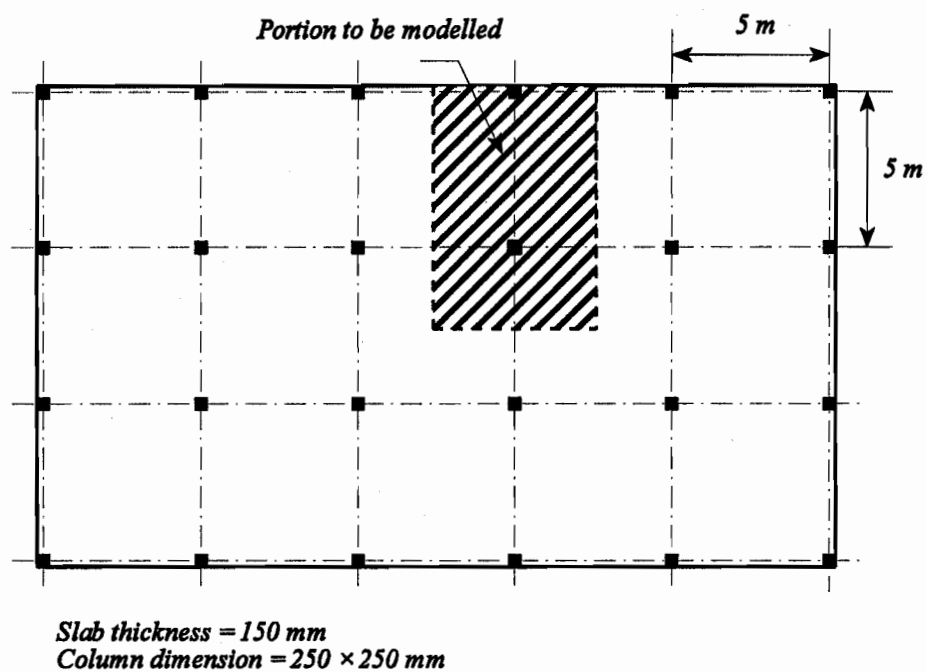


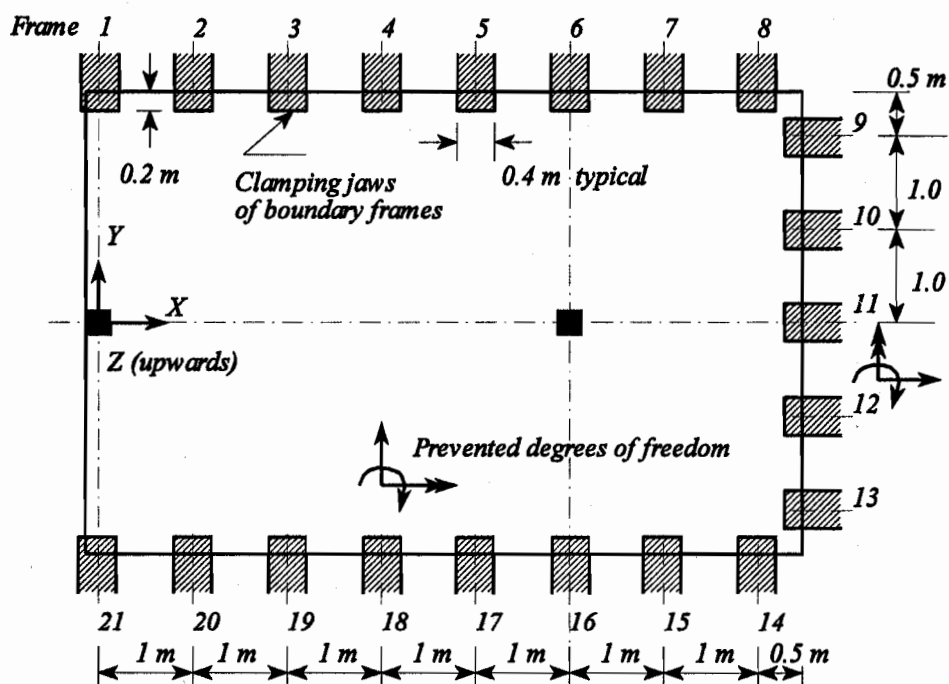
Fig. 10.1-7: Slab model tested by Long (1978)

10.1.3 Conclusions

In a single column test set-up the punching shear strength is increased by boundary forces created along the supported edges of the slab specimen. On the other hand, the compressive membrane action which enhances the punching strength of a real slab is not accounted for. However, in a real slab restrained shrinkage introduces tensile stresses which reduce the shear strength. Based on test results of full scale continuous slabs by Sherif (1996), it can be concluded that the punching shear strength of an interior slab-column connection in a real slab is about the same as for a single column test specimen. Thus, for design purposes it is recommended not to rely on the compressive membrane action in order to increase the punching shear strength of an interior slab-column connection.



(a) Plan of a typical flat slab floor



(b) Boundary conditions of symmetry provided by the boundary frames

Fig. 10.1-8: Full scale continuous slab tested by Sherif (1996)

[illegible]

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10.2 Punching of High Strength Concrete Slabs

Mikael Hallgren

10.2.1 Introduction

Design methods and empirical expressions in codes, for the calculation of the punching shear resistance of concrete slabs, are usually based on tests on slabs with concrete compressive strengths of 20 to 45 MPa, and in a few cases up to 55 MPa. However, modern concrete technology has made it possible to achieve compressive strengths of 100 MPa and higher, using conventional cement and conventional normal density aggregate (Helland et al. 1990). By adding water-reducing superplasticizers, the water-cement ratio can be reduced without affecting the workability of the fresh concrete. This is the main basis for high strength concrete (HSC). Furthermore, the addition of pozzolans, such as silica fume, increases the workability as well as the strength of concrete. HSC is usually defined as concrete with compressive strengths exceeding 60 MPa, measured on standard cylinders. This definition is also adopted here.

The compressive strength of HSC can, thus, today be more than three times higher than that of normal strength concrete (NSC). However, other mechanical properties, such as the tensile strength and the modulus of elasticity, do not increase proportional to the compressive strength, but increase at a lower rate (see e.g. Hallgren and Kinnunen 1993b). Neither does the shear strength of reinforced beams without shear reinforcement increase at the same rate as the compressive strength of concrete (see e.g. Hallgren 1994a).

Hence, as HSC now is feasible, the question arises whether the common design methods and empirical expressions for the assessment of punching shear resistance are applicable for concrete with strengths exceeding 55 MPa, which was the highest strength used in punching shear tests before 1990. In many codes the punching shear resistance is set proportional to the compressive strength of concrete to the power of n , where n is less than unity. In comparison with existing test data with compressive strengths up to 55 MPa, it has been found that expressions where $n \geq 1/2$ can overestimate the influence of concrete strength on the punching shear resistance (Braestrup 1989 and Gardner 1990). Nevertheless, the test data indicated that the punching shear strength increases with increasing concrete strength. The question is how much the punching shear strength can be increased by using HSC.

Many of the known accidental punching shear failures in buildings occurred during construction (Kaminetzky and Stivaros 1994). The reasons were mostly insufficient early-age strength and premature removal of the formwork. HSC can reach 1-day strength which is at the same level as the 28-days strength of NSC. Thus, with HSC placed in flat slabs it could be possible to speed up construction time by early removal of the formwork without jeopardising the safety of the structure.

10.2.2 Punching Shear Test of High Strength Concrete Slabs

Since 1990, several investigations on the punching shear strength of high strength concrete slabs have been conducted. Those are the tests by Marzouk and Hussein (1991), Hallgren and Kinnunen (1991, 1996), Tomaszewicz (1993), and Regan et al. (1993, 1996). In these tests, the compressive cylinder strength of concrete f_{cc} ranged up to 120 MPa. The tests by Tomaszewicz and Regan et al. also included HSC with light-weight aggregate.

In a comparison between test results and code predictions, the expression given in the CEB-FIP Model Code 1990, where the punching shear strength is set proportional to $f_{cc}^{1/3}$, gives consistent predictions for all slabs, see Fig. 10.2-1. Hence, the assumption that the punching shear strength increases proportional to $f_{cc}^{1/3}$ seems to agree with the available test data for compressive strengths of concrete up to 120 MPa. As previous research as well as most codes show, the tensile strength is proportional to f_{cc} to the power of 1/2 to 2/3, this indicates that the punching shear strength is not linearly proportional to the tensile strength of concrete, but increases at a lower rate.

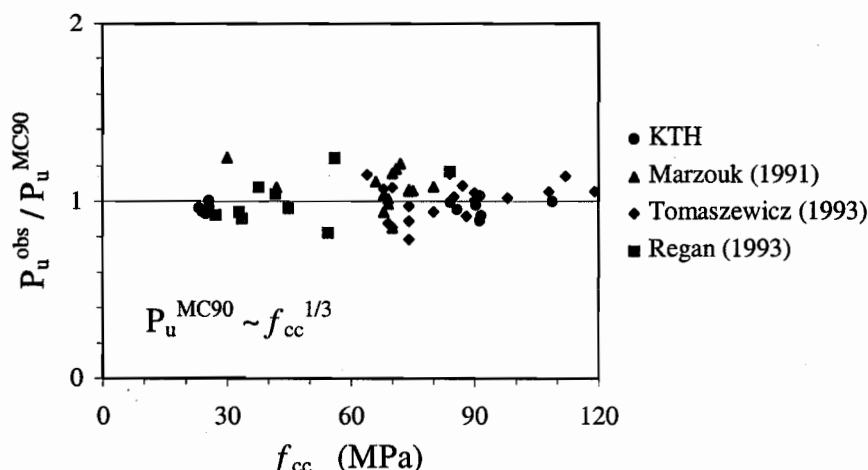


Fig. 10.2-1: Ratio between observed punching loads P_u^{obs} and calculated ultimate loads P_u^{MC90} , according to the Model Code 1990, versus the compressive strength of concrete f_{cc} . Safety factors removed [Hallgren and Kinnunen (1996)]

Nevertheless, a comparison between HSC slabs with compressive strength of concrete $f_{cc} \approx 90$ MPa, and identical previously tested NSC slab, with $f_{cc} \approx 25$ MPa, shows that a significant increase of the punching shear strength can be gained by using HSC (Hallgren 1996). The HSC slabs with flexural reinforcement ratio $\rho = 0.008$ had punching shear strengths which were about 50 to 60 percent higher than the punching shear strengths of the identical NSC slabs, see Fig. 10.2-2.

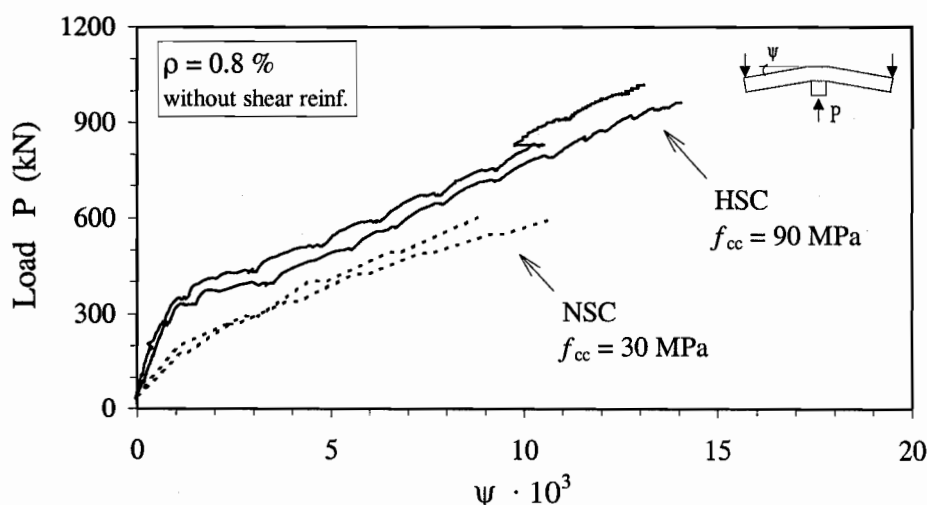


Fig. 10.2-2: Load P versus angle of rotation ψ of circular HSC and NSC slabs with reinforcement ratio $\rho = 0.008$ and without shear reinforcement [Hallgren (1996)]

For slabs with $\rho = 0.003$, the corresponding increase was only 20 percent, see Fig. 10.2-3. However, the HSC slab with $\rho = 0.003$ behaved in a ductile manner, reached its flexural load capacity, and failed in flexural punching. The identical NSC slab with $\rho = 0.003$ failed in brittle punching before reaching its flexural load capacity. This indicates a more efficient use of the flexural reinforcement in HSC slabs than in NSC slabs. The HSC slabs had an initially stiffer load-deflection behaviour, but failed at larger deflections than the identical NSC slabs. Thus, the HSC slabs had higher deformation capacities than the corresponding NSC slabs.

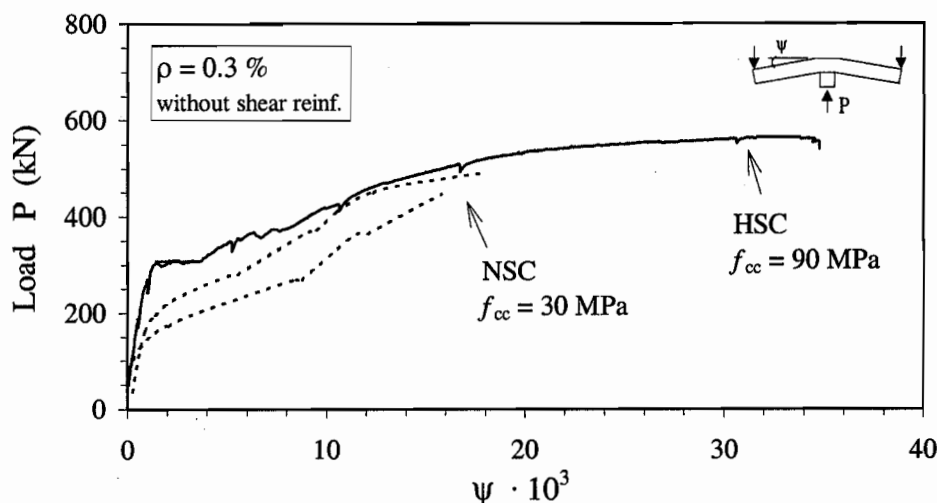


Fig. 10.2-3: Load P versus angle of rotation ψ of circular HSC and NSC slabs with reinforcement ratio $\rho = 0.003$ and without shear reinforcement [Hallgren (1996)]

10.2.3 Optimizing Flat Slabs by using HSC

The most obvious application of HSC so far has been in highly loaded columns. The cross-sectional dimensions of the columns can be reduced considerably by using HSC instead of NSC. A problem in connection to this is to transfer high compression stresses from slender HSC columns through slabs made of NSC. Tests by Gamble and Klinar (1991) showed that the ratio between the compressive strength of the column concrete and the compressive strength of the slab concrete should not exceed 1.4. In North America, a common solution to the problem is to cast the intervening slab portion between HSC columns with the same concrete as used in the columns (Moreno 1990). If the punching shear strength can be increased hereby, a dual benefit can be achieved.

Hallgren (1993) showed that major cost savings can be gained by using HSC in flat slabs assuming that current code expressions are applicable for HSC also. As higher concrete strength hardly increases the flexural capacity of an under-reinforced cross section, which is usually the case in slabs, and as HSC presumably is more expensive than NSC, it seems not to be very rational from an economical and structural point of view to cast the complete slab in HSC, at least not in ultimate limit state design. For design in the service limit state, the use of HSC in the complete slab may decrease the deformations [Favre et al. (1992)].

However, by using HSC in slabs locally over the columns as described above, it might be possible to reduce the slab thickness and/or preserve the punching shear resistance when thick NSC columns are replaced by slender HSC columns. Some full scale applications of this method have been tested in Finland [Ålander (1991) and Vahanen (1995)]. Punching shear test with prefabricated column heads made of concrete with higher strength than the strength

of the in-situ cast slab concrete have been reported by Ramm and Gastmeyer (1993) and by Ajdukiewicz and Kliszczewics (1993). Berge and Petterson (1983) increased the punching shear strength of a light-weight aggregate concrete test slab by casting a small slab portion above the column with normal density concrete.

In Hallgren (1996), one of the tested slabs was provided with HSC locally at the column support, while the remaining part of the slab was cast with normal strength concrete. The two different concretes were cast wet to wet, i.e. they were cast within one hour. The ratio of flexural reinforcement was 0.008 and the slab had no shear reinforcement. Fig. 10.2-4 shows the cross-section of the slab. It was possible to clearly see the difference between the HSC and the NSC. Due to the use of a different cement type and the addition of silica fume, the HSC had a significantly darker colour than the NSC. This difference could be seen for the fresh concrete as well as for the hardened concrete. The colour difference should be of help in the quality assurance of possible future applications of this method of casting.

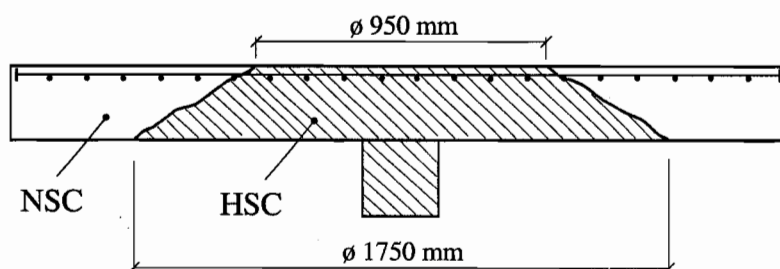


Fig. 10.2-4: Test slab with HSC placed locally over the column [Hallgren (1996)]

The test on the slab with HSC placed locally above the column and with NSC cast wet-to-wet around the HSC portion showed essentially the same good performance as the slabs made completely of HSC, see Figure 5. The punching shear strength of the slab was approximately equal to the punching shear strength of the identical slabs made completely of HSC, and was about 60 percent higher than the punching shear strength of the identical NSC slabs. The stiffness of the slab was slightly lower than the stiffness of the identical slabs made completely of HSC. The deflection of the slab at failure was larger than the failure deflections of the identical HSC slabs.

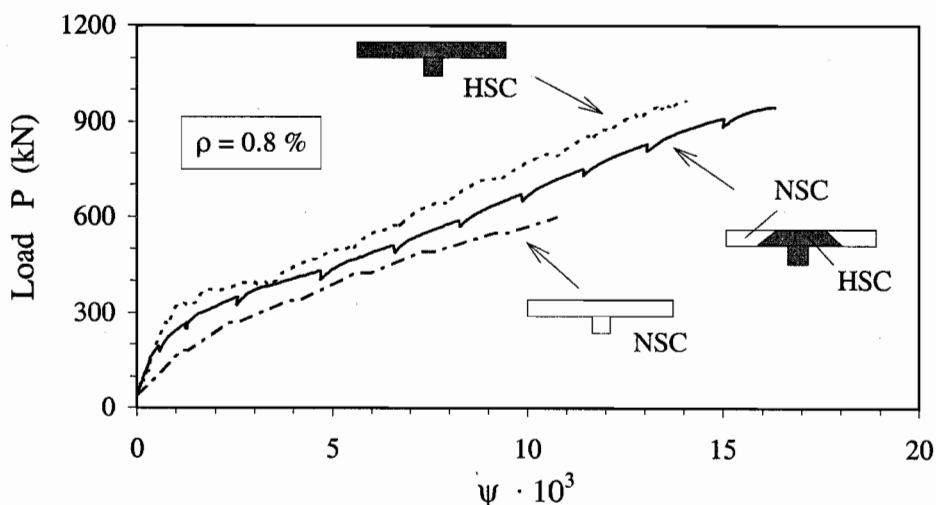


Fig. 10.2-5: Load P versus angle of rotation ψ of a HSC slab, a NSC slab and a slab with HSC locally placed over the column. All slabs with reinforcement ratio $\rho = 0.008$ and without shear reinforcement [Hallgren (1996)]

10.3 Slabs with shear reinforcement

Rüdiger Beutel, Josef Hegger

10.3.1 Introduction

The economic efficiency of each shear system in the punching zone is influenced by the following criteria:

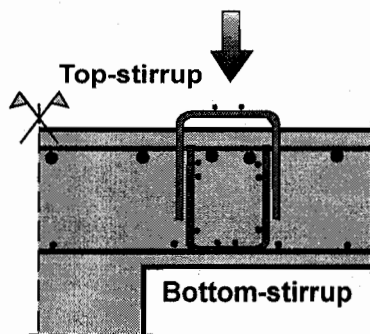
- (a) maximum utilisation of the material properties,
- (b) Anchorage behaviour within the tension and compression zone,
- (c) strengthening of the compression zone at the column face,
- (d) ductility,
- (e) cost in terms of labour for placing– simplicity and speed in assembling the flexural reinforcement bars and the shear system,
- (f) interaction between the arrangements of shear system and the flexural reinforcement,
- (g) material and production costs.

In practice there are several types of shear systems used e.g. stirrups, bent-up bars, double headed studs or stud rails ... etc. as shown in Fig. 9-1. The economic efficiency of conventional stirrups in the punching zone is reduced in case of flat slabs because of the following items:

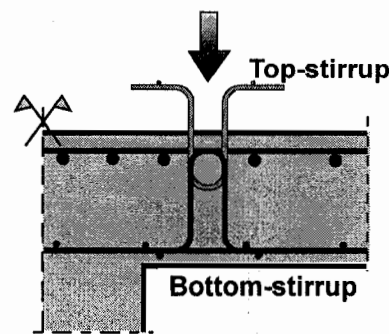
- The material yield strength cannot be reached as a result of the anchorage slip. Therefore, it is necessary that conventional stirrups enclose one layer of the tension and compression reinforcement to improve the anchorage behaviour.
- Caused by this detailing rule the placing is costly in terms of labour on site.

This section reports about one test without shear reinforcement, two tests with conventional stirrups (Type II: Fig. 9-1a (Type a)) and eight tests with new stirrup forms made of fabric reinforcement (Type: I and III, Fig. 10.3-1a and b) which are easy to place between the flexural reinforcement layers. The following points are investigated:

- (1) The efficiency of conventional stirrups in slabs with a full scale thickness of 230 mm.
- (2) The efficiency and the maximum shear capacity of stirrups made of fabric reinforcement.
- (3) Arrangement and detailing rules for stirrups.



a) Layout I: Stirrups made of fabric reinforcement with a vertical splice and without enclosing the bottom flexural reinforcement.

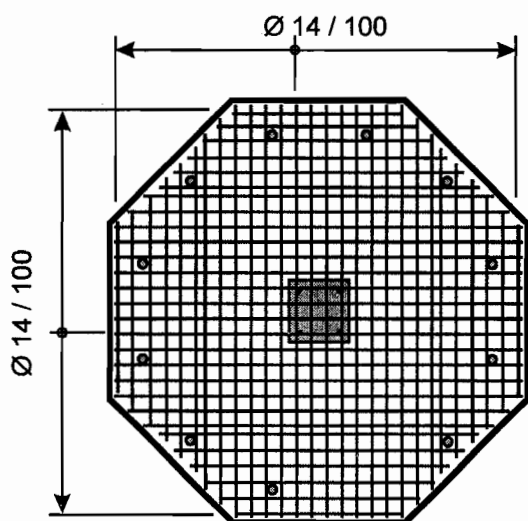


b) Layout III: Stirrups made of fabric reinforcement with a vertical splice and enclosing one direction of the top and bottom flexural reinforcement.

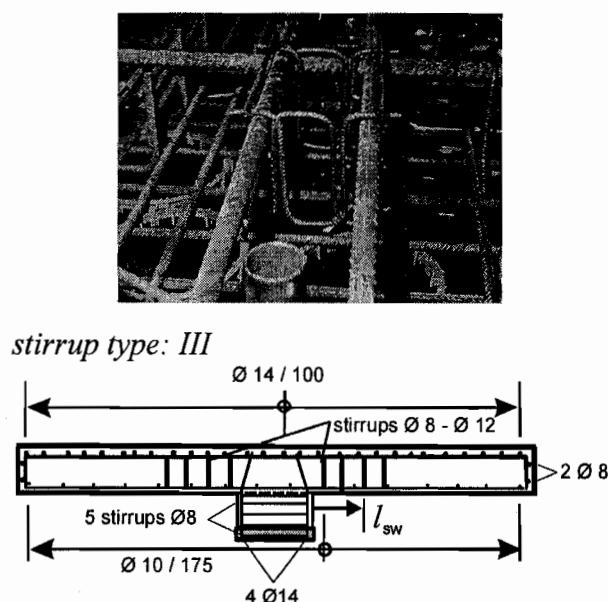
Figure 10.3-1: New stirrup types made out of fabric reinforcement in the punching zone.

10.3.2 Test Set-Up

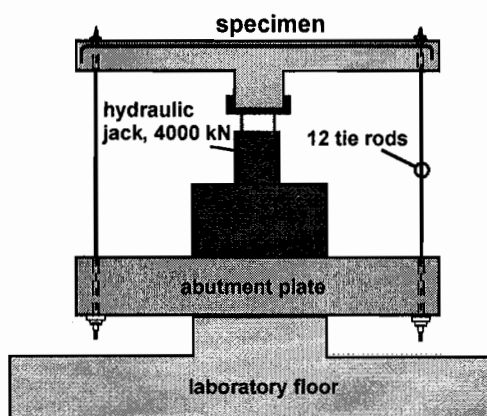
The test specimens represent the full scale part of a flat plate structure extending from an interior column to the line of contra flexure for a centric loaded interior column (Fig. 10.3-2c). The specimens had a slab thickness varying between 230 and 275 mm, a total diameter of 2750 mm, a loading diameter of 2400 mm and a square column with a side length of 400 and 320 mm (Fig. 10.3-2a and b). The mean yield strength of the reinforcement was 580 MPa. Each specimen was carefully instrumented with dial gauges and special photogrammetric points to determine deflections and rotations of the specimens. 100 electrical strain gauges were used to measure strains at selected locations, e.g. on the flexural reinforcement, the stirrups and on the concrete compression zone direct at the column face.



a) Top layer of the flexural reinforcement P1 - P2-III.



b) Cross section of the specimens P1 - P2-III.



c) Test set-up

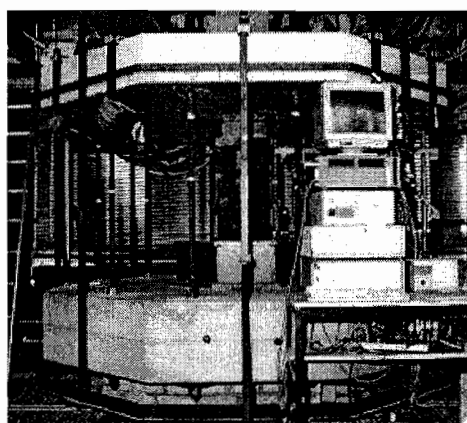


Figure 10.3-2: Reinforcement of the specimens P1 to P2-III and the test set-up.

10.3.3 Test Results

The failure loads of the specimens and further relevant information are given in Table 10.3-1.

1	2	3	4	5	6	7	8
no.	d	a/b	f_c	ρ_t, ρ_c	l_{sw} / A_{sw}	V_u	failure mode
	[m]	[m]	[MPa]	[%]	[m], [cm ²]	[kN]	
P1	0,19	0,4 / 0,4	21,9	0,806 0,235	-	615	punching
P1-I	0,19	0,4 / 0,4	27,3	0,806 0,235	0,110 / 11,0; 0,20 / 11,0 Σ 22,0	1151	punching outside the shear rein- forced area.
P1-II	0,19	0,4 / 0,4	26,2	0,806 0,235	0,100 / 12,57; 0,200 / 9,43 Σ 22,0	1055	punching outside the shear rein- forced area.
P2-I	0,19	0,4 / 0,4	37,9	0,806 0,230	0,075 / 7,04; 0,175 / 7,04 0,250 / 7,04; 0,350 / 10,05 Σ 31,2	1326	punching outside and inside the shear reinforced area.
P2-II	0,19	0,4 / 0,4	29,8	0,806 0,235	0,060 / 4,02; 0,172 / 13,07 0,362 / 15,08; Σ 32,2	1109	punching inside the shear rein- forced area
P2-III	0,19	0,4 / 0,4	37,5	0,806 0,235	0,065 / 7,04; 0,165 / 7,04 0,260 / 7,04; 0,365 / 10,05 Σ 31,2	1276	bending
P3-I	0,22	0,32 / 0,32	23,2	1,15 0,413	0,085 / 6,03; 0,189 / 9,05 0,355 / 8,04; 0,459 / 15,08 0,685 / 5,03; Σ 43,3	1624	punching inside the shear rein- forced area
P4-III	0,22	0,32 / 0,32	27,8	1,132 0,408	0,105 / 6,03; 0,205 / 10,50 0,365 / 8,04; 0,465 / 16,08; 0,615 / 4,02; Σ 44,2	1522	punching inside the shear rein- forced area, the
P5-I	0,22	0,32 / 0,32	45,3	1,354 0,409	0,115 / 15,71; 0,255 / 18,85 0,400 / 11,00; 0,540 / 25,13 0,690 / 18,85; Σ 44,2	1936	tests were fin- ished before the ultimate load was reached to study the internal shear crack pattern
P6-I	0,22	0,32 / 0,32	46,3	1,753 0,550	0,100 / 22,62; 0,268 / 31,67 0,428 / 22,62; 0,596 / 31,67 0,718 / 22,62; Σ 131,2	2349	punching inside the shear rein- forced area
P7-I	0,23	0,32 / 0,32	40,0	1,301 0,393	0,145 / 18,85; 0,285 / 25,13 0,455 / 15,71; 0,595 / 21,99 0,740 / 15,71; Σ 97,4	2117	punching inside the shear rein- forced area

1: Specimen number with indication of the stirrup type, Type I + III: (Fig. 10.3-1a and b), Type II: conventional stirrups (Fig. 9-1 a-left); 2: effective depth; 3: column side length; 4: concrete cylinder strength; 5: tension (ρ_t) and compression (ρ_c) flexural reinforcement ratio; 6: l_{sw} : distance between the column face and the stirrup row, A_{sw} : shear reinforcement cross section area (each row and sum of all rows); 7: failure load; 8: failure mode

Table 10.3-1: Failure load and details of the test specimens

10.3.3.1 Deformations

The results of the 3D photogrammetric deflection measurement show that the deformations of all specimens were axi-symmetric. All measurements above the serviceability limit state indicated that the diameter of the area with high rotations is approximately limited to the column diameter plus the effective depth (Fig. 10.3-3). Outside this area the deflection curve forms a straight line without any significant rotations.

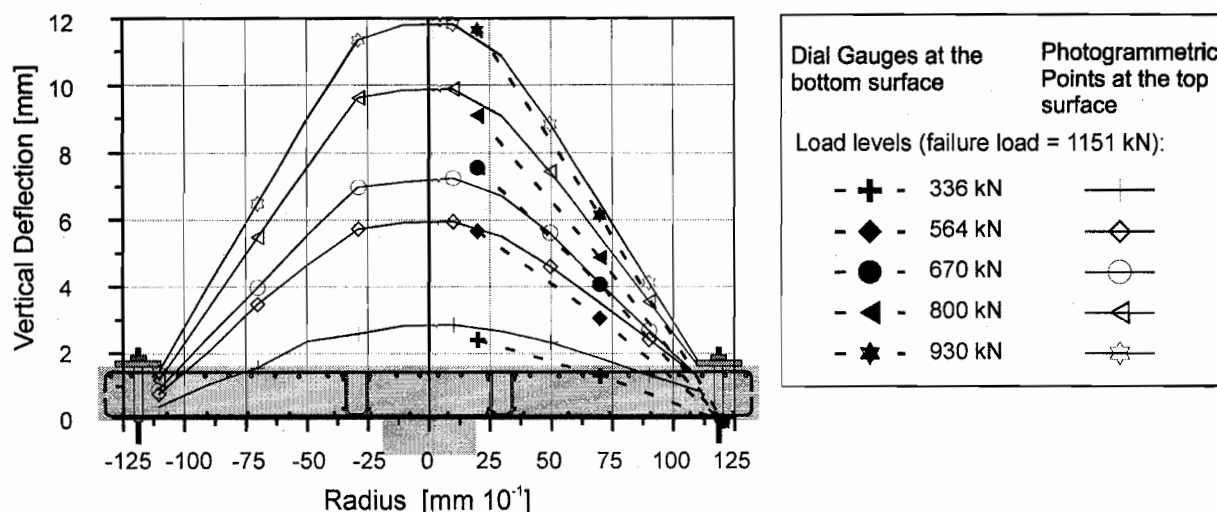


Figure 10.3-3: P1-I: Vertical Deflection versus Radius for selected load levels

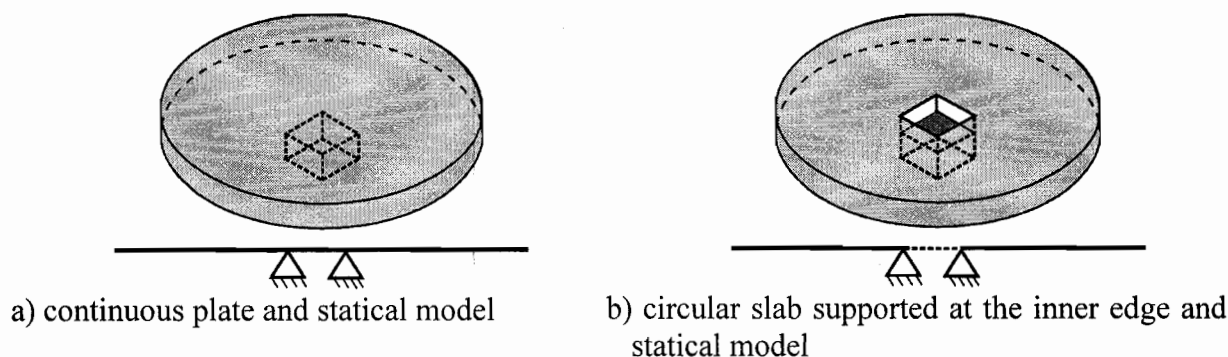


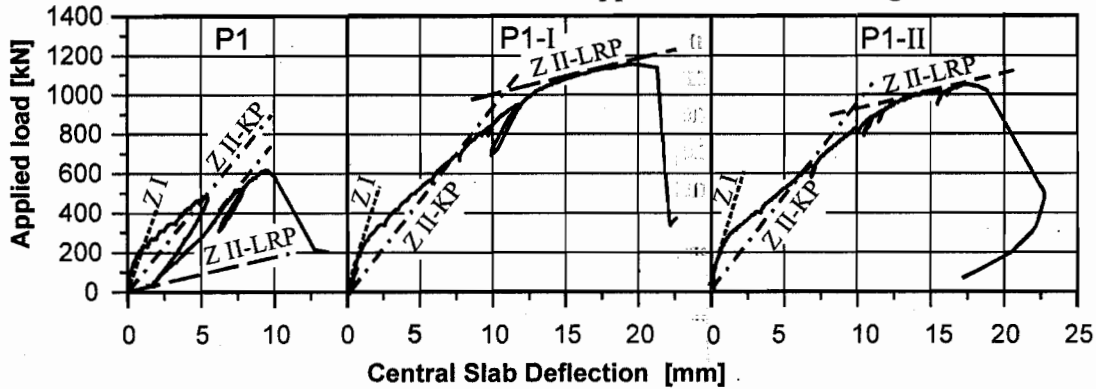
Figure 10.3-4: A continuous plate and a hole edge supported plate in the area of the column

This deflection behaviour and the tangential stiffness of the area of ultimate load (line: Z-II-LRP) (Fig. 10.3-5) indicated that the statical model at ultimate load can be described as a circular slab which is supported at the inner edge (Fig. 10.3-4). In addition to this, models with ridge sector elements e.g. Kinnunen and Nylander (1960), Andersson (1963) or Gomes (1991) are applicable.

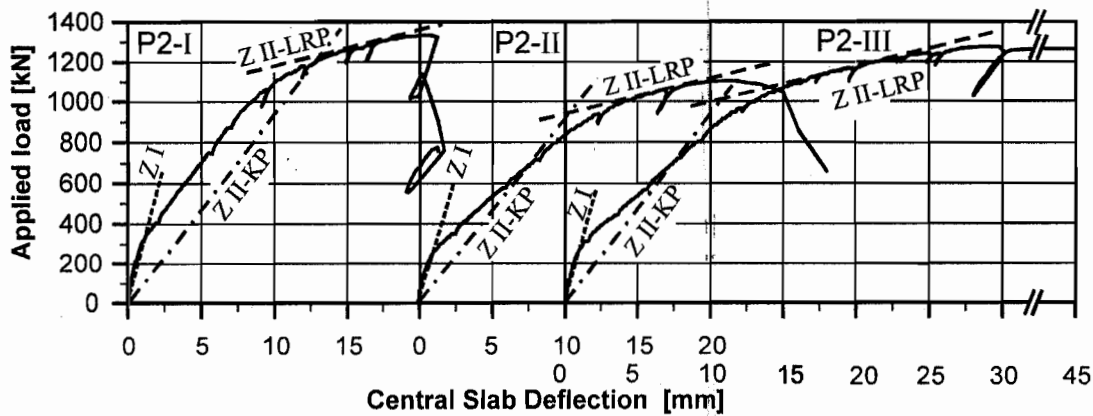
The model of a circular slab which is supported at the inner edge was already observed by Andrä (1982) for punching specimens with a loop flexural reinforcement and without shear reinforcement. The deflection curve of test P1 (without shear reinforcement, Fig. 10.3-5a) shows that in the case of an orthogonal flexure reinforcement the tangential stiffness above the serviceability limit state is equal to a cracked continuous plate (line: Z-II-KP, Fig. 10.3-5). This model of a continuous plate gives a good approximation for the secant stiffness up to serviceability load level (line: Z-II-KP, Fig. 10.3-5) for shear reinforced specimens.

This variation of the statical model during loading confirms the theory by Brändli (1984). He started from the assumption that the moment distribution can be described by a plasticity approach and he concluded that the punching shear capacity reaches a maximum value if the

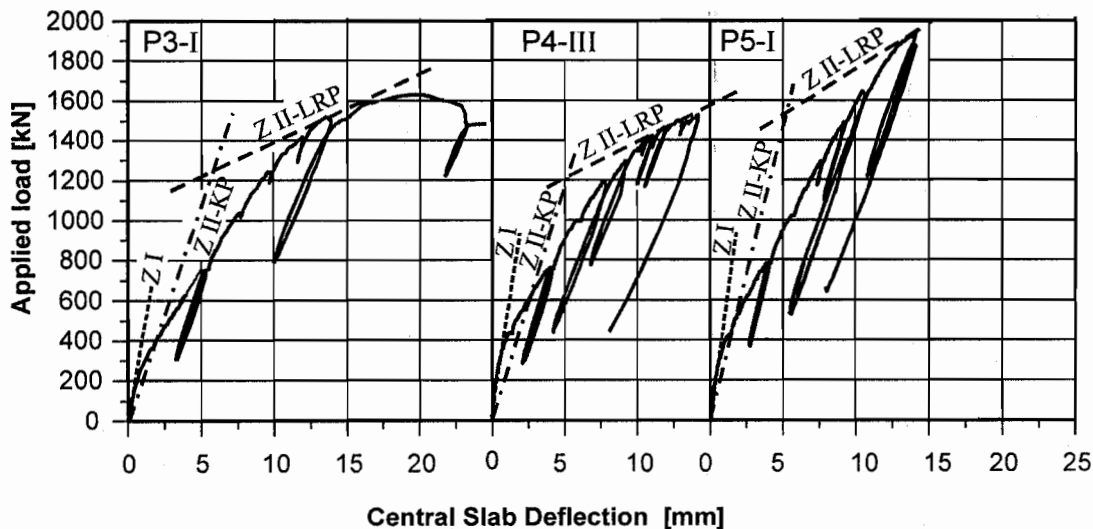
main part of the bending moments is carried in the tangential direction. This fits well with the moment distribution of a circular slab which is supported at the inner edge.



a) Test series P1: flexural reinforcement ratio of $\rho_l = 0,8\%$, P1-I and P1-II with two rows of stirrups within a radius of $1.2 d$ (measured from the column face).



b) Test series P2: flexural reinforcement ratio of $\rho_l = 0,8\%$, four rows of stirrups within a radius of $1,9 d$ (measured from the column face).



c) Test series 3: flexural reinforcement ratio of $\rho_l = 1,13\%$ to $1,35\%$, four rows of stirrups within a radius of $2,5 d$ (measured from the column face).

Figure 10.3.5: Applied load versus central slab deflection for nine specimens

10.3.3.2 Crack Formations

The development of the cracks on the surface in tension started with tangential cracks near the inner column face, followed by radial cracks from the column to the free edges and at last new tangential cracks were formed. Finally one of the tangential crack formed the punching cone (Fig. 10.3-7)

The design rules of Eurocode 2, Part 1 and DIN 1045(88) demand for stirrups to extend a distance of $1,2 \cdot d$ measured from the column face. Specimen P1-I and P1-II which have been reinforced according to this failed outside the shear reinforced area (Fig. 10.3-6 and 10.3-7) in a very brittle way (fig. 10.3-5a) before the stirrups had reached their ultimate strength. Tests with double headed studs and stud-rails which were placed up to a distance of $2,0 d$ from the column face failed in the same mode (Fig. 10.3-8). The crack formation of the saw-cut of specimen P1-I (Fig. 10.3-7) indicates that this failure can be explained by a simple strut and tie model.

From these observations it can be concluded that the design and detailing rules of Eurocode 2, Part 1 and DIN 1045(88) are not sufficient to avoid such a failure mode. Model Code 90 takes into account this failure mode but the ultimate loads of the specimens P1-I, P1-II and P2-I indicate that the design shear capacity in the exterior control perimeter of Model Code 90 is a mean value and not a characteristic one.

In the case of a staggered shear reinforcement within a radius of more than $2,0 d$ between the column face and the last stirrup row a multiple shear crack formation was observed (Fig.10.3-9 a). This crack pattern is similar to that of shear failure in beams. The crack pattern in the saw-cut of specimens P4-III and P5-I which were not loaded up to ultimate load indicated that the shear crack formation outside the distance $1,2 d$ began to start at a level of 80% of the ultimate load. It has to be pointed out that for all tests with a multiple shear crack formation the failure was combined with a concrete crushing of the compression zone at the column face (Fig. 10.3-9 b).

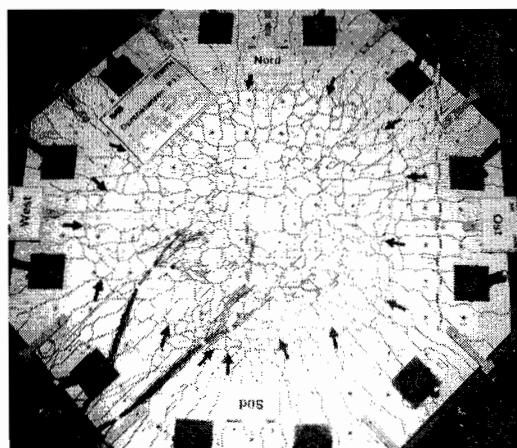


Figure 10.3-6:
P1-I: Crack pattern on the tension surface, the exterior punching cone is marked by arrows

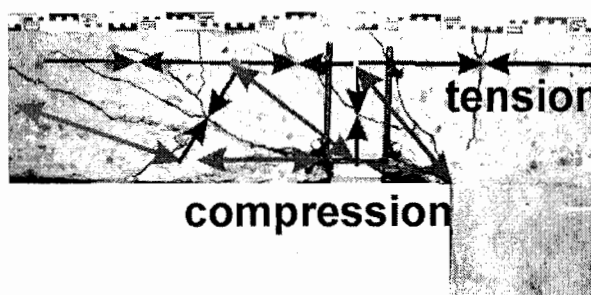


Figure 10.3-7:
P1-I: Punching in the Exterior Control Perimeter - strut and tie model based on the crack pattern.

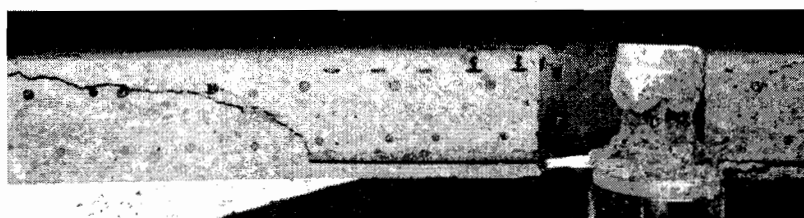
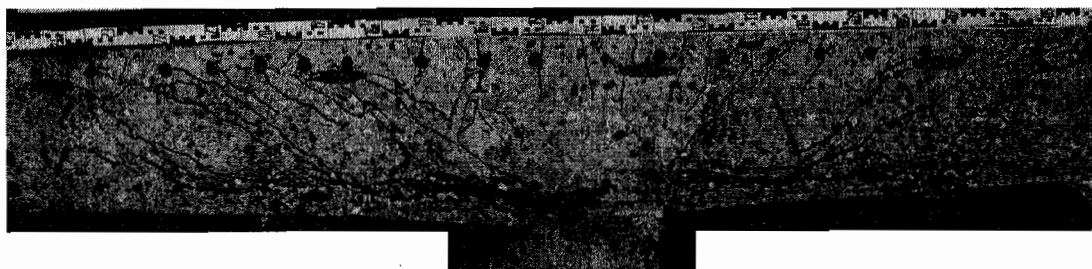
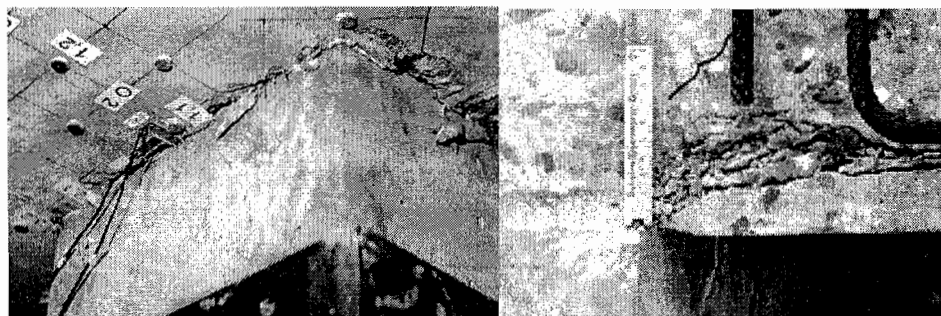


Figure 10.3-8: Punching in the Exterior Control Perimeter of a specimen with Stud Rails as shear reinforcement, Stangenberg (1992)



a) Saw-cut with a multiple shear crack formation in the punching zone



b) Concrete crushing of the compression zone at the column face

Figure 10.3-9: P7-I: Crack formation in a saw-cut after failure and concrete crushing at the column face at ultimate load

10.3.3.3 Flexural Reinforcement, Bond Stresses and Concrete Strain

Figure 11 shows the strain distribution of the tension reinforcement, the compression reinforcement and the concrete strain in the radial and tangential direction for specimen P4-III. This distribution of a brittle punching failure is characterised by the following points:

- (a) The tension reinforcement did not reach the yield strength.
- (b) The compression reinforcement was only strained at a low level.
- (c) The radial concrete strains were concentrated in the proximity of the column.
- (d) The radial concrete strains decreased at the column face before the ultimate load was reached.
- (e) The tangential concrete strains were higher and less concentrated than the radial concrete strains.

The calculation of the maximum bond stresses during loading according to Model Code 90 (Table 10.3-2) shows that a classical bond failure was not critical for all specimens.

By evaluating the critical bond stresses of splitting failure of the concrete cover according to Alexander and Simmonds (1992) (Table 10.3-3) this failure mode occurs for five out of eleven specimens. The saw-cut of specimen P7-I (Fig. 10.3-9a) supports the latter approach. It is visible that the shear cracks tend to be horizontal in the tension reinforcement layer. This fits well to the bond strength according to Alexander and Simmonds (1992). But the authors assume that this crack formation could be caused by the dowel shear strength of the tension reinforcement. However, it is shown that a splitting of the concrete cover can be critical and therefore stirrups have to enclose the tension reinforcement to avoid such a failure mode.

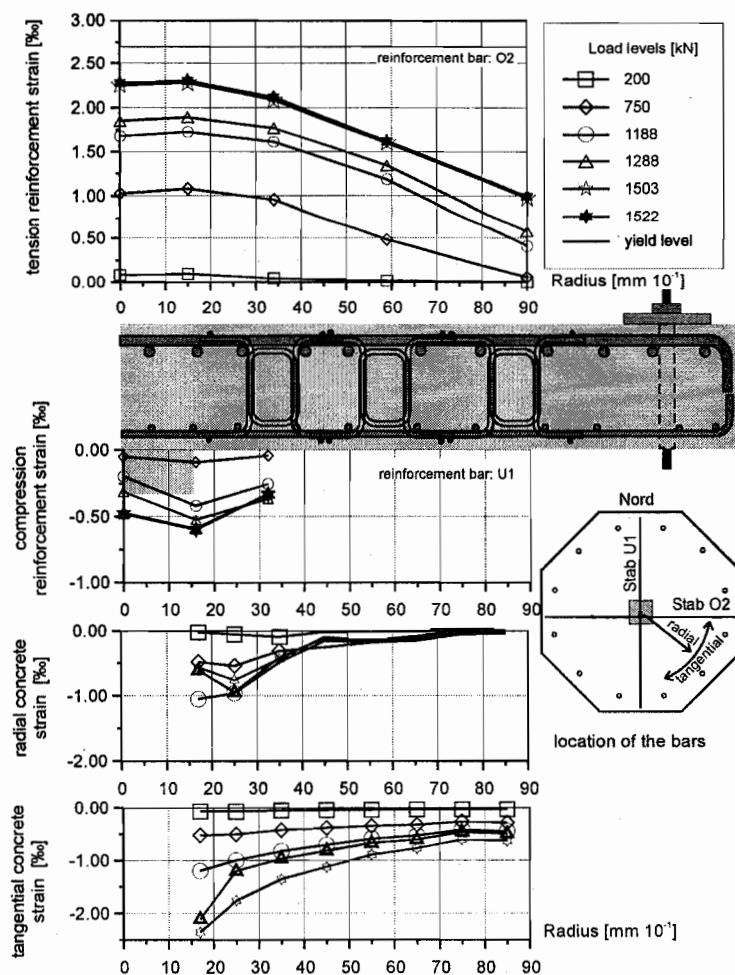


Figure 10.3-10: P4-III: reinforcement and concrete strains

1	2	3	4	5	6	7	8
no.	d_s	E_s	f_c	$\Delta\epsilon/10d_s$	τ_m	τ_u	γ_{bond}
	[mm]	[GPa]	[MPa]	[‰/m]	[MPa]	[MPa]	[-]
P1	14	218,5	21,9	3,6	2,8	9,4	3,4
P1-I	14	218,5	27,3	4,2	3,2	10,4	3,3
P1-II	14	218,5	26,2	3,5	2,7	10,2	3,8
P2-I	14	203,2	37,9	4,7	3,3	12,3	3,7
P2-II	14	203,2	29,8	5,3	3,8	10,9	2,9
P2-III	14	203,2	37,5	5,5	3,9	12,2	3,1
P3-I	20	203,2	23,2	3,6	3,6	9,6	2,7
P4-III	20	212,0	27,8	2,5	2,6	10,5	4,0
P5-I	20	212,0	45,3	4,4	4,7	13,5	2,9
P6-I	25	204,5	46,3	2,9	3,7	13,6	3,7
P7-I	20	212,0	40,0	4,4	4,6	12,6	2,7

1: specimen number including the stirrup type, 2: flexural reinforcement diameter, 3: Young's – modulus, 4: concrete cylinder strength, 5: observed maximum reinforcement strain at a distance of 10 d_s , 6: mean bond stress, 7: ultimate bond stress according to Model Code 90, 8: safety level

$$\tau_m = \frac{\Delta\epsilon}{10 \cdot d_s} \cdot \frac{E_s \cdot d_s}{4}, \quad \tau_u = 2,0 \sqrt{f_c}, \quad \gamma_{bond} = \frac{\tau_u}{\tau_m}$$

Table10.3-2: Safety factors of all performed tests regarding to a bond failure caused by splitting of the concrete cover according to Model Code 90

1	2	3	4	5	6	7	8	9	8
no.	d_s	s_f	c	E_s	f_c	$\Delta\epsilon$ /10 d_s	τ_m	τ_u	γ_{bond}
	[mm]	[mm]	[mm]	[GPa]	[MPa]	[%/m]	[MPa]	[MPa]	[-]
P1	14	100	25	218,5	21,9	3,6	2,8	3,4	0,82
P1-I	14	100	25	218,5	27,3	4,2	3,2	3,8	1,19
P1-II	14	100	25	218,5	26,2	3,5	2,7	3,7	1,38
P2-I	14	100	25	203,2	37,9	4,7	3,3	4,5	1,35
P2-II	14	100	25	203,2	29,8	5,3	3,8	4,0	1,04
P2-III	14	100	25	203,2	37,5	5,5	3,9	4,4	1,14
P3-I	20	125	30	203,2	23,2	3,6	3,6	3,1	0,85
P4-III	20	125	25	212,0	27,8	2,5	2,6	3,4	1,27
P5-I	20	105	30	212,0	45,3	4,4	4,7	3,6	0,78
P6-I	25	125	25	204,5	46,3	2,9	3,7	3,5	0,95
P7-I	20	105	26	212,0	40,0	4,4	4,6	3,4	0,74

1: specimen number including the stirrup type, 2: flexural reinforcement diameter, 3: spacing of the flexural reinforcement, 4 concrete cover 5: Young's -modulus, 6: concrete cylinder strength, 7: observed maximum reinforcement strain at a distance of 10 d_s , 8: mean bond stress, 9: ultimate bond stress according to Alexander & Simmonds, 10: safety level

$$\tau_m = \frac{\Delta\epsilon}{10 \cdot d_s} \cdot \frac{E_s \cdot d_s}{4}; \tau_u = \sqrt{f_c} \cdot (0,0961 \cdot b_i + 0,1337), b_i = \min\left(\frac{s_f}{d_s} - 1; \sqrt{3} \cdot \frac{2 \cdot (d_s + c)}{d_s}\right); \gamma_{bond} = \frac{\tau_u}{\tau_m}$$

Table 10.3-3: Safety factors of all performed tests regarding to a bond failure caused by splitting of the concrete cover according to Alexander and Simmonds (1992)

10.3.3.4 Stirrup Behaviour

The mechanical efficiency of stirrups and bent-up bars is usually defined as the ratio between the maximum strength reached and the yield strength. This efficiency is judged very differently in the literature and in design codes (Table 10.3-4). Therefore, on one hand the quantity of the provided shear reinforcement changes proportionally to the efficiency of the shear reinforcement, on the other hand the maximum shear capacity is influenced by this mechanical efficiency.

Author or code	Effectiveness of Stirrups and bent-up bars: $\kappa_s = \sigma_s / f_y \cdot 100\% = \sigma_s / 5$ [%]	
	stirrups	bent-up bars
Eurocode 2, Part 1, DIN 1045(88)	100 %	
ACI-318-95	82 %	
BS 8110-97	92 %	
Herzog (1974)	35 %	42 %
Georgopoulos (1987)	22 %	35 %
Kordina (1994)	from 32% to 50%	
Siao (1994)	100 %	

Table 10.3-4: Effectiveness of stirrups and bent-up bars from literature and design codes

The measurements of the stirrup strains and the anchor slip for all tests show that the anchorage behaviour in thin flat slabs is not sufficient to activate the yield strength of the stir-

stirrups. In addition, the comparisons of all tests with tests from the literature confirmed that for a practical slab thickness of more than 200 mm the anchorage behaviour of conventional stirrups is sufficient to cause a failure outside the shear reinforced zone (Fig. 10.3-7) or to activate the stirrups in a second, and third row (Fig. 10.3-12). The evaluation of the stirrups stresses in the highest strained stirrup rows 1 and 2 result in a characteristic value of 70% of the yield strength.

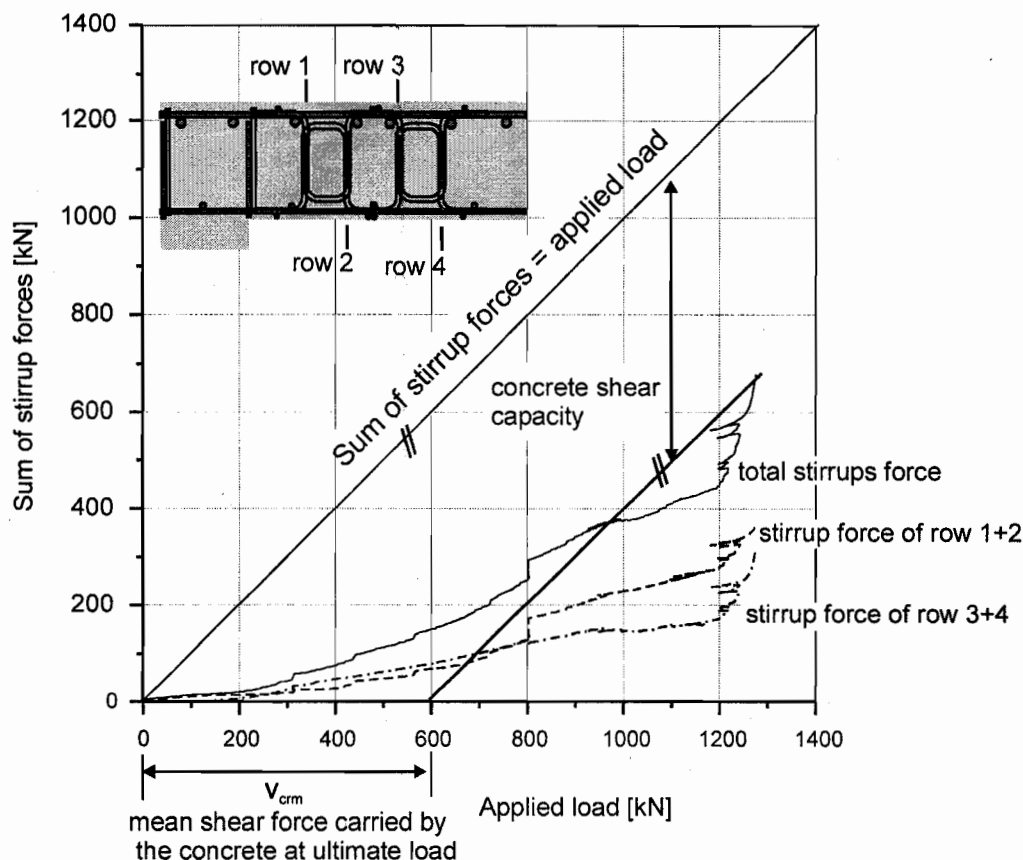


Figure 10.3-11: P2 III: Sum of stirrup forces versus applied load.

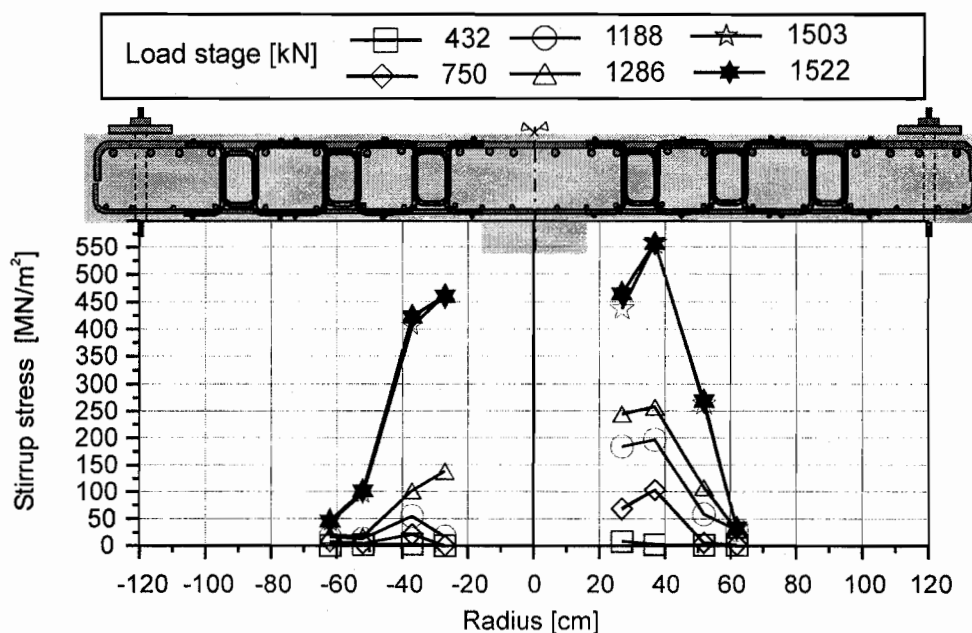


Figure 10.3-12: P4-III: Stirrup stress distribution at different load stages.

The simple truss model in Figure 10.3-7 is based on the assumption that the shear force was only carried by the stirrups. In Fig. 10.3-11 it is shown that this assumption is too conservative because in addition to the truss action there was a considerable contribution of the concrete. A comparison between e.g. the failure load of specimen P1 (without shear reinforcement, $V_u = 615$ kN) and the mean value of the shear force carried by the concrete of specimen P2-III (Fig. 10.3-11, $V_{crm} = 600$ kN) show that these shear capacities fit well together. Also, nonlinear 3D finite element simulations with MASA of the tests P2-II and P6-I confirm the mentioned shear mechanism, (see section 5.3).

Figure 10.3-12 shows the stirrup stress distribution at six load levels of specimen P4-III. From this figure the following conclusion can be drawn:

- (1) The stirrups in row 1+2 carried the main part of the applied shear force.
- (2) The stirrups in row 3+4 were less activated than in row 1+2.
- (3) The stirrups in row 3 were activated at a load stage of about 80% of the ultimate load. This indicates that the rows 1+2 react as a suspension reinforcement and the rows 3+4 as a shear reinforcement.
- (4) The main task of the reinforcement in 3+4 is to guarantee the concrete contribution of the shear capacity and to avoid an early brittle punching failure outside the first stirrup rows.
- (5) The characteristic stirrup stress capacity of the stirrup Types I - III is given by 70% of the yield strength (Fig. 10.3-12, $\sigma_s = 0,7 \cdot f_y = 0,7 \cdot 580 = 406$ MN/m² $\approx$ minimum of the stirrup stress in row 1+2.)
- (6) It has to be pointed out that an essential consequence of this stirrup arrangement is that the ductility of the punching failure was increased.

This measured stirrup behaviour gives a good explanation for the low mechanical effectiveness ratio according to Herzog (1974), Gergopoulos (1987) and Kordina (1994) (Table 10.3-4). It is shown in Fig. 10.3-12 that the shear force was suspended in row 1 and row 2. Therefore, the calculated effectiveness of stirrups result in 0,5 although the stirrups are yielding if a design approach takes into account only one control perimeter. Regarding the anchorage slip the effectiveness decreases to $0,5 \cdot 0,7 = 0,35$.

The high punching capacities of all slabs reinforced with stirrups of Type I (Fig. 10.3-13) had two reasons: (1) The improved anchorage behaviour by welded transverse bars, and (2) the double stirrup cross section area in the shear crack because of the vertical splice. It has to be pointed out that these items are arithmetically considered as a part of the concrete contribution of the shear capacity. The stirrups Type III failed more ductile than the stirrups Type I. The total shear capacity of Type III was in between of Type I and Type II.

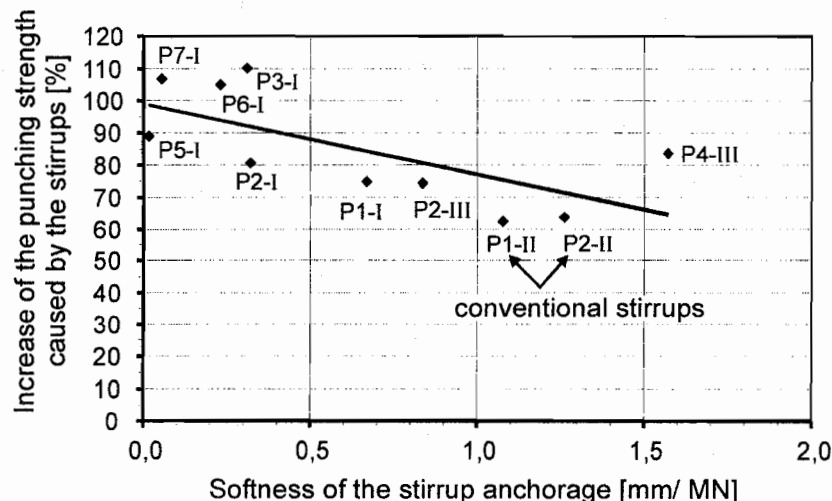


Figure 10.3-13: Increase of the punching shear strength ($V_{test} / V_{punch \text{ without shear reinforcement}} - 1$) * 100 % versus the stirrup anchorage slip gradient at failure.

The different total shear capacities of Type I, II and III (Fig. 10.3-13) were caused by the different concrete contribution to the shear capacities in stirrup rows 1 and 2. However, it has to be taken into account that the concrete contributions are not constant during loading. They decrease proportionally to the ultimate load. At the different ultimate load levels of Type I, II and III the concrete contribution was equal to the concrete contribution of conventional stirrups (Type II). Therefore, the stirrup cross section area of the Types I and III has to be designed like conventional stirrups but their total maximum shear capacities will be increased.

10.3.4 Summary

- (a) The moment distribution of punching specimens at ultimate load with an orthogonal flexural reinforcement and with shear reinforcement can be approximated by a hole edge supported flat slab.
- (b) The anchorage behaviour of conventional stirrups in slabs with a thickness of more than 200 mm is sufficient to cause a failure outside of a shear reinforced area of $1.2 d$ or to activate the following stirrups in the radial direction outside of this area.
- (c) Multiple shear cracks are formed in the punching zone if the shear reinforced area is greater than $1.2 d$. This crack pattern looks similar to that of a shear failure in beams.
- (d) A splitting of the concrete cover caused by the dowel action or by the bond stresses of the tension reinforcement can not be excluded. Therefore, the shear reinforcement has to enclose the tension reinforcement to avoid this type of failure.
- (e) The failure loads show that the investigated shear reinforcement types made of fabric reinforcement have been effective. This is also valid for the stirrup Type I without enclosing the bottom layer of the flexural reinforcement.
- (f) The investigated anchorage types slipped over-proportionally when the flexural reinforcement began to yield. This can be explained by the wide opening of flexural cracks (1,5 to 2 mm) which causes a destruction of the bond at the stirrup anchor elements. Therefore, it was not surprising that the electrical strain gauges never indicated yielding of the stirrups. A characteristic value of the design stirrup stress is given by 70% of the yield strength.
- (g) The results of stirrup Type I showed that stirrups do not have to enclose the bottom flexural reinforcement if welded transverse bars are used as anchor elements. This statement has to be limited to punching tests at interior columns under symmetrical loading. It has to be checked whether this is also applicable in the punching zone of edge or corner columns with moment transfer. Such investigations were started in May 1999 at the Institute for Structural Concrete (IMB) at the Aachen University of Technology (Germany).
- (h) The production of the specimens confirmed that stirrups made out of fabric reinforcement (Type I+III) are easier to place than conventional stirrups and economic on site. Therefore, it is expected that these stirrups find use on the building site although more reinforcement weight is placed.

10.4 Relationship between flexural-punching failure

Ph. Menétrey

10.4.1 Preliminaries

Three types of failures are differentiated for reinforced concrete slabs supported on columns:

- 1) Flexural failure is due to the formation of a yield-line mechanism. Yield-lines are lines about which adjacent parts of the plate exclusively experience a relative rotation. The yield-line method determines the limit load as described by Johansen (1962), Hognestad (1953), and Gesund and Kaushik (1970).
- 2) Punching failure occurs when a conical plug of concrete suddenly perforates the slab above the column.
- 3) Bond failure which is characterized by a slip of the reinforcement and is due to a lack of bond between steel and concrete. This type of failure was observed experimentally in footings by Richart (1948).

The bond failure is not considered here, as the use of high-bond reinforcement can overcome it. Flexural and the punching failure are treated here.

The effect of the flexural strength on the punching capacity of slabs has been clearly demonstrated by the tests of Elstner and Hognestad (1956). Following these results, it has been suggested that the flexural strength be introduced as one of the parameters governing the punching strength of slabs as considered by the models of Yitzhaki (1966), Long (1975) and more recently Long and Rankin (1987). However, the magnitude of the flexural strength has in itself no direct physical relation to the punching failure mechanism. It reflects, rather, several other important influences, such as distribution of cracking, amount or elongation of the tensile reinforcement, magnitude of compressive stresses, depth of the neutral axis at failure as already mentioned by Moe (1961).

The relationship between flexural and punching failure is addressed here based on experimental results performed by Menétrey (1998) on similar slabs differing only in their reinforcements.

10.4.2 Description of the experimental setup

The punching tests performed by Menétrey (1998) is presented in fig. 10.4-1. Octagonal slabs supported at the perimeter by rectangular hollow steel tube (RHS) and loaded at the center through a circular steel column were used. The tests were performed by controlling the vertical displacement of the column. A continuous monitoring of the vertical displacement at the four corners of the slabs and the force under the column was maintained.

The slabs were prepared in two series resulting in slightly different concrete characteristics: first series for slabs 1 to 6 and second series for slabs 7 to 12. For both series, the concrete was made with 350 kg of cement per m^3 and an aggregate size 0-16 mm. Once the slabs were built, they were covered for 48 hours and tested after 14 to 16 days. The mean compressive strength determined on three cylinders ($\phi = 160$ mm, height 320 mm given by SIA 162/1 (1989)) is respectively $f_c = 40.3, 39.1$ MPa and the tensile strength is $f_{ct} = 2.5, 2.7$ MPa obtained with a direct tensile test on similar

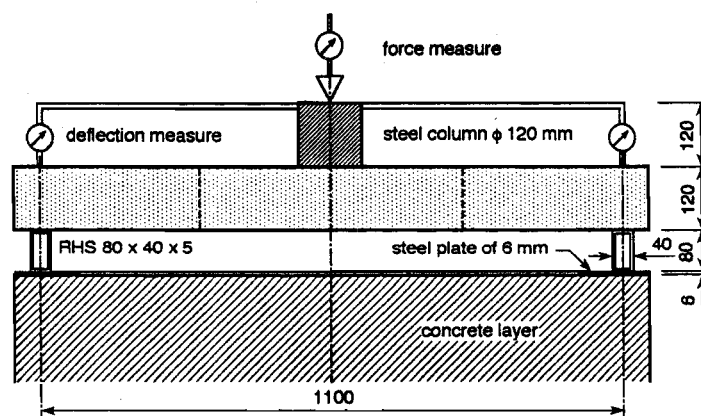


Figure 10.4-1: Cross-section of the tested octagonal slabs

cylinders (SIA 162/1 (1989)) after 14 days. The characteristics of steel reinforcement were determined based on the test 33 of the SIA 162/1 (1989). The mean results on three samples are summarized as follows: the yield strength of the steel at 0.2 percent offset is $f_t = 620$ MPa and the uniaxial tensile strength is $f_s = 660$ MPa.

All slabs were reinforced with a two-way wire mesh of $\phi=2.2$ mm. The wire mesh is used to decrease the influence of imperfections in concrete and to keep the different parts of the slabs together after failure has occurred but without influencing the slabs response justifying the low percentage of reinforcement over the column ($\rho=0.06\%$). The effective depth of the wire mesh is variable so that the effective depth of the main reinforcement with 105 mm is constant. The main reinforcement inside the slabs was varied as presented in fig. 10.4-2 so that three parameters are modified: the cross-section of ring reinforcement (4, 6, 8, and 10 mm), the cross-section of two-way reinforcement (4, 6, 8, and 10 mm), and the size of ring reinforcement (120, 242, 330, and 484 mm).

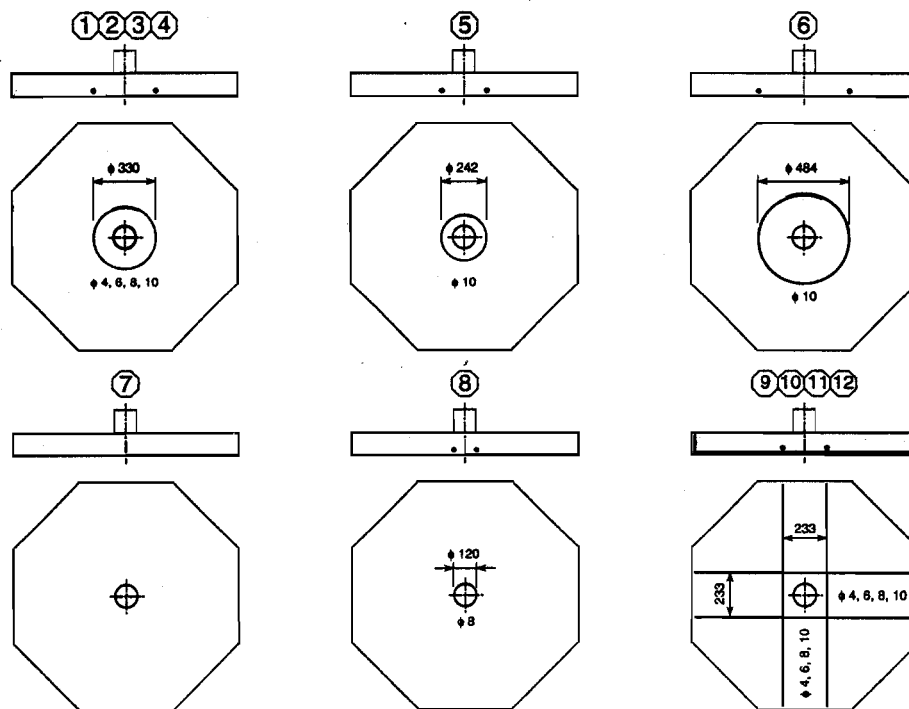


Figure 10.4-2: Cross-section and overview of the main reinforcement

10.4.3 Experimental results

A reference slab (slab 7) made only with the wire mesh was tested to generate a typical flexural failure. The maximum failure load was reached at 77.3 kN followed by a smooth decrease of the load carrying capacity. The crack pattern at the bottom of the slab is similar to the standard flexural failure mechanism composed of one tangential crack above the column and radial cracks.

The load-displacement curves of four slabs with similar ring reinforcement (except for the cross-section of the ring) are shown in fig. 10.4-3. The observed crack pattern for slab 1 (with $\phi = 4$ mm) is analogous to the one of a standard flexural failure (slab 1 was loaded and unloaded by mistake at the beginning of the testing process). For slabs 2, 3, and 4 (with $\phi = 6, 8$ and 10 mm), a punching failure is generated, as the bottom of each slab clearly exhibits the extremity of the punching crack. The load-displacement curves illustrates that after the maximum load (peak-load) has been reached, a flexural failure (slab 1) is characterized by a smooth decrease of the carrying load with increasing displacement. In the opposite, a punching failure results in a sudden decrease of the carrying load after the peak-load has been reached (vertical branch of the response).

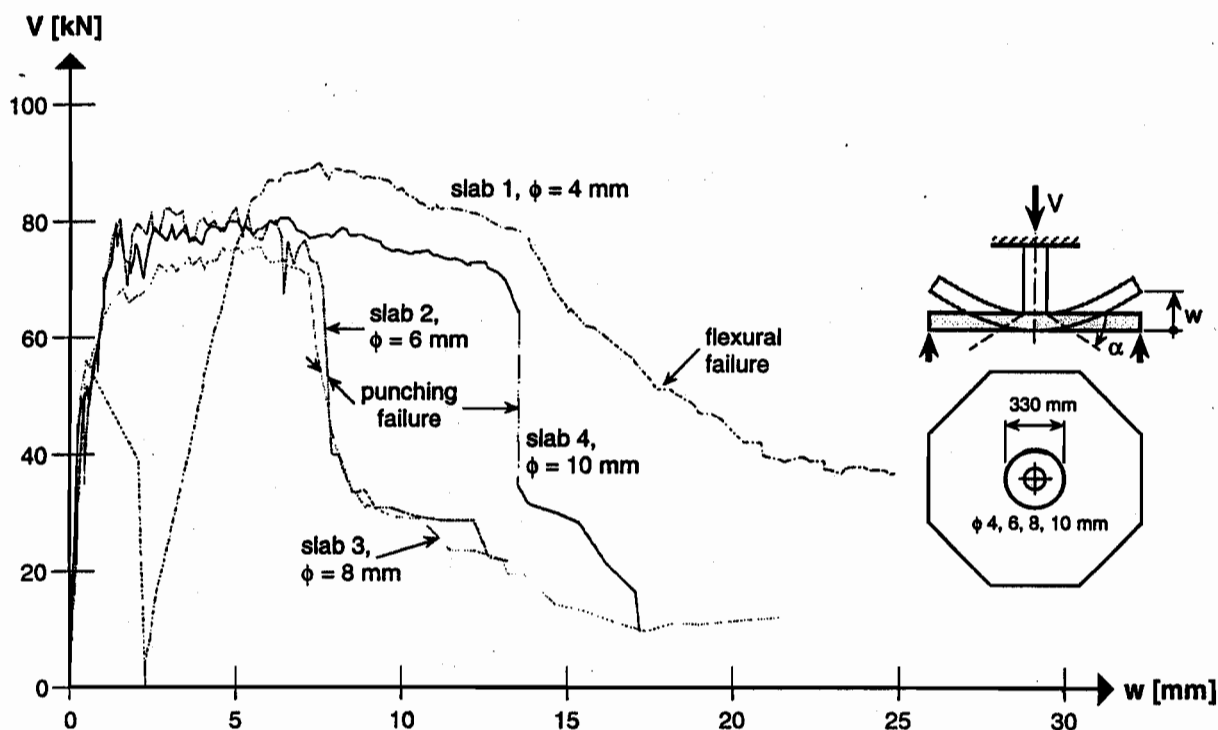


Figure 10.4-3: Load-displacement curves of slabs with ring reinforcement (slabs 1, 2, 3 and 4)

The load-displacement curves of four slabs with similar two-way reinforcement (except for the cross-section) are shown in fig. 10.4-4. It can be observed that the failure load is raised with increasing cross-section of the reinforcement. This is due to dowel effect (shear transfer mechanism between concrete and reinforcement) which is a function of the cross-section of the bars as expressed for example by Menétrey (1996) and reported in sec. 4.7.3. Two slabs failed by flexural failure (slabs 9 and 10 with reinforcement $\phi = 4$ and 6 mm) characterized by a yield-line mechanism. Two slabs failed by punching (slabs 11 and 12 with reinforcement $\phi = 8$ and 10 mm) characterized by a punching cone. The same remark concerning the post-peak behavior (response after failure) can be stated for slab with orthogonal reinforcement.

These experiments were performed by monitoring the post-peak response of the slab.

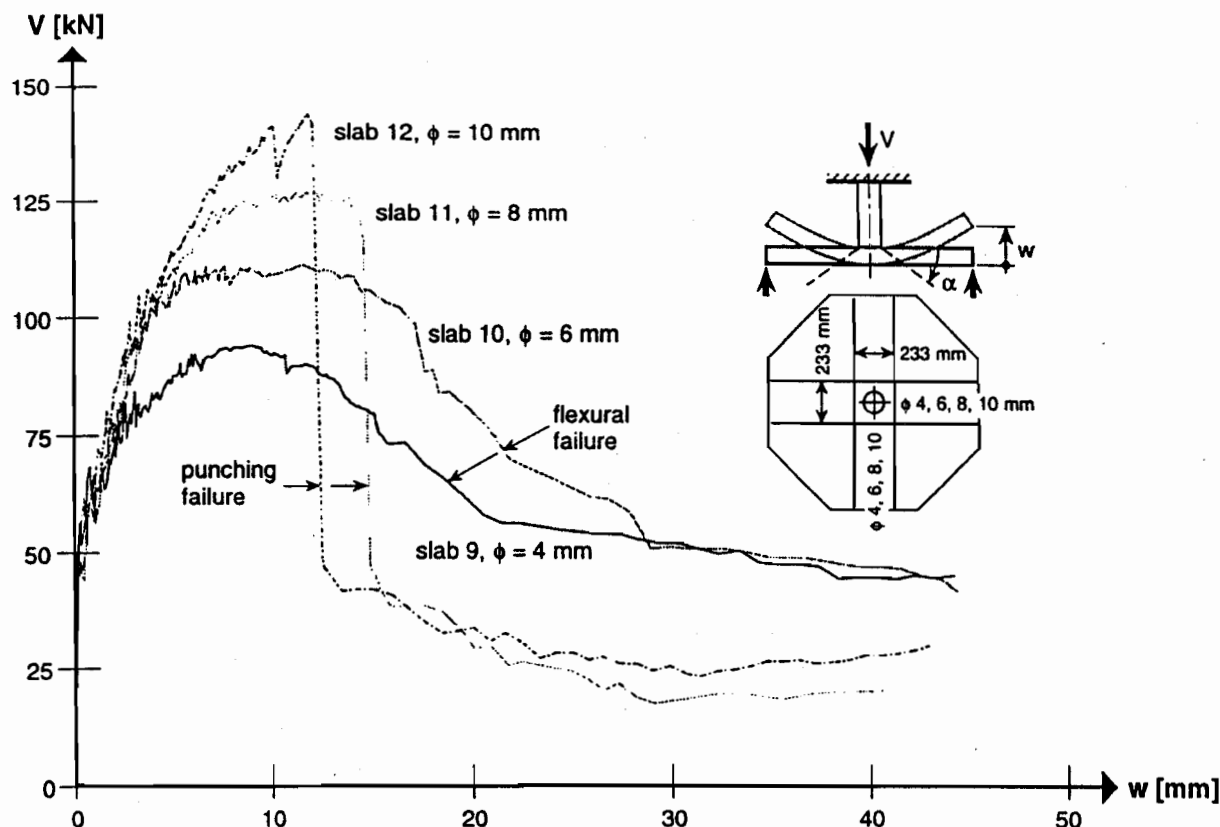


Figure 10.4-4: Load-displacement curves of slabs with two-way reinforcement (slabs 9, 10, 11 and 12)

This monitoring allow to generated the load-displacement curves also after the peak-load has been reached (as presented in fig. 10.4-3 and 10.4-4), so that the descending branch of the response is captured. The descending branch of a flexural and a punching failure are clearly different:

Flexural failure: smooth decrease of the load carrying capacity.

Punching failure: sudden decrease of the load carrying capacity.

This result is important and also allow to further justify why punching failure is so dangerous.

Finally, it should be mentioned that, the shape of the reinforcement influences the shape of the punching cone. For slab with ring reinforcement, the punching cone is clearly confined within the ring. For slab with two-way reinforcement, the punching cone is located inside the cross created by the two-way reinforcement.

10.4.4 Condition for punching failure

These tests illustrate that the punching cone forms only in slabs with high diameter reinforcement inside the cone. It can be inferred that, if the reinforcement is not sufficient, the punching cone cannot form and a flexural failure occurs. The resulting condition for punching failure to occur is that the internal strength of the cone V_{cone} is greater than the punching strength V_{pun} so that:

$$\text{Condition for punching failure: } V_{cone} > V_{pun}$$

The internal strength of the punching cone V_{cone} is determined based on simple forces decomposition as presented in fig. 10.4-5. The strut-and-tie model presented in fig. 4-10.3a) and 4.7-1 is again considered but this time only inside the punching cone. The

forces are decomposed into an inclined radial compression force F_b per unit length inside the concrete strut and a horizontal tensile force inside the reinforcement. The integral of these two forces are in equilibrium with the vertical force V arising from the column. The strength of the cone is controlled by both the strength of the compressive strut and the reinforcement. However, the tensile force inside the reinforcement is critical for the commonly used percentage of reinforcement.

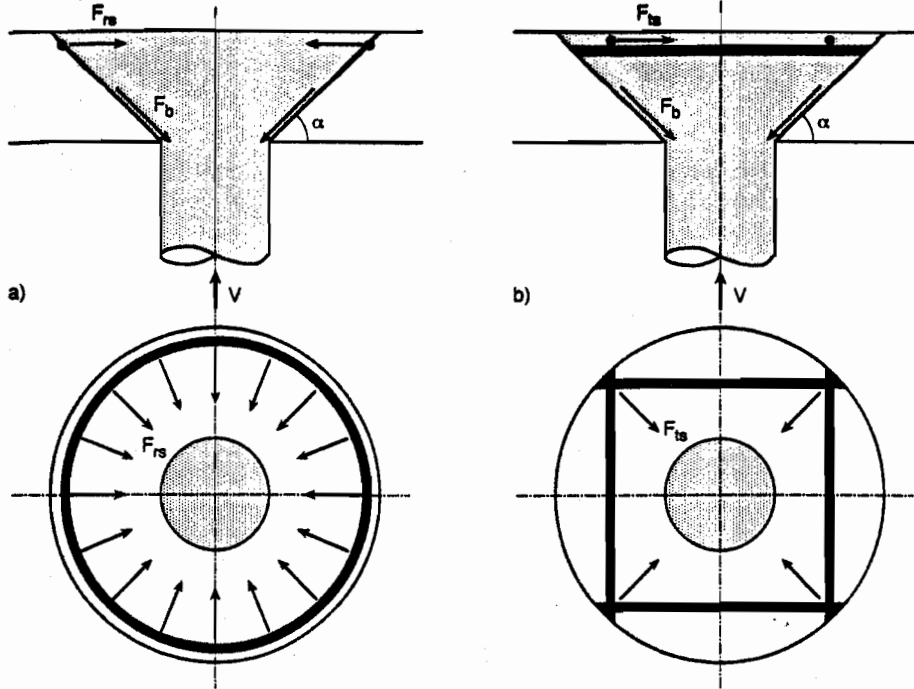


Figure 10.4-5: Force decomposition for slabs with: a) ring, b) two-way reinforcements

The tensile force is computed by projection of the compression force inside the concrete strut which is determined by simple equilibrium considerations:

$$\frac{V}{\sin(\alpha)} = \int_0^{2\pi} F_b r d\phi = 2\pi F_b r, \quad (10.4-1)$$

where $r d\phi$ is an infinitesimal length around the punching crack at the level of the reinforcement. For slab with ring reinforcement, the traction force is equilibrated by hoop stresses (see Popov (1990)) inside the ring. At the ultimate state, the integration of the hoop stresses are expressed as: $F_{rs} = (A_s f_s)/r$, where A_s is the cross-section of the ring reinforcement and f_s the tensile strength. The horizontal force projection results in:

$$F_{rs} = F_b \cos(\alpha) = \frac{V}{2\pi r} \cot(\alpha). \quad (10.4-2)$$

The strength of the cone V_{cone} for slab with ring reinforcement is:

$$V_{cone} = 2\pi A_s f_s \tan(\alpha). \quad (10.4-3)$$

For slabs with two-way reinforcement the horizontal traction force is equilibrated with the radial force contributions F_{ts} which are radial decomposition of the forces in the reinforcement. The resulting strength of the cone is:

$$V_{cone} = \sum F_{ts} \tan(\alpha) = 4\sqrt{2} A_s f_s \tan(\alpha), \quad (10.4-4)$$

for two-way reinforcement.

The internal strength of the cone V_{cone} is compared to the experimental failure load in table 10.4-1 for the performed experiments (the contribution of the wire mesh to the strength of the cone is neglected). It can be verified that for all slabs failing by punching failure, the maximum strength of the cone V_{cone} (computed with eq. 10.4-3 or 10.4-4) is larger than the experimental failure load V_{fail} , therefore, inferring the condition for punching failure. Similarly, for all slabs failing by flexural failure, the maximum strength of the cone V_{cone} is lower than the experimental failure load.

# slab	ϕ mm	shape reinf.	V_{fail} (experim.) kN		V_{cone} eq. 10.4-3 or 10.4-4 kN	Type of failure
1	4	ring	90.2	>	52.2	flexural
9	4	two-way	94.2	>	46.9	flexural
10	6	two-way	111.1	>	105.2	flexural
2	6	ring	82.7	<	116.9	punching
3	8	ring	76	<	206.1	punching
4	10	ring	80.8	<	326.1	punching
11	8	two-way	127.1	<	185.5	punching
12	10	two-way	144.3	<	293.6	punching

Table 10.4-1: Comparison between internal strength of punching cone and type of failure

The experimental investigation of Bollinger (1985) on slabs without reinforcement around the punching cone can be used to further justify the condition for punching failure presented here. By setting one ring reinforcement, located so that the punching crack is inclined at 33° , the failure mode was changed from flexural (slab 9a without the ring) to punching failure (slab 10a with the ring). It can be controlled that for the slab without the ring, characterized by a flexural failure, the strength of the cone which is close to zero is smaller than the experimental punching load $V_{pun}=41.5$ kN. Similarly, for slab with the ring, the strength of the cone computed with eq. 10.4-3 is $V_{cone}=212$ kN which is larger than the experimental punching load $V_{pun}=41.5$ kN, inferring the condition for punching failure to occur.

10.4.5 Link between flexural and punching failures

The geometry of the failure surface plays an important role, especially when the failure surface is forced into a shape different from that giving the normal punching resistance (this may occur for example if a load is close to a support) as showed by Regan (1984) and Azad et al. (1993). The influence of the geometry of the failure surface, and more especially the punching crack inclination is investigated here. The punching crack inclination can be controlled by the size of ring reinforcement which was varied so that the punching crack is inclined at 30° , 45° , 60° and 90° to the middle plane of the slab (corresponding to slabs 6, 4, 5 and 8 respectively). A view of the resulting punching cone inclined at : 30° , 45° , and 60° is presented in fig. 10.4-6.

The load-displacement curves of these four slabs with different punching cone inclination is presented in fig. 10.4-7. A typical punching failure is illustrated with slab 6 ($\alpha = 30^\circ$) which is characterized by the highest failure load and a sudden decrease of the load-carrying capacity. The slab with a foreseen punching inclination at 90° fails by flexural failure as a gradually decrease of the load-carrying capacity and yield-line mechanism are reported. The failure of slabs with a punching crack inclination at 45° , and

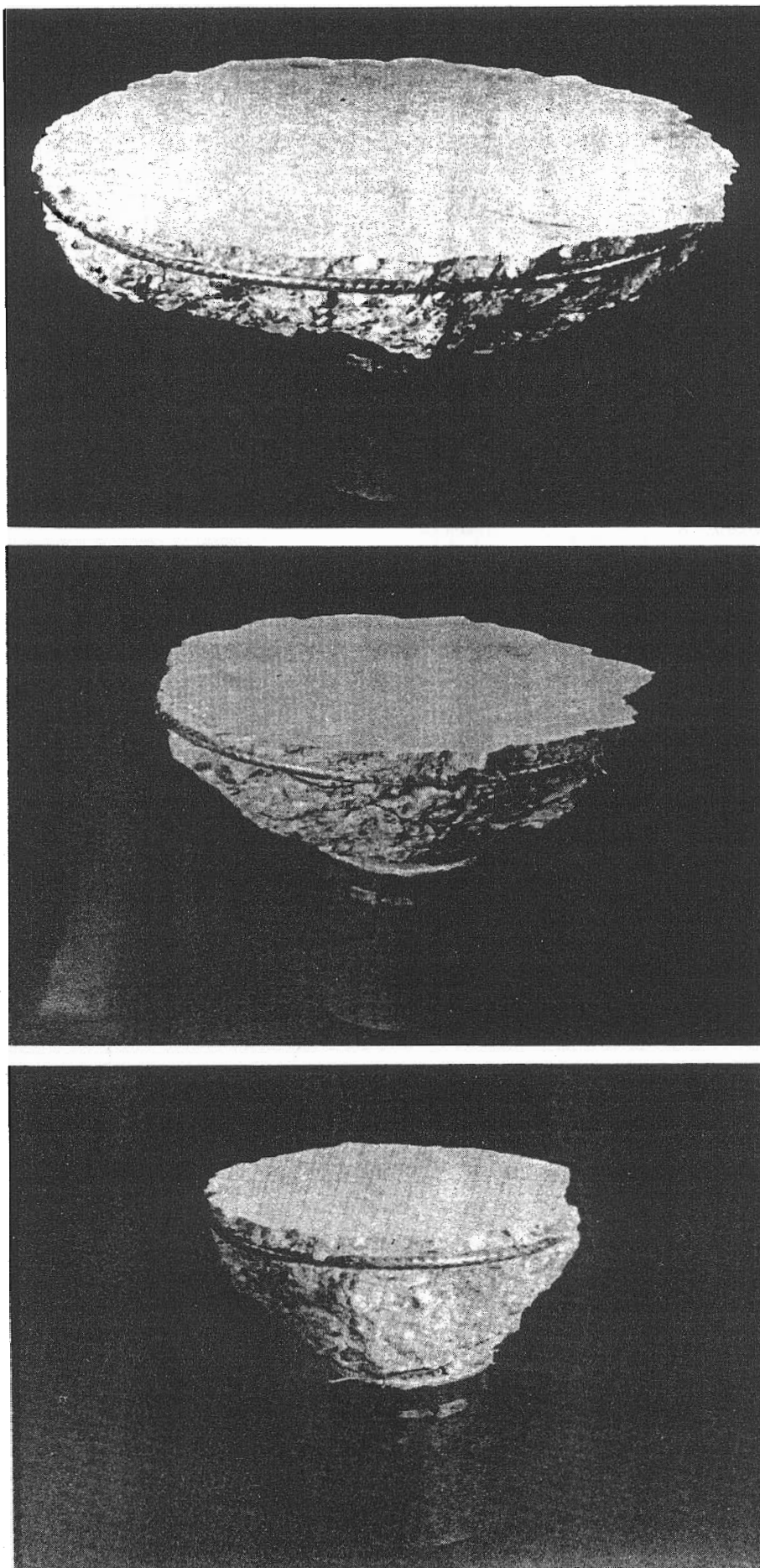


Figure 10.4-6: Punching cone with different inclination: 30° , 45° , and 60° .

60° is not a typical punching failure (even though a sudden decrease of the load-carrying capacity and punching crack was formed) and is also not a typical flexural failure.

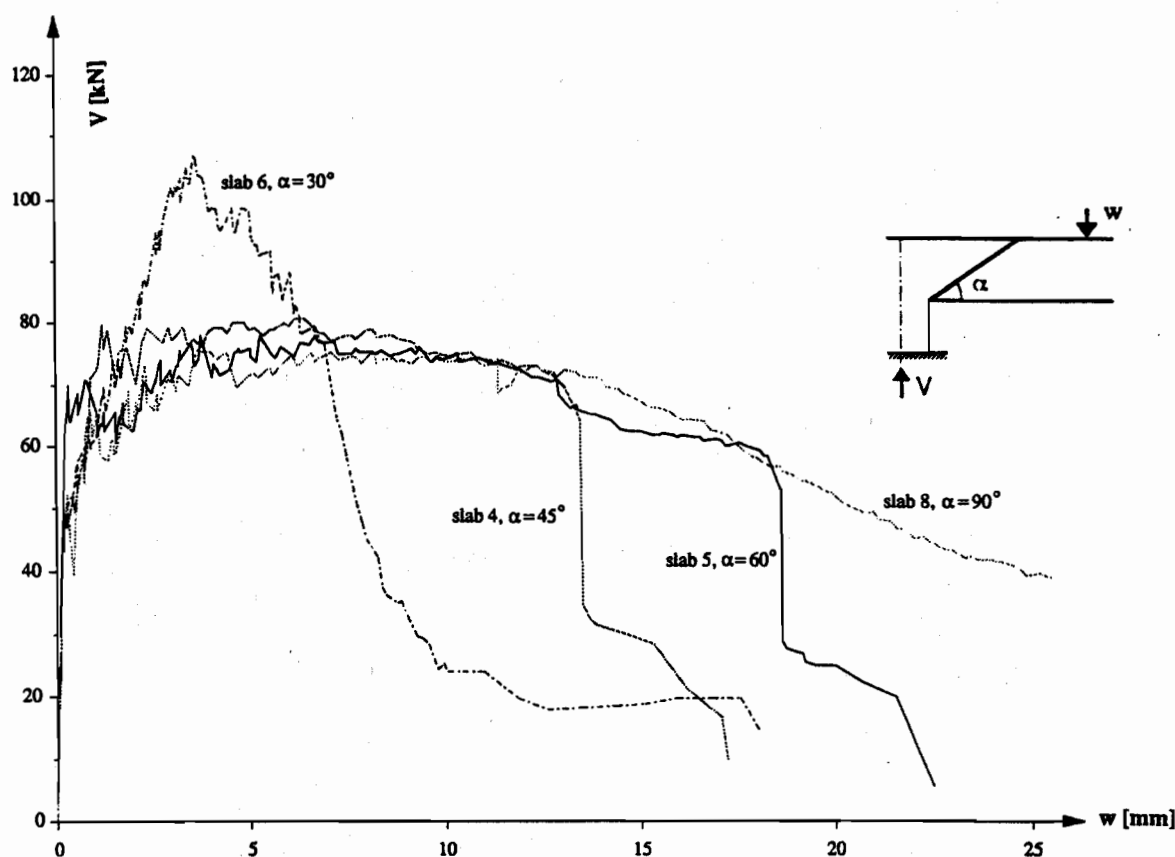


Figure 10.4-7: Load-displacement curves of slabs with various punching crack inclinations.

These tests reveal a transition between punching and flexural failure which is illustrated by the fact that with increasing the punching crack inclination ($\alpha : 30^\circ \rightarrow 90^\circ$): the failure load approaches the flexural failure load and the sudden decrease of the load-carrying capacity ΔV^- is reduced, as summarized in table 10.4-2.

# slab	α	V_{max} kN	ΔV^- kN	failure
6	30°	107	39	punching
4	45°	80.8	31	flexural-punching
5	60°	77.9	24	flexural-punching
8	90°	75.6	0	flexural

Table 10.4-2: Influence of punching crack inclination on failure load and on decrease of load-carrying capacity

Consequently, an analytical expression is proposed to link both failure loads which is:

$$V_{fail} = V_{pun} + (V_{flex} - V_{pun}) \left\{ \sin \left[\frac{\pi}{\pi - 2\alpha_0} (\alpha - \alpha_0) \right] \right\}^{1/2}, \quad (10.4-5)$$

where α_0 is the inclination of the reference punching crack (usually adopted to be $\alpha_0 = 30^\circ$ leading to eq. 4.7-16).

10.4.6 Conclusion

Flexural and punching failure experiments were performed on flat slabs supported at the perimeter and loaded at the center by controlling the vertical displacement under the column. The difference between these two failures are established as follows: (1) a flexural failure is due to formation of a yield-line mechanism and on the contrary a punching failure is due to formation of a punching cone, (2) the load-carrying capacity is progressively reduced after flexural failure and is suddenly decreased after punching failure.

A condition for punching failure to occur is proposed which is expressed as the internal strength of the punching cone being larger than the punching strength itself. By simple force equilibrium, the strength of the punching cone is determined. This condition is successfully compared to experimental results.

The use of ring reinforcements of different size generates different punching crack inclination. The punching load is reached for a punching crack inclination of about 30° to the middle plane of the slab. By increasing this inclination, the maximum load approaches the flexural failure load and the sudden decrease of the load-carrying capacity is reduced. These tests reveal a transition between punching and flexural failure which is controlled by the punching crack inclination and an interaction formula is proposed.

10.5 Punching of Footings

Mikael Hallgren

Current models and codes formulas for the assessment of the punching shear strength are based on tests on relatively slender slabs with shear-span to depth ratios of more than 3 to 4. Column footings have usually much smaller values of the shear-span to depth ratio. The structural behaviour of footings loaded to punching shear failure is not clear.

The previous investigations by Dieterle (1978), and also a more recent investigation by Hallgren, Kinnunen, and Nylander (1998), indicate that the mechanism of punching shear failure of column footings differ from the that of more slender slabs. The test reported in Hallgren et al. (1998) show that the angle of the punching shear crack is much steeper in column footings with low shear-span to depth ratios than observed in previous test on more slender slabs, see Fig. 10.5-1.

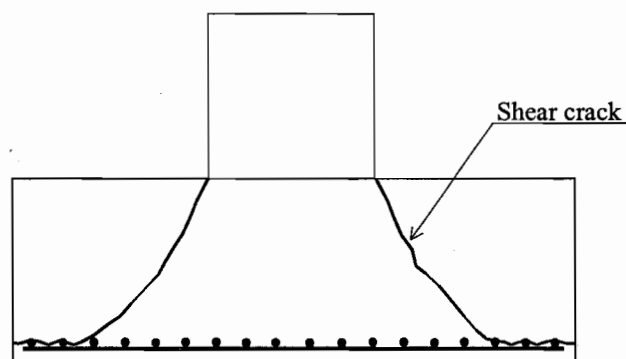


Fig. 10.5-1: Typical profile of the failure surface of a column footing (Hallgren et al. 1998)

Furthermore, the compressive strength had a stronger influence on the punching shear strength of column footings than observed in previous tests on slender slabs, see Fig. 10.5-2.

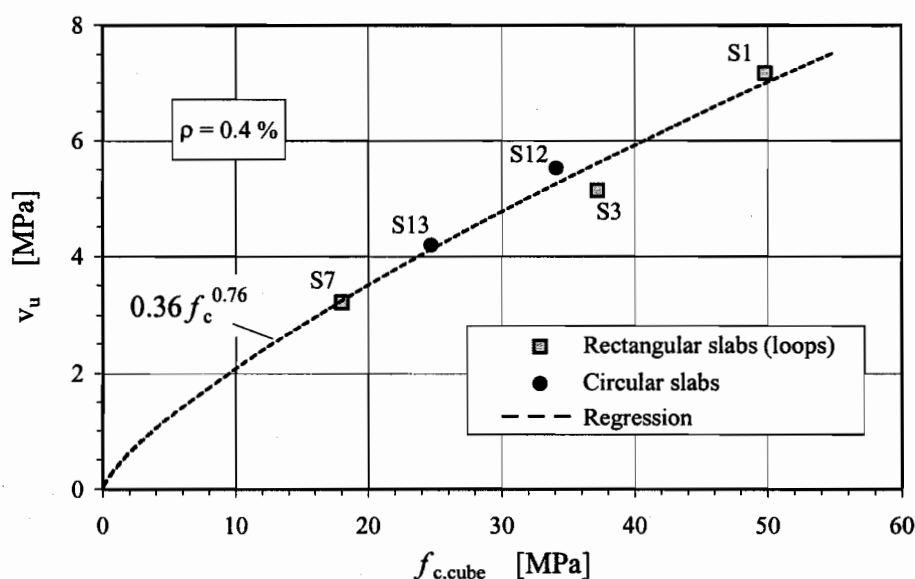


Fig. 10.5-2: Punching shear strength v_u as function of the compressive cube strength $f_{c,cube}$ (Hallgren et al. 1998)

Theoretical research work for the development of a relevant method for the design of column footings is needed. Further experimental research is also needed. The column footings tested hitherto were usually much smaller than real footings, found in e.g. bridges. In order to check the size effect, test on larger slabs with small shear-span to depth ratios are of great interest. As high strength concrete is available now, and tests on column footings indicate a strong influence of the compressive strength, high strength concrete footings should also be tested.

11 Conclusions

The review of models showed that still no consensus has been reached in the last 15 years since the CEB Bulletin 168 (1985) was published. There were very few new efforts based on the theory of plasticity and some further developments of the Kinnunen/Nylander approach. Instead of efforts for reconciling some of the mentioned theories, new approaches were developed using fracture mechanics and nonlinear Finite Element Methods.

The Kinnunen/Nylander approach may be characterized as a failure mechanism approach, where rigid bodies separate at defined failure surfaces. At the inclined crack surface bordering the punching cone there were no forces applied, apart from the dowel action in the recent work by Hallgren. The same failure mechanism approach was applied by other researchers but with completely different shear transfer actions, like especially residual tensile stresses across the cracks as justified by fracture mechanics. This corresponds to a strut-and-tie model with concrete ties, but such models have not been used by the researchers in fracture mechanics as an explanation for the punching phenomenon.

When reviewing the shear transfer actions it becomes obvious that no further actions are involved in punching as compared to the shear force transfer in one-way members without transverse reinforcement, because no load is transferred in ring direction in case of symmetrical punching. Of course, the ring direction is highly stressed, especially in the compression zone at the column. Despite of the fact that the shear transfer actions are the same for one-way members in shear and punching shear, the unsatisfactory state remains that the models for both problems are still quite different and there is no transition between them, and this also applies to most of the empirical formulae used in codes.

There is still no general agreement on the role of the concrete tensile strength. However, in the new approaches using fracture mechanics and nonlinear Finite Element Methods the concrete tensile properties are modelled and this can be regarded as an increasing awareness of the role of the concrete tensile strength in punching. It is noteworthy that the new developments since the CEB Bulletin 168 was published in 1985 include the concrete tensile properties. This is a step forward to unifying the models for modelling punching failures and shear failures in one-way members, because for the latter concrete tension plays a decisive role in most models for slender members by means of the shear transfer actions friction at cracks or residual tensile stresses across cracks.

This finding is supported by the results gained from nonlinear Finite Element Methods (FEM), where all models indicate a strong influence of the concrete tensile strength in the fracture energy. However, likewise the properties of concrete in compression have to be modelled properly, and this especially refers to the triaxial stress conditions in the compression zone of the slab near the column.

Furthermore, the review of nonlinear FEM applied to punching demonstrated the merits of these tools, which may realistically predict the failure mode in punching and the load deformation behaviour of the slab column connection. An insight into the behaviour of the slab may be gained, which experimentally is difficult or impossible to gain. The influence of geometrical and material parameters may be studied and this is helpful for planning expensive test, which still cannot be replaced by FEM.

In Chapter 10 the different test set-ups used for testing slab - column connections and their effects on the punching behaviour were studied. It was concluded that the punching strength of an interior slab - column connection is about the same as for the usually tested single column test specimen. Thus, for design purposes it is not recommended to rely on compressive membrane action in order to increase the punching capacity of an interior slab - column connection.

In older models and code formulae the size effect on the punching shear strength is usually represented by an empirical expression, or even omitted. However, the theory of non-linear fracture mechanics can now be used to explain the size-effect on punching.

An important feature of the present *fib* Bulletin is that a data bank on symmetrical punching tests was established for slabs without and with shear reinforcement. The data of 250 tests on slabs without shear reinforcement and of 150 test with shear reinforcement were collected and critically reviewed, e.g. with respect to eliminating flexural failures. The strength values of the different control specimens for the compressive and tensile strength of concrete were collected and converted in terms of uniaxial strength values, so that only a unique value for each quantity is used for any further evaluations and comparisons with codes or models. These evaluations showed that large differences could occur for the prism strength, i.e. the uniaxial strength, in case different control specimens were used for testing the compressive or tensile strength, so that further research is needed to clear up these differences. This especially refers to high strength concrete where the presently used conversion factors do obviously not apply, e.g. between the cylinder strength or the cube strength and the prism strength, i.e. the uniaxial compressive strength.

The comparisons carried out in the chapters 8 and 9 between the test data of the data bank and the predicted results from either models or empirical formulae of codes showed that remarkable scatter could occur. This means that for the investigated codes the characteristic values for the punching resistances vary in a wide range. In addition the comparison revealed that the role of the parameters influencing the punching resistance could be represented very differently by the different models or code equations. Of course, most empirical formulae should not be compared with tests slabs made out of high strength concrete, because these formulae were developed at times where high strength concrete was not used. It must be pointed out that the different models and empirical formulae were compared with the same set of test data and that no further checks or special "treatment" of data was possible. Such unbiased and neutral comparisons are a major step forward in deriving design models and empirical formulae.

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- Abbreviations:
- ACI = American Concrete Institute
 - ASCE = American Society of Civil Engineering
 - CEB = Comité Eurointernational du Béton
 - CJCE = Canadian Journal of Civil Engineering
 - fib* = Fédération Internationale du Béton
 - IABSE = International Association for Bridge and Structural Engineering
 - RILEM = Réunion Internationale des Laboratoires d'Essais et de Recherches sur les Matériaux et les Constructions (Zeitschrift: Materials and Structures)
 - SIA = Schweizer Ingenieur und Architekt
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I - 1 Databank on slabs without shear reinforcement – Collected data

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Hallgren 1996 [Source: Hallgren/Kinnunen (1993a), Hallgren/Kinnunen (1991), Hallgren (1996)]

		geometry of slab and column				reinforcement			material properties										failure																						
serial no.	slab no.	total diameter of circular slab		diameter of circle along the load was distributed		total height of slab		diameter of circular column stub ¹		effective depth			diameter of reinforcing steel ²			layout of reinforcement (x-direction)		layout of reinforcement (y-direction)		concrete compressive cylinder strength ³		concrete compressive cube strength		tensile splitting strength of concrete ⁴		tensile flexural strength of concrete ⁵		modulus of elasticity of concrete ⁶		maximum aggregate size		fracture energy ⁷		yield stress of reinforcing steel		tensile strength of reinforcing steel		modulus of elasticity of reinforcing steel		failure load	type of failure ⁸
		c	mm	c ₁	mm	h	mm	c ₂	mm	d	mm	d _s	mm	s _x	mm	s _y	mm	f _{c,cyl}	MPa	f _{c,cube}	MPa	f _{ct,sp}	MPa	f _{ct,fl}	MPa	E _c	GPa	d _a	mm	G _f	N/m	f _y	MPa	f _t	MPa	E _s	GPa	P _u	MN		
1-1	HSC 0	2540	2540	2400	2400	240	240	250	250	200	16	60/120	65/130					-	112.9	106.8	112.9	4.5	-	-	51.8	18	-	807	643	807	-	0,965	P								
1-2	HSC 1	2540	2540	2400	2400	245	245	250	250	200	16	60/120	65/130					91.3	106.8	106.8	6.9	10.0	10.0	42.9	18	179	750	627	750	-	1,021	P									
1-3	HSC 2	2540	2540	2400	2400	240	240	250	250	194	16	60/120	65/130					85.7	98.8	98.8	5.7	8.5	8.5	37.2	18	168	743	620	743	-	0,889	P									
1-4	HSC 4	2540	2540	2400	2400	240	240	250	250	200	20	63/125	70/140					91.6	98.7	98.7	6.6	12.3	12.3	41.3	18	154	715	596	715	195	1,041	P									
1-5	HSC 6	2540	2540	2400	2400	239	239	250	250	201	16	160	85/170					108.8	105.8	105.8	7.6	12.7	12.7	46.0	18	161	765	633	765	210	0,960	P									
1-6	N/HSC8 ⁹	2540	2540	2400	2400	242	242	250	250	198	16	60/120	65/130					94.9	100.1	100.1	6.9	10.2	10.2	43.5	18	-	754	631	754	213	0,944	P									
1-7	HSC 9	2540	2540	2400	2400	239	239	250	250	202	16	150/300/ 350/200	150/300/ 350/200					84.1	86.7	86.7	6.7	9.1	9.1	39.0	18	195	761	634	761	231	0,565	FP									

¹ monolithic² anchored by steel plates (50/50/20 mm)³ cylinder $\varnothing=150$ mm, $h=300$ mm⁴ measured at time of test; cubes 150 mm⁵ measured at time of test; four-point bending test on plain beams 100/150/800 mm⁶ measured at time of test; cylinder $\varnothing=150$ mm, $h=300$ mm⁷ three-point bending test on notched beams 100/100/800 mm acc. to RILEM 1985⁸ FP = Flexural Punching, P = Brittle Punching⁹ values for HSC (NSC in brackets)

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I - 1 Databank on slabs without shear reinforcement – Collected data

Tomaszewicz (1993)

serial no.	slab no. ¹⁰	geometry of slab and column				reinforcement				material properties					failure	
		side length of quadratic slab	span of quadratic slab	total height of slab	side length of quadratic column stub ¹¹	effective depth	diameter of reinforcing steel ¹²	layout of reinforcement (x- and y-direction)	layout of compressive reinforcement (x- and y-direction)	concrete compressive cylinder strength ¹³	concrete compressive cube strength ¹⁴	tensile splitting strength of concrete ¹⁵	maximum aggregate size	yield stress of reinforcing steel	failure load	type of failure
notation	unit	a	a ₁	h	a ₂	d	d _s	s _x /s _y	s _{x2} /s _{y2}	f _{ccyl} MPa	f _{ccube} MPa	f _{ct,sp} MPa	d _a mm	f _y MPa	P _u MN	-
2-1	ND65-1-1	3000	2500	320	200	275	25	60/120	-	64.3	72.0	4.3	16	500	2,050	P
2-2	ND65-2-1	2600	2200	240	150	200	20	45/90	-	70.2	86.1	4.3	16	500	1,200	P
2-3	ND95-1-1	3000	2500	320	200	275	25	60/120	-	83.7	102.8	5.2	16	500	2,250	P
2-4	ND95-1-3	3000	2500	320	200	275	25	35/70	-	89.9	102.4	5.0	16	500	2,400	P
2-5	ND95-2-1	2600	2200	240	150	200	20	45/90	-	88.2	103.1	5.2	16	500	1,100	P
2-6	ND95-2-1D	2600	2200	240	150	200	20	45/90	180	86.7	104.9	5.0	16	500	1,300	P
2-7	ND95-2-3	2600	2200	240	150	200	20	30/60	-	89.5	100.5	5.2	16	500	1,450	P
2-8	ND95-2-3D	2600	2200	240	150	200	20	30/60	180	80.3	95.1	4.7	16	500	1,250	P
2-9	ND95-2-3D+	2600	2200	240	150	200	20	30/60	180	98.0	111.8	5.4	16	500	1,450	P
2-10	ND95-3-1	1500	1100	120	100	88	12	35/70	-	85.1	95.9	5.0	16	500	0,330	P

¹⁰ concrete density; ND: 2400 – 2500 kg/m³; LWA: 2000 – 2050 kg/m³¹¹ quadratic steel loading plates¹² anchored by long hooks¹³ cylinder Ø=100, h = 300 mm; measured at time of test¹⁴ cubes 100 mm; stored and tested acc. to N 3473 (1989)¹⁵ cylinder Ø=150, h = 300 mm; measured at time of test

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I - 1 Databank on slabs without shear reinforcement – Collected data

2-11	ND115-1-1	3000	2500	320	200	275	25	60/120	-	<u>112.0</u>	<u>122.1</u>	<u>5.9</u>	16	500	2,450	P
2-12	ND115-2-1	2600	2200	240	150	200	20	45/90	-	<u>119.0</u>	<u>117.1</u>	<u>5.8</u>	16	500	1,400	P
2-13	ND115-2-3	2600	2200	240	150	200	20	30/60	-	<u>108.1</u>	<u>111.3</u>	<u>5.8</u>	16	500	1,550	P

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I - 1 Databank on slabs without shear reinforcement – Collected data

3-10	21	1700	1372	125	150	98	10	63	52,4	-	-	-	20	650	0,286	P
3-11	22	1700	1372	125	150	98	10	63	105,3	6,8	10,5	-	20	650	0,405	P
3-12	23	1700	1372	125	150	100	10	90	70,5	4,5	6,4	-	20	650	0,341	DP
3-13	25	1700	1372	125	150	100	10	63	41,1	-	-	-	10	650	0,244	P
3-14	26	1700	1372	125	150	100	10	63	47,0	-	-	-	20	650	0,294	P
3-15	27	1700	1372	125	150	102	10	75 ²⁰	42,1	-	-	-	20	650	0,227	P

²⁰ additional compressive reinforcement: $d_s = 6$ mm, $s_{x,y} = 75$ mm

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I - 1 Databank on slabs without shear reinforcement – Collected data

Marzouk / Hussein (1991)

serial no.	slab no.	geometry of slab and column			reinforcement			material properties				failure	
		side length of quadratic slab	span of quadratic slab	total height of slab	side length of quadratic column ²¹	effective depth	diameter of reinforcing steel ²²	layout of reinforcement (x- and y-direction)	concrete compressive strength	maximum aggregate size	yield stress of reinforcing steel	failure load	type of failure
	notation	a	a _l	h	a ₂	d	d _s	s _x /s _y	f _{coyl}	d _a	f _y	P _u	
	unit	mm	mm	mm	mm	mm	mm	mm	MPa	mm	MPa	MN	
4-1	I. HS1	1700	1500	120	150	95	10	214	67.0	20	490	0,178	P
4-2	I. HS2	1700	1500	120	150	95	10	125	70.0	20	490	0,249	P
4-3	I. HS3	1700	1500	120	150	95	10	71	69.0	20	490	0,356	P
4-4	I. HS4	1700	1500	120	150	90	15	94	66.0	20	490	0,418	P
4-5	I. HS7	1700	1500	120	150	95	10	88	74.0	20	490	0,356	P
4-6	II. HS5	1700	1500	150	150	125	10	125	68.0	20	490	0,365	P
4-7	II. HS6	1700	1500	150	150	120	10	125	70.0	20	490	0,489	P
4-8	II. HS8	1700	1500	150	150	120	15	150	69.0	20	490	0,436	P
4-9	II. HS9	1700	1500	150	150	120	15	100	74.0	20	490	0,543	P
4-10	II. HS10	1700	1500	150	150	120	15	71	80.0	20	490	0,645	P

²¹ monolithic²² anchored by hooks

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4-11	III. HS11	1700	1500	90	150	70	10	150	<u>70.0</u>	20	490	0,196	FP
4-12	III. HS12	1700	1500	90	150	70	10	94	<u>75.0</u>	20	490	0,258	P
4-13	III. HS13	1700	1500	90	150	70	10	71	<u>68.0</u>	20	490	0,267	P
4-14	IV. HS14	1700	1500	120	220	95	10	71	<u>72.0</u>	20	490	0,498	P
4-15	IV. HS15	1700	1500	120	300	95	10	71	<u>71.0</u>	20	490	0,560	P
4-16	I. NS1	1700	1500	120	150	95	10	71	<u>42.0</u>	20	490	0,320	P
4-17	II. NS2	1700	1500	150	150	120	10	125	<u>30.0</u>	20	490	0,396	P

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Lovrovich / McLean (1990)

serial no.	slab no.	geometry of slab and column				reinforcement				material properties					failure			
		total diameter of circular slab	diameter of circle along the load was distrib. ²³	total height of slab	diameter of circular column stub ²⁴	effective depth	diameter of reinforcing steel ²⁵	s_x/s_y	layout of reinforcement	layout of compressive reinforcement	concrete compressive cylinder strength	tensile splitting strength of concrete ²⁶	maximum aggregate size	yield stress of reinforcing steel	tensile stress of reinforcing steel	failure load	type of failure	
5-1 5-2 5-3 5-4 5-5	notation unit	c	c ₁	h	c ₂	d	d _s	s_x/s_y	s_{x2}/s_{y2}	$f_{c,cyl}$ MPa	$f_{c,sp}$ MPa	d _a mm	f _y MPa	f _t MPa	P _u MN	P P P P P P	type of failure	
		mm	mm	mm	mm	mm	mm	mm	mm	mm	mm	mm	MPa	MPa	MPa			
		500	200	100	100	83	9,5	49+1	49+1	49+1	39,3	3,6	19	531	621			0,480
		700	400	100	100	83	9,5	49+1	49+1	49+1	39,3	3,6	19	531	621			0,204
		900	600	100	100	83	9,5	49+1	49+1	49+1	39,3	3,6	19	531	621			0,149
		1100	800	100	100	83	9,5	49+1	49+1	49+1	39,3	3,6	19	531	621			0,129
		1500	1200	100	100	83	9,5	49+1	49+1	39,3	3,6	19	531	621	0,139			

²³ supported on 25 mm diameter solid steel bars bent into circular configuration²⁴ thin leather pad located between steel cylinder and slab²⁵ No. 3 ASTM Grade 60; anchored by straight ends²⁶ split cylinder test

I - 1 Databank on slabs without shear reinforcement – Collected data

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Tolf (1988)

serial no.	slab no.	geometry of slab and column				reinforcement				material properties				failure	
		total diameter of circular slab	c_1	h	c_2	effective depth	diameter of reinforcing steel ²⁸	layout of reinforcement (x-direction)	layout of reinforcement (y-direction)	concrete compressive prism strength ²⁹	concrete compressive cube strength ³⁰	maximum aggregate size	yield stress of reinforcing steel	failure load	type of failure
notation	unit	c	c_1	h	c_2	d	d_s	s_x	s_y	$f_{c,prism}$	$f_{c,cube}$	d_a	f_y	P_u	
6-1	S2.1	2540	2378	240	250	200	16	60/120	65/130	19.4	30.3	32	657	0,603	
6-2	S2.2	2540	2378	240	250	199	16	60/120	65/130	18.9	28.6	32	670	0,600	
6-3	S2.3	2540	2378	240	250	200	16	100/350/ 2-300	100/350/ 2-300	19.3	31.7	32	668	0,489	
6-4	S2.4	2540	2378	240	250	197	16	100/350/ 2-300	100/350/ 2-300	21.6	30.2	32	664	0,444	
6-5	S1.1	1270	1189	120	125	100	8	30/60	33/65	30.1	35.7	16	706	0,216	
6-6	S1.2	1270	1189	120	125	99	8	30/60	33/65	23.8	28.6	16	701	0,194	
6-7	S1.3	1270	1189	120	125	98	8	50/175/ 2-150	50/175/ 2-150	26.0	33.3	16	720	0,145	
6-8	S1.4	1270	1189	120	125	99	8	50/175/ 2-150	50/175/ 2-150	25.5	31.4	16	712	0,148	

²⁷ monolithic²⁸ anchored by steel plates²⁹ measured at time of test; Serie S1: prisms 240/240/540 mm; Serie S2: prisms 120/120/240 mm³⁰ measured at time of test; cubes 150 mm

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I - 1 Databank on slabs without shear reinforcement – Collected data

Regan 1986 [Source: Regan (1986), Regan (1981)]

serial no.	slab no	geometry of slab and column				reinforcement				material properties			failure	
		side length of quadratic slab	span of quadratic slab	total height of slab	side length of quadratic column stub ³¹	effective depth	diameter of reinforcing steel ³²	layout of reinforcement (x-direction)	layout of reinforcement (y-direction)	concrete compressive strength	maximum aggregate size	yield stress of reinforcing steel	failure load	type of failure
notation	unit	a	a ₁	h	a ₂	d	d _s	s _x	s _y	f _{c,cube}	d _a	f _y	P _u	-
7-1	I/1	2000	1830	100	200	77	10	20/2-40/50/60/80/90/120/3-150	2-35/40/2-50/2-70/90/100/3-140+1	32.2	10	500	0,194	P
7-2	I/2	2000	1830	100	200	77	10	45/90	40/80	29.3	10	500	0,176	P
7-3	I/3	2000	1830	100	200	77	10	50/60/70/90/120/170/2-200+1	40/2-50/60/100/130/170/2-180+1	34.3	10	500	0,194	P
7-4	I/4	2000	1830	100	200	77	10	35/70	53/105	40.4	10	500	0,194	P
7-5	I/5	2000	1830	100	200	79	8	20/2-40/50/60/80/90/120/3-150	2-35/40/2-50/2-70/90/100/3-140+1	35.2	10	480	0,165	P
7-6	I/6	2000	1830	100	200	79	8	45/90	40/80	27.4	10	480	0,165	P
7-7	I/7	2000	1830	100	200	79	8	45/90	40/80	38.0	10	480	0,186	F
7-8	II/1	2745	250	250	250	200	25	250	250	43.6	20	530	0,825	P
7-9	II/2	1800	160	160	160	128	16	160	160	41.6	20	485	0,390	P
7-10	II/3	1800	160	160	160	128	16	160	160	42.9	10	485	0,365	P
7-11	II/4	900	80	80	80	64	8	80	80	41.6	20	480	0,117	P
7-12	II/5	900	80	80	80	64	8	80	80	42.9	10	480	0,105	P
7-13	II/6	900	80	80	80	64	8	80	80	45.2	5	480	0,105	P

³¹ monolithic (slabs I/1 - I/7) and diameter of circular steel disk (slabs II/1 - II/6; III/1 - III/6; V-1 - V-5)³² anchored by straight ends (slabs I/1 - I/7) and anchored with end hooks (slabs II/1 - II/6; III/1 - III/6; V-1 - V-5)

I - 1 Databank on slabs without shear reinforcement – Collected data

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7-14	III/1	1500	1370	120	150	95	10	50/100	50/100	29,0	-	494	0,197	P
7-15	III/2	1500	1370	120	150	95	10	50/100	50/100	11,9	-	494	0,123	P
7-16	III/3	1500	1370	120	150	95	10	50/100	50/100	47,3	-	494	0,214	P
7-17	III/4	1500	1370	120	150	93	12	40/80	40/80	14,9	-	464	0,154	P
7-18	III/5	1500	1370	120	150	93	12	40/80	40/80	33,5	-	464	0,214	P
7-19	III/6	1500	1370	120	150	93	12	40/80	40/80	53,3	-	464	0,248	P
7-20	V/1	1600	1500	150	54	118	12	120	120	42,9	-	628	0,170	P
7-21	V/2	1600	1500	150	170	118	12	120	120	40,2	-	628	0,280	P
7-22	V/3	1600	1500	150	110	118	12	120	120	40,5	-	628	0,265	P
7-23	V/4	1600	1500	150	102	118	12	120	120	45,3	-	628	0,285	P
-	V/5	1600	1500	150	150 ³³	118	12	120	120	41,1	-	628	0,285	P ³⁴

³³ diameter precast concrete cylinder³⁴ load arrangement penetrated slab

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I - 1 Databank on slabs without shear reinforcement – Collected data

Swamy / Ali (1982)

serial no.	slab no	geometry of slab and column				reinforcement					material				failure	
		side length of quadratic slab	span of quadratic slab	total height of slab	side length of quadratic column stub ³⁵	effective depth	diameter of reinforcing steel ³⁶	layout of reinforcement	diameter of compressive reinforcing steel	layout of compressive reinforcement	concrete compressive cylinder strength ³⁷	maximum aggregate size	yield stress of reinforcing steel	tensile stress of reinforcing steel	failure load	type of failure
notation	unit	a	a_1	h	a_2	d	d_s	s_x / s_y	d_{s2}	s_{x2} / s_{y2}	$f_{c,cyl}$	d_a	f_y	f_t	P_u	-
8-1	S1	1800	1690	125	150	100	10	138	8	300/ 2.180+1	40,1	10	462	568	0,198	P
8-2	S7	1800	1690	125	150	100	10	61/3-113/ 2.180	8	2.180/ 300+1	37,4	10	462	568	0,222	P
-	S19	1800	1690	125	150	100	10	113/225/ 300	8	300	37,6	10	462	568	0,131 ³⁸	F

³⁵ monolithic³⁶ anchored by end hooks³⁷ acc. to Noeling (DAfStb-Heft 371) calculated for β_c (cylinder $\varnothing=150$ mm, $h=300$ mm)³⁸ calculated flexural load

I - 1 Databank on slabs without shear reinforcement – Collected data

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ETH 1979 / 77 [Source: Marti/Pralong/Thürlimann (1977), Pralong/Brändli/Thürlimann (1979)]

serial no.	slab no.	geometry of slab and column				reinforcement				material properties										failure	
		total diameter of circular slab ³⁹	diameter of circle along the load was distributed	total height of slab	diameter of circular column stub ⁴⁰	effective depth	diameter of reinforcing steel ⁴¹	s_x/s_y	diameter of compressive reinforcing steel	layout of compressive reinforcing steel	concrete compressive prism strength ⁴²	concrete compressive cube strength ⁴³	tensile flexural strength of concrete ⁴⁴	maximum aggregate size	modulus of elasticity of concrete ⁴⁵	yield stress of reinforcing steel	tensile strength of reinforcing steel	modulus of elasticity of reinforcing steel	failure load	type of failure	
notation	unit	c	c ₁	h	c ₂	d	d _s	s _x /s _y	d _s	s _{x2} /s _{y2}	f _{c,prism}	f _{c,cube}	f _{ct,fl}	d _a	E _c	f _y	f _y	E _s	P _u	-	
9-1	P 2 ⁴⁶	2750	2600	180	300	143	18	120	10	120	35,4	43,2	6,3	16	36,4	558	711	206	0,628	P	
9-2	P 5 ⁴⁷	2750	2600	191	300	171	16	100	8	100	27,1	32,8	4,7	16	36,5	515	683	204	0,626	P	

³⁹ octagonal⁴⁰ monolithic⁴¹ anchored by end hooks⁴² measured at t = 47 d (≠ time of test); prism 120/120/360 mm⁴³ measured at t = 47 d (≠ time of test); cube 150 mm⁴⁴ measured at time of test (P2: t = 47 d; P5: t = 195 d)⁴⁵ measured at time of test; cylinder P2: Ø=150 mm, h = 300 mm; cylinder P5: Ø=50 mm, h = 150 mm⁴⁶ test by Marti/Pralong/Thürlimann 1977⁴⁷ test by Pralong/Brändli/Thürlimann 1979

I - 1 Databank on slabs without shear reinforcement – Collected data

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Schaefer 1978 [Source: Schaefer (1984)]

serial no.	slab no.	geometry of slab and column				reinforcement		material properties										failure	
		total diameter of circular slab	diameter of circle along the load was distributed	total height of slab	diameter of circular column stub ⁴⁸	effective depth	diameter of reinforcing steel ⁴⁹	layout of reinforcement (x-direction)	concrete compressive cylinder strength ⁵⁰	concrete compressive cube strength ⁵¹	tensile splitting strength of concrete ⁵²	tensile flexural strength of concrete ⁵³	maximum aggregate size	modulus of elasticity of concrete ⁵⁴	yield stress of reinforcing steel ⁵⁵	failure load	type of failure		
notation	unit	c	c ₁	h	c ₂	d	d _s	s _x /s _y	f _{c,cyl} MPa	f _{c,cube} MPa	f _{ct,sp} MPa	f _{ct,fl} MPa	d _a mm	E _c GPa	f _y MPa	P _u MN	-		
10-1	0	1960	1680	143	210	113	12	120	23.1	27.8	-	5.4	32	30.0	420	0,280	P		
10-2	3	1960	1680	200	210	170	12	120	23.3	26.6	2.6	4.0	32	28.3	450	0,460	P		

⁴⁸ monolithic⁴⁹ anchored by straight ends⁵⁰ measured at time of test (t = 28 d); cylinder Ø=150 mm, h = 300 mm⁵¹ measured at time of test (t = 28 d); cube 200 mm⁵² measured at time of test (t = 28 d); cylinder Ø=150 mm, h = 450 mm⁵³ measured at time of test (t = 28 d); beam 150/100/700 mm⁵⁴ measured at time of test; cylinder Ø=150 mm, h = 300 mm⁵⁵ BS 420/500 S

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I - 1 Databank on slabs without shear reinforcement – Collected data

Ladner / Schaeidt / Gut (1977)

serial no.	slab no	geometry of slab and column				reinforcement			material				failure	
		total diameter of circular slab	diameter of circle along the load was distributed	total height of slab	side length of quadratic column stub ⁵⁶	effective depth	diameter of reinforcing steel ⁵⁷	layout of reinforcement	concrete compressive strength ⁵⁸	maximum aggregate size	yield stress of reinforcing steel	tensile stress of reinforcing steel	failure load	type of failure
notation	unit	c	c _l	h	a ₂	d	d _s	s _x /s _y	f _{c,cube}	d _a	f _y	f _t	P _u	
11-1	DA 6	1260	1056	110	100	80	16	140	37,5	16	550	752	0,183	P
11-2	DA 7	1260	1056	110	200	80	16	140	41,9	16	550	752	0,288	P
11-3	DA 10	1260	1056	110	240	80	16	140	40,0	16	550	752	0,281	P
11-4	DA 11	1260	1056	110	320	80	16	140	38,0	16	550	752	0,324	P

⁵⁶ monolithic⁵⁷ anchored by steel plates⁵⁸ measured at time of test; cube 150 mm

I - 1 Databank on slabs without shear reinforcement – Collected data

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Ladner et al. 1973 / 70 [Source: Schaeidt/Ladner/Rösli (1970), Ladner (1973)]

serial no.	slab no.	geometry of slab and column					reinforcement			material properties					failure	
		total diameter of circular slab	circular slab	diameter of circle along the load was distributed	total height of slab	diameter of circular column stub ⁵⁹	effective depth	diameter of reinforcing steel ⁶⁰	layout of reinforcement (x-direction)	concrete compressive cube strength	tensile flexural strength of concrete	maximum aggregate size	yield stress of reinforcing steel	failure load	type of failure	
notation	unit	c	c ₁	h	c ₂	d	d _s	s _x /s _y	f _{c,cube}	f _{ct,fl}	d _a	f _y	P _u	-		
12-1	P1 ⁶¹	2900	2650	280	500	240	20	100	34.9	5.8	32	544	1,662	P		
12-2	M1 ⁶²	1400	1200	127	226	109	10	60	39.7	5.3	32	541	0,362	P		

⁵⁹ monolithic⁶⁰ anchored by steel plates⁶¹ test by Schaeidt/Ladner/Rösli 1970⁶² test by Ladner 1973

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Corley / Hawkins (1968)

serial no.	slab no.	geometry of slab and column				reinforcement			material		failure	
		side length of quadratic slab	span of quadratic slab	total height of slab	side length of quadratic column stub ⁶³	effective depth	diameter of reinforcing steel	layout of reinforcement	concrete compressive cylinder strength	yield stress of reinforcing steel	failure load	type of failure
notation	unit	a	a_1	h	a_2	d	d_s	s_x/s_y	$f_{c,cyl}$	f_y	P_u	.
13-1	AN-1	mm	1820	146	254	111	15,9	118	44,4	404	0,334	P
13-2	AN-2	mm	1820	146	203	111	15,9	177	44,4	444	0,266	P

⁶³ monolithic

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I - 1 Databank on slabs without shear reinforcement – Collected data

Base 1966 [Source: Bernaert/Puech (1966)]

serial no.		geometry of slab and column					reinforcement		material		failure	
slab no		total diameter of circular slab	diameter of circle along the load was distributed	total height of slab	side length of quadratic column	effective depth	longitudinal reinforcement ratio	concrete compressive strength	yield stress of reinforcing steel	failure load	type of failure	
notation		c	c ₁	h	a ₂	d	ρ _l	f _{c,cyl}	f _y	P _u	-	
unit		mm	mm	mm	mm	mm	-	MPa	MPa	MN	-	
14-1	A1/M1	1370	1370	140	203	114	0,011	16,3	255	0,322	F	
14-2	A1/M2	1370	1370	140	203	117	0,015	15,5	282	0,346	P	
14-3	A1/M3	1370	1370	140	203	121	0,019	14,2	282	0,307	P	
14-4	A1/M4	1370	1370	140	203	124	0,010	14,0	432	0,259	P	
14-5	A1/M5	1370	1370	140	203	117	0,012	21,0	432	0,346	P	
14-6	A2/M1	1370	1370	140	203	124	0,010	35,4	255	0,409	F	
14-7	A2/M2	1370	1370	140	203	117	0,015	32,8	282	0,419	P	
14-8	A2/M3	1370	1370	140	203	121	0,019	32,5	282	0,430	P	
14-9	A2/T1	1370	1370	140	203	124	0,010	39,3	432	0,419	P	
14-10	A2/T2	1370	1370	140	203	124	0,017	41,4	432	0,439	P	

⁶⁴ missing data ($c \approx c_1$)

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14-11	A3/M1	1370	1370	140	203	124	0,010	<u>18,8</u>	<u>255</u>	0,247	P
14-12	A3/M2	1370	1370	140	203	102	0,017	<u>19,3</u>	<u>282</u>	0,336	P
14-13	A3/M3	1370	1370	140	203	117	0,019	<u>27,3</u>	<u>282</u>	0,298	P
14-14	A3/T1	1370	1370	140	203	121	0,010	<u>20,6</u>	<u>432</u>	0,328	P
14-15	A3/T2	1370	1370	140	203	119	0,012	<u>16,0</u>	<u>432</u>	0,298	P
14-16	A4/M1	1370	1370	140	203	114	0,011	<u>38,3</u>	<u>255</u>	0,259	P
14-17	A4/M2	1370	1370	140	203	119	0,015	<u>29,2</u>	<u>282</u>	0,341	P
14-18	A4/M3	1370	1370	140	203	117	0,019	<u>32,2</u>	<u>322</u>	0,541	P
14-19	A4/T1	1370	1370	140	203	114	0,011	<u>32,8</u>	<u>432</u>	0,384	P
14-20	A4/T2	1370	1370	140	203	117	0,012	<u>29,3</u>	<u>432</u>	0,402	P

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I - 1 Databank on slabs without shear reinforcement – Collected data

Manterola (1966)

		geometry of slab and column				reinforcement				material		failure		
serial no.	slab no.	side length of quadratic slab	span of quadratic slab	total height of slab	side length of quadratic column stub ⁶⁵	effective depth	diameter of reinforcing steel ⁶⁶	d _x / s _y	s _{x2} / s _{y2}	layout of compressive reinforcement	concrete compressive cylinder strength	yield stress of reinforcing steel	failure load	type of failure
notation	unit	a	a _l	h	a ₂	d	d _s	s _x / s _y	s _{x2} / s _{y2}		f _{c,cyl}	f _y	P _u	-
15-1	P1-S1	3250	3000	125	100	107	12	100	-		25.6	304	0,216	P
15-2	P2-S1	3250	3000	125	250	107	12	100	-		33.8	304	0,257	P
15-3	P3-S1	3250	3000	125	450	107	12	100	-		29.7	304	0,301	P
15-4	P1-S2	3250	3000	125	100	107	12	100	200		24.2	324	0,196	P
15-5	P2-S2	3250	3000	125	250	107	12	100	200		33.1	324	0,283	P
15-6	P3-S2	3250	3000	125	450	107	12	100	200		31.9	324	0,397	P
15-7	P1-S3	3250	3000	125	100	107	12	100	100		39.7	324	0,184	P
15-8	P2-S3	3250	3000	125	100	107	12	75	100		35.8	324	0,211	P
15-9	P3-S3	3250	3000	125	100	107	12	200	100		39.2	324	0,165	P
15-10	P1-S4	3250	3000	125	100	107	8	100	-		26.4	451	0,175	F
15-11	P2-S4	3250	3000	125	250	107	8	100	-		31.3	451	0,246	F
15-12	P3-S4	3250	3000	125	450	107	8	100	-		34.2	451	0,294	F

⁶⁵ steel plates⁶⁶ anchored by small end hooks

I - 1 Databank on slabs without shear reinforcement – Collected data

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Yitzhaki (1966)

serial no.	slab no.	geometry of slab and column				reinforcement		material		failure	
		total diameter of circular slab	diameter of circle along the load was distributed	total height of slab	diameter of circular (rectangular) column stub ⁶⁷	effective depth	longitudinal reinforcement ratio ⁶⁸	concrete compressive strength ⁶⁹	yield stress of reinforcing steel	failure load	type of failure
notation	unit	c	c ₁	h	c ₂ , a ₂ /a ₂	d	ρ_l	f _{c,cube} MPa	f _y MPa	P _u MN	-
16-1	II-5	1163	1067	102	221	82	0,005	21,5	471	0,152	F
16-2	II-8	1163	1067	102	333	82	0,006	23,4	456	0,218	F
16-3	IIB20-1	1707	1600	102	201/201	78	0,007	13,7	403	0,128	F
16-4	II-1	1163	1067	102	221	82	0,012	13,1	457	0,181	P
16-5	II-4a	1163	1067	102	221	82	0,009	22,4	559	0,245	P
16-6	II-4b	1163	1067	102	201/201	82	0,009	12,3	466	0,162	P
16-7	II-4c	1163	1067	102	201/201	82	0,009	17,4	510	0,215	P
16-8	IIB20-2	1163	1067	102	201	83	0,009	18,7	500	0,307	P
16-9	IIB30-1	1707	1600	102	300	80	0,020	22,0	403	0,239	P

⁶⁷ monolithic; specimens 7 and II-10 loaded by means of a steel ball⁶⁸ plain steel bars⁶⁹ cubes 120 mm

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I - 1 Databank on slabs without shear reinforcement – Collected data

16-10	II-2	1707	1600	102	221	82	0,013	<u>12,2</u>	<u>373</u>	0,152	P
16-11	II-3	1707	1600	102	201/402 ⁷⁰	82	0,013	<u>16,9</u>	<u>491</u>	0,244	P
16-12	II-6	1707	1600	102	221	82	0,013	<u>27,0</u>	<u>456</u>	0,240	P
16-13	II-9	1163	1067	102	201/201	79	0,085	<u>11,6</u>	<u>550</u>	0,157	P
16-14	III-3	1163	1067	102	221	82	0,012	<u>22,6</u>	<u>559</u>	0,201	P
16-15	7	1163	1067	102	119	82	0,007	<u>12,5</u>	<u>456</u>	0,117	P
16-16	II-10	1707	1600	102	119	82	0,010	<u>14,6</u>	<u>385</u>	0,098	P

⁷⁰ calculated as quadratic slab (with the same area)

I - 1 Databank on slabs without shear reinforcement – Collected data

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Moe (1961)

		geometry of slab and column				reinforcement		material properties					failure		
serial no.	slab no	side length of quadratic slab	span of quadratic slab	total height of slab	side length of quadratic column ⁷¹	effective depth	diameter of reinforcing steel ⁷²	layout of reinforcement ⁷³	concrete compressive cylinder strength ⁷⁴	maximum aggregate size	yield stress of reinforcing steel	tensile strength of reinforcing steel	modulus of elasticity of reinforcing steel	failure load	type of failure
		a mm	a_1 mm	h mm	a_2 mm	d mm	d_s mm	s_x/s_y mm	$f_{c,cyl}$ MPa	d_a mm	f_y MPa	f_t MPa	E_s GPa	P_u MN	-
17-1	S1-60	1830	1780	152	254	114	15,9	165	<u>23,3</u>	38	<u>399</u>	<u>705</u>	<u>179</u>	0,389	P
17-2	S2-60	1830	1780	152	254	114	15,9	2-115/ 210+1	<u>22,1</u>	38	<u>399</u>	<u>705</u>	<u>179</u>	0,356	P
17-3	S3-60	1830	1780	152	254	114	15,9	3-75/ 325+1	<u>22,6</u>	38	<u>399</u>	<u>705</u>	<u>179</u>	0,364	P
17-4	S4-60	1830	1780	152	254	114	15,9	4-50/ 355+1	<u>23,8</u>	38	<u>399</u>	<u>705</u>	<u>179</u>	0,334	P
17-5	S1-70	1830	1780	152	254	114	15,9	165	<u>24,5</u>	38	<u>483</u>	<u>897</u>	<u>171</u>	0,393	P
17-6	S3-70	1830	1780	152	254	114	15,9	3-75/ 325+1	<u>25,4</u>	38	<u>483</u>	<u>897</u>	<u>171</u>	0,378	P
17-7	S4-70	1830	1780	152	254	114	15,9	4-50/ 355+1	<u>35,2</u>	38	<u>483</u>	<u>897</u>	<u>171</u>	0,374	P
17-8	S4-70A	1830	1780	152	254	114	15,9	4-50/ 355+1	<u>20,5</u>	38	<u>483</u>	<u>897</u>	<u>171</u>	0,312	P

⁷¹ monolithic⁷² anchored by straight ends⁷³ acc. to Moe, table 3.1⁷⁴ $f_{c,cyl} = (f_c)_m$ acc. to Moe, table 4.3 (cylinder $\varnothing = 150$ mm, $h = 300$ mm)

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I - 1 Databank on slabs without shear reinforcement – Collected data

17-9	S5-60	1830	1780	152	203	114	15,9	165	22,2	38	399	705	179	0,343	P
17-10	S5-70	1830	1780	152	203	114	15,9	165	23,0	38	483	897	171	0,378	P
17-11	R1	1830	1780	152	152	114	15,9	125	26,6	10	328	541	195	0,312	P
17-12	R2	1830	1780	152	152/ ⁷⁵ ₄₅₇	114	15,9	125	27,6	10	328	541	195	0,394	P
17-13	H1	1830	1780	152	254	114	15,9	152	26,1	38	328	541	195	0,372	P
17-14	M1A	1830	1780	152	305	114	19,1	165	20,8	38	481	861	195	0,433	P

⁷⁵ calculated as quadratic slab (with the same area)

I - 1 Databank on slabs without shear reinforcement – Collected data

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Kinnunen / Nylander (1960)

serial no.	slab no. ⁷⁶	geometry of slab and column				reinforcement			material properties				failure	type of failure ⁸²
		total diameter of circular slab	diameter of circle along the load was distributed	total height of slab	diameter of circular column stub ⁷⁷	effective depth	diameter of reinforcing steel ⁷⁸	s_x/s_y	concrete compressive cube strength ⁸¹	maximum aggregate size	yield stress of reinforcing steel	modulus of elasticity of reinforcing steel	failure load	
notation	unit	c	c_1	h	c_2	d	d_s	s_x/s_y	$f_{c,cube}$	d_a	f_y	E_s	P_u	-
18-1	IA15a-5	1840	1710	149	150	117	12	115	34,9	32	441	210	0,255	P
18-2	IA15a-6	1840	1710	151	150	118	12	115	32,2	32	454	210	0,275	P
-	IA15b-9	1840	1710	150	150	117	12	115 (8.57/115+1)	31,7	32	446	210	0,275	A
-	IA15b-10	1840	1710	150	150	117	12	115 (8.57/115+1)	31,9	32	448	210	0,275	A
18-3	IA15c-11	1840	1710	153	150	121	12	6.57/115+1	39,3	32	436	210	0,334	P
18-4	IA15c-12	1840	1710	154	150	122	12	6.57/115+1	36,0	32	439	210	0,332	P

⁷⁶ only slabs with orthogonal layout of reinforcement⁷⁷ monolithic⁷⁸ Type Ks 40 deformed Swedish Kam steel (ribbed); anchored by straight ends⁷⁹ for slabs IA15b and IA30b the value for the additional reinforcement of central area with straight ends (not anchored outside the punching cone) is given in brackets⁸⁰ for slabs IA15c, IA30c and IA30e additional reinforcement of central area is bent down at a distance of 425 mm from the center of the slab⁸¹ cubes 150 mm; age 28 days⁸² P = Punching, A = Anchorage failure

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I - 1 Databank on slabs without shear reinforcement – Collected data

18-5 IA30a-24	1840	1710	158	300	128	12	43/85	<u>32.4</u>	32	<u>456</u>	<u>210</u>	0,430	P
18-6 IA30a-25	1840	1710	154	300	124	12	43/85	<u>30.8</u>	32	<u>451</u>	<u>210</u>	0,408	P
- IA30b-28	1840	1710	151	300	119	12	$\frac{43/85}{(11,43/85+1)}$	<u>31.8</u>	32	<u>437</u>	<u>210</u>	0,368	A
- IA30b-29	1840	1710	151	300	119	12	$\frac{43/85}{(11,43/85+1)}$	<u>31.8</u>	32	<u>444</u>	<u>210</u>	0,417	A
18-7 IA30c-30	1840	1710	151	300	120	12	9,43/85+1	<u>36.9</u>	32	<u>436</u>	<u>210</u>	0,491	P
18-8 IA30c-31	1840	1710	151	300	119	12	9,43/85+1	<u>36.9</u>	32	<u>448</u>	<u>210</u>	0,540	P
18-9 IA30d-32	1840	1710	155	300	123	12	85/171	<u>32.3</u>	32	<u>448</u>	<u>210</u>	0,258	P
18-10 IA30d-33	1840	1710	156	300	125	12	85/171	<u>32.7</u>	32	<u>462</u>	<u>210</u>	0,258	P
18-11 IA30e-34	1840	1710	150	300	120	12	5,85/171+1	<u>33.6</u>	32	<u>461</u>	<u>210</u>	0,332	P
18-12 IA30e-35	1840	1710	153	300	122	12	5,85/171+1	<u>30.7</u>	32	<u>459</u>	<u>210</u>	0,332	P

notation ⁸³ unit	c mm	c _l mm	h mm	c ₂ mm	d mm	d _s mm	ρ_r / ρ_t ⁸⁴ %	f _{c,cube} MPa	d _a mm	f _y MPa	E _s GPa	P _u MN	-
- IC15a-18	1840	1710	150	150	125	12	0,15 / 0,64	<u>33.2</u>	32	<u>443</u>	<u>210</u>	0,203	P
- IC15a-19	1840	1710	149	150	126	12	0,15 / 0,63	<u>34.9</u>	32	<u>443</u>	<u>210</u>	0,198	P
- IC15b-20	1840	1710	152	150	127	12	0,30 / 1,32	<u>33.2</u>	32	<u>441</u>	<u>210</u>	0,267	P
- IC15b-21	1840	1710	150	150	124	12	0,31 / 1,35	<u>33.1</u>	32	<u>441</u>	<u>210</u>	0,285	P
- IA30a-41	1840	1710	153	300	125	12	0,25 / 0,96	<u>32.4</u>	32	<u>434</u>	<u>210</u>	0,300	P
- IA30b-43	1840	1710	153	300	130	12	0,49 / 0,99	<u>33.6</u>	32	<u>452</u>	<u>210</u>	0,429	P

⁸³ slabs 18-17 to 18-22: ring reinforcement in conjunction with radial reinforcement⁸⁴ for slabs 18-17 to 18-22 the ratio of reinforcement in radial and tangential direction is given:

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I - 1 Databank on slabs without shear reinforcement – Collected data

Elstner / Hognestad (1956)

serial no.	slab no.	geometry of slab and column				reinforcement					material properties			failure	
		side length of quadratic slab	span of quadratic slab	total height of slab	side length of quadratic column	effective depth	diameter of compressive reinforcing steel	layout of reinforcement	diameter of reinforcing steel	layout of compressive reinforcement	concrete compressive strength	maximum aggregate size	yield stress of reinforcing steel	failure load	type of failure
notation	unit	a	a ₁	h	a ₂	d	d _s	s _x /s _y	d _s	s _{x2} /s _{y2}	f _{c,cube}	d _a	f _y	P _u	-
19-1	I A-1a	1830	1780	152	254	118	19,1	210	12,7	190	14,1	25	332	0,303	P
19-2	I A-1b	1830	1780	152	254	118	19,1	210	12,7	190	25,2	25	332	0,365	P
19-3	I A-1c	1830	1780	152	254	118	19,1	210	12,7	190	29,0	25	332	0,356	P
19-4	I A-1d	1830	1780	152	254	118	19,1	210	12,7	190	36,8	25	332	0,351	P
19-5	I A-1e	1830	1780	152	254	118	19,1	210	12,7	190	20,3	25	332	0,356	P
19-6	I A-2a	1830	1780	152	254	114	25,4	180	19,1	210	13,6	25	321	0,334	P
19-7	I A-2b	1830	1780	152	254	114	25,4	180	19,1	210	19,5	25	321	0,400	P
19-8	I A-2c	1830	1780	152	254	114	25,4	180	19,1	210	37,4	25	321	0,467	P
19-9	I A-7b	1830	1780	152	254	114	25,4	180	19,1	210	27,9	25	321	0,512	P
19-10	I A-3a	1830	1780	152	254	114	25,4	120	19,1	210	12,8	25	321	0,356	P
19-11	I A-3b	1830	1780	152	254	114	25,4	120	19,1	210	22,6	25	321	0,445	P
19-12	I A-3c	1830	1780	152	254	114	25,4	120	19,1	210	26,5	25	321	0,534	P
19-13	I A-3d	1830	1780	152	254	114	25,4	120	19,1	210	34,5	25	321	0,547	P

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I - 1 Databank on slabs without shear reinforcement – Collected data

19-14	II A-4	1830	1780	152	356	118	19,1	210	12,7	190	26,1	25	332	0,400	P
19-15	II A-5	1830	1780	152	356	114	25,4	180	19,1	210	27,8	25	321	0,534	P
-	II A-6 ⁸⁵	1830	1780	152	356	114	25,4	120	19,1	210	25,0	25	321	0,498	P
19-16	V A-9	1830	1780	152	254	114	25,4	64 ⁸⁶	19,1	210	29,9	25	321	0,445	P
19-17	V A-10	1830	1780	152	356	114	25,4	64 ⁸⁷	19,1	210	29,7	25	321	0,489	P
19-18	VII A-13	1830	1780	152	356	121	12,7	194	9,5	215	26,2	25	294	0,236	F
19-19	VIII B-1	1830	1780	152	254	114	12,7	225	-	-	14,2	38	324	0,178	F
19-20	VIII B-2	1830	1780	152	254	114	12,7	225	-	-	47,6	38	321	0,200	F
19-21	VIII B-4	1830	1780	152	254	114	15,9	200	-	-	47,7	38	303	0,334	F
19-22	VIII B-9	1830	1780	152	254	114	22,2	170	-	-	43,9	38	341	0,505	P
19-23	VIII B-11	1830	1780	152	254	114	25,4	150	-	-	13,5	38	409	0,329	P
19-24	VIII B-14	1830	1780	152	254	114	25,4	150	-	-	50,5	38	325	0,578	P

⁸⁵ delete because of "abnormal" behaviour⁸⁶ outer area $s_{x,y} = 267$ mm⁸⁷ outer area $s_{x,y} = 267$ mm

I - 2 Databank on slabs without shear reinforcement – Input data for calculation

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running number	notations		geometry		reinforcement			material properties				failure																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																								
	serial no.	slab no.	author	slab	column	dimensions	ratio	concrete	reinforcing steel	failure load	failure load over flexure load (plasticity)	lower bound of concrete strength																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																								
no.	notat.	n	A	BC	c_1/a_1	h	shape of column	diameter of circular / side length of quadratic column stub	effective depth	d	d_s	d_{z2}	ρ_{t1}	reinforcement ratio (inner section)	ρ_{t2}	compression reinforcement ratio (inner section)	compression reinforcement ratio (outer section)	compressive uniaxial strength (prism strength)	f_{ic}	MPa	$f_{1\sigma}$	MPa	E_c	GPa	d_a	mm	yield stress of reinforcing steel	f_y	MPa	f_t	MPa	E_s	modulus of elasticity of reinforcing steel	tensile strength of reinforcing steel	f_t	MPa	E_s	modulus of elasticity of reinforcing steel	failure load	P_u	MN	P_u/P_{flex}	f_{lc-12}	MPa																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																								
unit		-	-	-	mm	mm	-	mm	mm	mm	mm	mm	-	-	-	-	-	-	MPa	MPa	MPa	MPa	GPa	GPa	mm	MPa	GPa	MPa	MPa	MPa	GPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa</

I - 2 Databank on slabs without shear reinforcement – Input data for calculation

(2 of 7)

no.	notat.	n	A	BC	c_1/a_1	h	SC	c_2/a_2	d	d_s	d_{s2}	ρ_{li}	ρ_{la}	$\rho_{2,l}$	$\rho_{2,a}$	f_{ic}	f_{icr}	E_t	d_a	f_y	f_i	E_s	P_u	P_u/P_{flex}	f_{ic-12}
unit	unit	-	-	-	mm	mm	-	mm	mm	mm	mm	-	-	-	-	MPa	MPa	GPa	mm	MPa	MPa	GPa	MN	-	MPa
21	3-1	1	Randane	k	1372	125	k	150	98	10		0,006	0,006			87,1	4,8	48,9	10	550			0,224	1,07	75,1
22	3-2	2	Randane	k	1372	125	k	150	98	10		0,006	0,006			55,5	3,9	40,6	10	550			0,212	1,01	43,5
23	3-3	3	Randane	k	1372	125	k	150	98	10		0,006	0,006			26,5	2,3	30,8	10	550			0,169	0,81	14,5
24	3-4	4	Randane	k	1372	125	k	150	98	10		0,006	0,006			58,0	4,1	36,5	10	550			0,233	1,11	46,0
25	3-5	6	Randane	k	1372	125	k	150	98	10		0,006	0,006			100,3	4,9	52,8	10	550			0,233	1,11	88,3
26	3-6	12	Randane	k	1372	125	k	150	98	10		0,013	0,013			59,6	3,4	41,2	10	550			0,319	0,70	47,6
27	3-7	13	Randane	k	1372	125	k	150	98	10		0,013	0,013			43,1	2,3	33,7	10	550			0,297	0,65	31,1
28	3-8	14	Randane	k	1372	125	k	150	98	10		0,013	0,013			60,0	3,4	35,7	10	550			0,341	0,75	48,0
29	3-9	16	Randane	k	1372	125	k	150	98	10		0,013	0,013			97,2	4,6	43,2	10	550			0,362	0,80	85,2
30	3-10	21	Randane	k	1372	125	k	150	98	10		0,013	0,013			41,4	3,3	35,1	20	650			0,286	0,53	29,4
31	3-11	22	Randane	k	1372	125	k	150	98	10		0,013	0,013			83,2	6,1	41,2	20	650			0,405	0,75	71,2
32	3-12	23	Randane	k	1372	125	k	150	100	10		0,009	0,009			55,7	4,1	36,5	20	650			0,341	0,88	43,7
33	3-13	25	Randane	k	1372	125	k	150	100	10		0,012	0,012			32,5	2,7	32,4	10	650			0,244	0,47	20,5
34	3-14	26	Randane	k	1372	125	k	150	100	10		0,012	0,012			37,1	3,0	33,9	20	650			0,294	0,57	25,1
35	3-15	27	Randane	k	1372	125	k	150	102	10		0,010	0,010	0,004	0,004	33,3	2,7	32,6	20	650			0,227	0,51	21,3
36	4-1	I. HS1	Marz./Hus.	q	1500	120	q	150	95	10		0,004	0,004			63,7	4,3	38,0	20	490			0,178	0,98	51,7
37	4-2	I. HS2	Marz./Hus.	q	1500	120	q	150	95	10		0,007	0,007			66,5	4,4	38,5	20	490			0,249	0,78	54,5
38	4-3	I. HS3	Marz./Hus.	q	1500	120	q	150	95	10		0,012	0,012			65,6	4,4	38,4	20	490			0,356	0,65	53,6
39	4-4	I. HS4	Marz./Hus.	q	1500	120	q	150	90	15		0,021	0,021			62,7	4,3	37,9	20	490			0,418	0,49	50,7
40	4-5	I. HS7	Marz./Hus.	q	1500	120	q	150	95	10		0,009	0,009			70,3	4,5	39,2	20	490			0,356	0,87	58,3
41	4-6	II. HS5	Marz./Hus.	q	1500	150	q	150	125	10		0,005	0,005			64,6	4,4	38,2	20	490			0,365	0,93	52,6
42	4-7	II. HS6	Marz./Hus.	q	1500	150	q	150	120	10		0,005	0,005			66,5	4,4	38,5	20	490			0,489	1,34	54,5
43	4-8	II. HS8	Marz./Hus.	q	1500	150	q	150	120	15		0,010	0,010			65,6	4,4	38,4	20	490			0,436	0,60	53,6
44	4-9	II. HS9	Marz./Hus.	q	1500	150	q	150	120	15		0,015	0,015			70,3	4,5	39,2	20	490			0,543	0,50	58,3
45	4-10	II. HS10	Marz./Hus.	q	1500	150	q	150	120	15		0,021	0,021			76,0	4,7	40,1	20	490			0,645	0,42	64,0
46	4-11	III. HS11	Marz./Hus.	q	1500	90	q	150	70	10		0,007	0,007			66,5	4,4	38,5	20	490			0,196	1,13	54,5
47	4-12	III. HS12	Marz./Hus.	q	1500	90	q	150	70	10		0,012	0,012			71,3	4,5	39,4	20	490			0,258	0,87	59,3
48	4-13	III. HS13	Marz./Hus.	q	1500	90	q	150	70	10		0,016	0,016			64,6	4,4	38,2	20	490			0,267	0,67	52,6
49	4-14	IV. HS14	Marz./Hus.	q	1500	120	q	220	95	10		0,012	0,012			68,4	4,5	38,9	20	490			0,498	0,85	56,4
50	4-15	IV. HS15	Marz./Hus.	q	1500	120	q	300	95	10		0,012	0,012			67,5	4,4	38,7	20	490			0,560	0,88	55,5
51	4-16	I. NS1	Marz./Hus.	q	1500	120	q	150	95	10		0,012	0,012			39,9	3,2	34,7	20	490			0,320	0,59	27,9
52	4-17	II. NS2	Marz./Hus.	q	1500	150	q	150	120	10		0,005	0,005			28,5	2,4	31,0	20	490			0,396	1,09	16,5

I - 2 Databank on slabs without shear reinforcement – Input data for calculation

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no.	notat.	n	A	BC	c_1/a_1	h	SC	c_2/a_2	d	d_s	d_{s2}	ρ_{li}	ρ_{sa}	$\rho_{2,i}$	$\rho_{2,s}$	f_{ic}	$f_{i\sigma}$	E_s	d_a	f_y	f_i	E_s	P_u/P_{fcr}	P_u	f_{ic-12}
unit	unit	-	-	-	mm	mm	-	mm	mm	mm	mm	-	-	-	-	MPa	MPa	MPa	mm	MPa	MPa	MPa	-	MN	MPa
53	5-1	F1	Lov/McL.	k	200	100	k	100	83	10	10	0,017	0,017	0,017	0,017	37,3	3,2	33,9	19	531	621		0,480	1,02	25,3
54	5-2	F2	Lov/McL.	k	400	100	k	100	83	10	10	0,017	0,017	0,017	0,017	37,3	3,2	33,9	19	531	621		0,204	0,47	25,3
55	5-3	F3	Lov/McL.	k	600	100	k	100	83	10	10	0,017	0,017	0,017	0,017	37,3	3,2	33,9	19	531	621		0,149	0,35	25,3
56	5-4	F4	Lov/McL.	k	800	100	k	100	83	10	10	0,017	0,017	0,017	0,017	37,3	3,2	33,9	19	531	621		0,129	0,31	25,3
57	5-5	F5	Lov/McL.	k	1200	100	k	100	83	10	10	0,017	0,017	0,017	0,017	37,3	3,2	33,9	19	531	621		0,139	0,35	25,3
58	6-1	S2.1	Tolf	k	2378	240	k	250	200	16		0,008	0,008			19,4	1,6	27,3	32	657			0,603	0,44	7,4
59	6-2	S2.2	Tolf	k	2378	240	k	250	199	16		0,008	0,008			18,9	1,6	27,0	32	670			0,600	0,43	6,9
60	6-3	S2.3	Tolf	k	2378	240	k	250	200	16		0,005	0,004			19,3	1,6	27,2	32	668			0,489	0,56	7,3
61	6-4	S2.4	Tolf	k	2378	240	k	250	197	16		0,005	0,004			21,6	1,8	28,3	32	664			0,444	0,52	9,6
62	6-5	S1.1	Tolf	k	1189	120	k	125	100	8		0,008	0,008			30,1	2,5	31,6	16	706			0,216	0,58	18,1
63	6-6	S1.2	Tolf	k	1189	120	k	125	99	8		0,008	0,008			23,8	2,0	29,2	16	701			0,194	0,54	11,8
64	6-7	S1.3	Tolf	k	1189	120	k	125	98	8		0,004	0,004			26,0	2,2	30,1	16	720			0,145	0,80	14,0
65	6-8	S1.4	Tolf	k	1189	120	k	125	99	8		0,004	0,004			25,5	2,1	29,9	16	712			0,148	0,81	13,5
66	7-1	I/1	Regan	q	1830	100	q	200	77	10		0,024	0,008			25,4	2,1	29,9	10	500			0,194	0,26	13,4
67	7-2	I/2	Regan	q	1830	100	q	200	77	10		0,012	0,012			23,1	1,9	28,9	10	500			0,176	0,47	11,1
68	7-3	I/3	Regan	q	1830	100	q	200	77	10		0,014	0,006			27,1	2,3	30,5	10	500			0,194	0,45	15,1
69	7-4	I/4	Regan	q	1830	100	q	200	77	10		0,012	0,012			31,9	2,6	32,2	10	500			0,194	0,52	19,9
70	7-5	I/5	Regan	q	1830	100	q	200	79	8		0,015	0,005			27,8	2,3	30,8	10	480			0,165	0,35	15,8
71	7-6	I/6	Regan	q	1830	100	q	200	79	8		0,008	0,008			21,6	1,8	28,3	10	480			0,165	0,66	9,6
72	7-7	I/7	Regan	q	1830	100	q	200	79	8		0,008	0,008			30,0	2,5	31,6	10	480			0,186	0,74	18,0
73	7-8	II/1	Regan	q	2745	250	q	250	200	25		0,010	0,010			34,4	2,8	33,0	20	530			0,825	0,38	22,4
74	7-9	II/2	Regan	q	1800	160	q	160	128	16		0,010	0,010			32,9	2,7	32,5	20	485			0,390	0,48	20,9
75	7-10	II/3	Regan	q	1800	160	q	160	128	16		0,010	0,010			33,9	2,8	32,9	10	485			0,365	0,45	21,9
76	7-11	II/4	Regan	q	900	80	q	80	64	8		0,010	0,010			32,9	2,7	32,5	20	480			0,117	0,59	20,9
77	7-12	II/5	Regan	q	900	80	q	80	64	8		0,010	0,010			33,9	2,8	32,9	10	480			0,105	0,53	21,9
78	7-13	II/6	Regan	q	900	80	q	80	64	8		0,010	0,010			35,7	2,9	33,4	5	480			0,105	0,53	23,7
79	7-14	III/1	Regan	q	1370	120	q	150	95	10		0,008	0,008			22,9	1,9	28,8		494			0,197	0,53	10,9
80	7-15	III/2	Regan	q	1370	120	q	150	95	10		0,008	0,008			9,4	0,5	21,4		494			0,123	0,33	-2,6
81	7-16	III/3	Regan	q	1370	120	q	150	95	10		0,008	0,008			37,4	3,0	33,9		494			0,214	0,57	25,4
82	7-17	III/4	Regan	q	1370	120	q	150	93	12		0,015	0,015			11,8	0,8	23,1		464			0,154	0,24	-0,2
83	7-18	III/5	Regan	q	1370	120	q	150	93	12		0,015	0,015			26,5	2,2	30,3		464			0,214	0,34	14,5
84	7-19	III/6	Regan	q	1370	120	q	150	93	12		0,015	0,015			42,1	3,3	35,3		464			0,248	0,39	30,1
85	7-20	V/1	Regan	q	1500	150	q	54	118	12		0,008	0,008			33,9	2,8	32,9		628			0,170	0,26	21,9
86	7-21	V/2	Regan	q	1500	150	q	170	118	12		0,008	0,008			31,8	2,6	32,1		628			0,280	0,38	19,8
87	7-22	V/3	Regan	q	1500	150	q	110	118	12		0,008	0,008			32,0	2,6	32,2		628			0,265	0,38	20,0
88	7-23	V/4	Regan	q	1500	150	q	102	118	12		0,008	0,008			35,8	2,9	33,5		628			0,285	0,41	23,8

I - 2 Databank on slabs without shear reinforcement – Input data for calculation

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no.	notat.	n	A	BC	c_1/a_1	h	SC	c_2/a_2	d	d_s	d_{s2}	ρ_{s1}	ρ_{s2}	ρ_{s3}	ρ_{s4}	f_{tc}	f_{tr}	E_t	d_a	f_y	f_t	E_s	P_u	P_u/P_{max}	f_{te-12}
unit	unit	-	-	-	mm	mm	-	mm	mm	mm	mm	-	-	-	-	MPa	MPa	GPa	mm	MPa	MPa	GPa	MN	-	MPa
89	8-1	S1	Swamy/Alf	q	1690	125	q	150	100	10		0,006	0,006	0,001	0,001	40,1	3,2	34,7	10	462	568		0,198	0,70	28,1
90	8-2	S7	Swamy/Alf	q	1690	125	q	150	100	10		0,007	0,005	0,003	0,001	37,4	3,0	33,9	10	462	568		0,222	0,68	25,4
91	9-1	P2	ETH	k	2600	180	k	300	143	18		0,015	0,015	0,005	0,005	35,4	2,9	36,4	16	558	711	206	0,628	0,55	23,4
92	9-2	P5	ETH	k	2600	191	k	300	171	16		0,012	0,012	0,003	0,003	27,1	2,3	36,5	16	515	683	204	0,626	0,52	15,1
93	10-1	0	Schaeffers	k	1680	143	k	210	113	12		0,008	0,008			21,9	1,8	30,0	32	420			0,280	0,97	9,9
94	10-2	3	Schaeffers	k	1680	200	k	210	170	12		0,006	0,006			22,1	2,3	28,3	32	450			0,460	0,88	10,1
95	11-1	DA 6	Ladner	k	1056	110	q	100	80	16		0,018	0,018	0,010	0,010	29,6	2,5	31,4	16	550	752		0,183	0,44	17,6
96	11-2	DA 7	Ladner	k	1056	110	q	200	80	16		0,018	0,018	0,010	0,010	33,1	2,7	32,6	16	550	752		0,288	0,63	21,1
97	11-3	DA 10	Ladner	k	1056	110	q	240	80	16		0,018	0,018	0,010	0,010	31,6	2,6	32,1	16	550	752		0,281	0,58	19,6
98	11-4	DA 11	Ladner	k	1056	110	q	320	80	16		0,018	0,018	0,010	0,010	30,0	2,5	31,6	16	550	752		0,324	0,60	18,0
99	12-1	P1	Ladner	k	2650	280	k	500	240	20		0,013	0,013			27,6	2,3	30,7	32	544			1,662	0,56	15,6
100	12-2	M1	Ladner	k	1200	127	k	226	109	10		0,012	0,012			31,4	2,6	32,0	32	541			0,362	0,65	19,4
101	13-1	AN-1	Corl/Hawk.	q	1820	146	q	254	111	16		0,015	0,015			42,2	3,3	35,3		404			0,334	0,41	30,2
102	13-2	AN-2	Corl/Hawk.	q	1820	146	q	203	111	16		0,010	0,010			42,2	3,3	35,3		444			0,266	0,46	30,2
103	14-1	A1/M1	Base	k	1370	140	q	203	114			0,011	0,011			15,5	1,2	25,3		255			0,322	0,80	3,5
104	14-2	A1/M2	Base	k	1370	140	q	203	117			0,015	0,015			14,7	1,2	24,9		282			0,346	0,54	2,7
105	14-3	A1/M3	Base	k	1370	140	q	203	121			0,019	0,019			13,5	1,0	24,2		282			0,307	0,35	1,5
106	14-4	A1/M4	Base	k	1370	140	q	203	124			0,010	0,010			13,3	1,0	24,1		432			0,259	0,35	1,3
107	14-5	A1/M5	Base	k	1370	140	q	203	117			0,012	0,012			20,0	1,7	27,5		432			0,346	0,44	8,0
108	14-6	A2/M1	Base	k	1370	140	q	203	124			0,010	0,010			33,6	2,7	32,8		255			0,409	0,95	21,6
109	14-7	A2/M2	Base	k	1370	140	q	203	117			0,015	0,015			31,2	2,6	31,9		282			0,419	0,66	19,2
110	14-8	A2/M3	Base	k	1370	140	q	203	121			0,019	0,019			30,9	2,5	31,8		282			0,430	0,50	18,9
111	14-9	A2/T1	Base	k	1370	140	q	203	124			0,010	0,010			37,3	3,0	33,9		432			0,419	0,57	25,3
112	14-10	A2/T2	Base	k	1370	140	q	203	124			0,017	0,017			39,3	3,1	34,5		432			0,439	0,35	27,3
113	14-11	A3/M1	Base	k	1370	140	q	203	124			0,010	0,010			17,9	1,5	26,5		255			0,247	0,57	5,9
114	14-12	A3/M2	Base	k	1370	140	q	203	102			0,017	0,017			18,3	1,5	26,8		282			0,336	0,61	6,3
115	14-13	A3/M3	Base	k	1370	140	q	203	117			0,019	0,019			25,9	2,2	30,0		282			0,298	0,37	13,9
116	14-14	A3/T1	Base	k	1370	140	q	203	121			0,010	0,010			19,6	1,6	27,4		432			0,328	0,47	7,6
117	14-15	A3/T2	Base	k	1370	140	q	203	119			0,012	0,012			15,2	1,2	25,1		432			0,298	0,37	3,2
118	14-16	A4/M1	Base	k	1370	140	q	203	114			0,011	0,011			36,4	2,9	33,6		255			0,259	0,64	24,4
119	14-17	A4/M2	Base	k	1370	140	q	203	119			0,015	0,015			27,7	2,3	30,7		282			0,341	0,52	15,7
120	14-18	A4/M3	Base	k	1370	140	q	203	117			0,019	0,019			30,6	2,5	31,7		322			0,541	0,59	18,6
121	14-19	A4/T1	Base	k	1370	140	q	203	114			0,011	0,011			31,2	2,6	31,9		432			0,384	0,56	19,2
122	14-20	A4/T2	Base	k	1370	140	q	203	117			0,012	0,012			27,8	2,3	30,8		432			0,402	0,51	15,8

I - 2 Databank on slabs without shear reinforcement – Input data for calculation

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no.	notat.	n	A	BC	c_1/a_1	h	SC	c_2/a_2	d	d_s	d_{s2}	ρ_{li}	ρ_{ls}	ρ_{2li}	ρ_{2ls}	f_{ic}	f_{icr}	E_s	d_a	f_y	f_t	E_s	P_u/P_{fcr}	P_u	f_{cr-12}
unit	unit	-	-	-	mm	mm	-	mm	mm	mm	mm	-	-	-	-	MPa	MPa	MPa	mm	MPa	MPa	MPa	-	MN	MPa
123	15-1	P1-S1	Manterola	q	3000	125	q	100	107	12		0,011	0,011			24,3	2,0	29,4		304			0,216	0,60	12,3
124	15-2	P2-S1	Manterola	q	3000	125	q	250	107	12		0,011	0,011			32,1	2,6	32,3		304			0,257	0,67	20,1
125	15-3	P3-S1	Manterola	q	3000	125	q	450	107	12		0,011	0,011			28,2	2,3	30,9		304			0,301	0,71	16,2
126	15-4	P1-S2	Manterola	q	3000	125	q	100	107	12		0,011	0,011	0,005	0,005	23,0	1,9	28,9		324			0,196	0,51	11,0
127	15-5	P2-S2	Manterola	q	3000	125	q	250	107	12		0,011	0,011	0,005	0,005	31,4	2,6	32,0		324			0,283	0,69	19,4
128	15-6	P3-S2	Manterola	q	3000	125	q	450	107	12		0,011	0,011	0,005	0,005	30,3	2,5	31,6		324			0,397	0,88	18,3
129	15-7	P1-S3	Manterola	q	3000	125	q	100	107	12		0,011	0,011	0,011	0,011	37,7	3,0	34,0		324			0,184	0,48	25,7
130	15-8	P2-S3	Manterola	q	3000	125	q	100	107	12		0,014	0,014	0,011	0,011	34,0	2,8	32,9		324			0,211	0,43	22,0
131	15-9	P3-S3	Manterola	q	3000	125	q	100	107	12		0,005	0,005	0,011	0,011	37,2	3,0	33,9		324			0,165	0,94	25,2
132	15-10	P1-S4	Manterola	q	3000	125	q	100	107	8		0,005	0,005			25,1	2,1	29,7		451			0,175	0,72	13,1
133	15-11	P2-S4	Manterola	q	3000	125	q	250	107	8		0,005	0,005			29,7	2,5	31,4		451			0,246	0,95	17,7
134	15-12	P3-S4	Manterola	q	3000	125	q	450	107	8		0,005	0,005			32,5	2,7	32,4		451			0,294	1,03	20,5
135	16-1	II-5	Yitzhaki	k	1067	102	k	221	82			0,005	0,005			16,0	1,3	25,6		471			0,152	1,29	4,0
136	16-2	II-8	Yitzhaki	k	1067	102	k	333	82			0,006	0,006			17,4	1,4	26,3		456			0,218	1,38	5,4
137	16-3	II-20-1	Yitzhaki	k	1600	102	q	201	78			0,007	0,007			10,2	0,6	22,0		403			0,128	1,11	-1,8
138	16-4	II-1	Yitzhaki	k	1067	102	k	221	82			0,012	0,012			9,7	0,5	21,7		457			0,181	0,66	-2,3
139	16-5	II-4a	Yitzhaki	k	1067	102	k	221	82			0,009	0,009			16,6	1,4	25,9		559			0,245	0,97	4,6
140	16-6	II-4b	Yitzhaki	k	1067	102	q	201	82			0,009	0,009			9,1	0,4	21,2		466			0,162	0,79	-2,9
141	16-7	II-4c	Yitzhaki	k	1067	102	q	201	82			0,009	0,009			12,9	1,0	23,8		510			0,215	0,96	0,9
142	16-8	II-20-2	Yitzhaki	k	1067	102	k	201	83			0,009	0,009			13,9	1,1	24,4		500			0,307	1,36	1,9
143	16-9	II-30-1	Yitzhaki	k	1600	102	k	300	80			0,020	0,020			16,3	1,3	25,8		403			0,239	0,64	4,3
144	16-10	II-2	Yitzhaki	k	1600	102	k	221	82			0,013	0,013			9,1	0,4	21,2		373			0,152	0,68	-2,9
145	16-11	II-3	Yitzhaki	k	1600	102	q	284	82			0,013	0,013			12,5	0,9	23,6		491			0,244	0,79	0,5
146	16-12	II-6	Yitzhaki	k	1600	102	k	221	82			0,013	0,013			20,1	1,7	27,6		456			0,240	0,88	8,1
147	16-13	II-9	Yitzhaki	k	1067	102	q	201	79			0,085	0,085			8,6	0,3	20,8		550			0,157	0,07	-3,4
148	16-14	III-3	Yitzhaki	k	1067	102	k	221	82			0,012	0,012			16,8	1,4	26,0		559			0,201	0,60	4,8
149	16-15	7	Yitzhaki	k	1067	102	k	119	82			0,007	0,007			9,3	0,4	21,3		456			0,117	0,82	-2,7
150	16-16	II-10	Yitzhaki	k	1600	102	k	119	82			0,010	0,010			10,8	0,7	22,5		385			0,098	0,59	-1,2

I - 2 Databank on slabs without shear reinforcement – Input data for calculation

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no.	notat.	n	A	BC	c_1/a_1	h	SC	c_2/a_2	d	d_s	d_{s2}	$\rho_{1,i}$	$\rho_{1,a}$	$\rho_{2,i}$	$\rho_{2,a}$	f_{ic}	$f_{ic\alpha}$	E_c	d_a	f_y	f_i	E_s	P_u	P_u/P_{uex}	f_{ic-12}
unit	unit	-	-	-	mm	mm	-	mm	mm	mm	mm	-	-	-	-	MPa	MPa	GPa	mm	MPa	MPa	GPa	MM	-	MPa
151	17-1	S1-60	Moe	q	1780	152	q	254	114	16		0,011	0,011			22,1	1,9	28,5	38	399	705	179	0,389	0,62	10,1
152	17-2	S2-60	Moe	q	1780	152	q	254	114	16		0,015	0,003			21,0	1,8	28,0	38	399	705	179	0,356	0,42	9,0
153	17-3	S3-60	Moe	q	1780	152	q	254	114	16		0,020	0,003			21,5	1,8	28,2	38	399	705	179	0,364	0,32	9,5
154	17-4	S4-60	Moe	q	1780	152	q	254	114	16		0,026	0,003			22,6	1,9	28,7	38	399	705	179	0,334	0,23	10,6
155	17-5	S1-70	Moe	q	1780	152	q	254	114	16		0,011	0,011			23,3	2,0	29,0	38	483	897	171	0,393	0,52	11,3
156	17-6	S3-70	Moe	q	1780	152	q	254	114	16		0,020	0,003			24,1	2,0	29,3	38	483	897	171	0,378	0,27	12,1
157	17-7	S4-70	Moe	q	1780	152	q	254	114	16		0,026	0,003			33,4	2,7	32,7	38	483	897	171	0,374	0,21	21,4
158	17-8	S4-70A	Moe	q	1780	152	q	254	114	16		0,026	0,003			19,5	1,6	27,3	38	483	897	171	0,312	0,17	7,5
159	17-9	S5-60	Moe	q	1780	152	q	203	114	16		0,011	0,011			21,1	1,8	28,0	38	399	705	179	0,343	0,57	9,1
160	17-10	S5-70	Moe	q	1780	152	q	203	114	16		0,011	0,011			21,9	1,8	28,4	38	483	897	171	0,378	0,52	9,9
161	17-11	R1	Moe	q	1780	152	q	152	114	16		0,014	0,014			25,3	2,1	29,8	10	328	541	195	0,312	0,52	13,3
162	17-12	R2	Moe	q	1780	152	q	254	114	16		0,014	0,014			26,2	2,2	30,2	10	328	541	195	0,394	0,60	14,2
163	17-13	H1	Moe	q	1780	152	q	264	114	16		0,011	0,011			24,8	2,1	29,6	38	328	541	195	0,372	0,72	12,8
164	17-14	M1A	Moe	q	1780	152	q	305	114	19		0,015	0,015			19,8	1,7	27,4	38	481	861	195	0,433	0,40	7,8
165	18-1	IA15a-5	Kinn./Nyl.	k	1710	149	k	150	117	12		0,008	0,008			27,6	2,3	30,7	32	441		210	0,255	0,82	15,6
166	18-2	IA15a-6	Kinn./Nyl.	k	1710	151	k	150	118	12		0,008	0,008			25,4	2,1	29,9	32	454		210	0,275	0,84	13,4
169	18-3	IA15c-11	Kinn./Nyl.	k	1710	153	k	150	121	12		0,018	0,004			31,0	2,6	31,9	32	436		210	0,334	0,45	19,0
170	18-4	IA15c-12	Kinn./Nyl.	k	1710	154	k	150	122	12		0,017	0,004			28,4	2,4	31,0	32	439		210	0,332	0,46	16,4
171	18-5	IA30a-24	Kinn./Nyl.	k	1710	158	k	300	128	12		0,010	0,010			25,6	2,1	29,9	32	456		210	0,430	0,81	13,6
172	18-6	IA30a-25	Kinn./Nyl.	k	1710	154	k	300	124	12		0,011	0,011			24,3	2,0	29,4	32	451		210	0,408	0,75	12,3
175	18-7	IA30c-30	Kinn./Nyl.	k	1710	151	k	300	120	12		0,021	0,006			29,2	2,4	31,2	32	436		210	0,491	0,52	17,2
176	18-8	IA30c-31	Kinn./Nyl.	k	1710	151	k	300	119	12		0,021	0,006			29,2	2,4	31,2	32	448		210	0,540	0,57	17,2
177	18-9	IA30d-32	Kinn./Nyl.	k	1710	155	k	300	123	12		0,005	0,005			25,5	2,1	29,9	32	448		210	0,258	1,07	13,5
178	18-10	IA30d-33	Kinn./Nyl.	k	1710	156	k	300	125	12		0,005	0,005			25,8	2,2	30,0	32	462		210	0,258	1,00	13,8
179	18-11	IA30e-34	Kinn./Nyl.	k	1710	150	k	300	120	12		0,010	0,006			26,5	2,2	30,3	32	461		210	0,332	0,70	14,5
180	18-12	IA30e-35	Kinn./Nyl.	k	1710	153	k	300	122	12		0,010	0,006			24,3	2,0	29,4	32	459		210	0,332	0,68	12,3

I - 2 Databank on slabs without shear reinforcement – Input data for calculation

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no.	notat.	n	A	BC	c_1/a_1	h	SC	c_2/a_2	d	d_t	d_{s2}	ρ_{li}	$\rho_{li,a}$	$\rho_{li,l}$	$\rho_{li,a}$	f_{ic}	f_{ice}	E_c	d_a	f_y	f_t	E_s	P_u/P_{flex}	P_u	f_{ic-12}
unit	unit	-	-	-	mm	mm	-	mm	mm	mm	mm	-	-	-	-	MPa	MPa	GPa	mm	MPa	MPa	GPa	-	MN	MPa
181	19-1	I A-1a	Elst./Hog.	q	1780	152	q	254	118	19		0,012	0,012	0,006	0,006	11,1	0,7	22,7	25	332			0,303	0,50	-0,9
182	19-2	I A-1b	Elst./Hog.	q	1780	152	q	254	118	19		0,012	0,012	0,006	0,006	19,9	1,7	27,5	25	332			0,365	0,60	7,9
183	19-3	I A-1c	Elst./Hog.	q	1780	152	q	254	118	19		0,012	0,012	0,006	0,006	22,9	1,9	28,8	25	332			0,356	0,59	10,9
184	19-4	I A-1d	Elst./Hog.	q	1780	152	q	254	118	19		0,012	0,012	0,006	0,006	29,1	2,4	31,2	25	332			0,351	0,58	17,1
185	19-5	I A-1e	Elst./Hog.	q	1780	152	q	254	118	19		0,012	0,012	0,006	0,006	16,0	1,3	25,6	25	332			0,356	0,59	4,0
186	19-6	I A-2a	Elst./Hog.	q	1780	152	q	254	114	25		0,025	0,025	0,012	0,012	10,7	0,7	22,4	25	321			0,334	0,29	-1,3
187	19-7	I A-2b	Elst./Hog.	q	1780	152	q	254	114	25		0,025	0,025	0,012	0,012	15,4	1,2	25,3	25	321			0,400	0,35	3,4
188	19-8	I A-2c	Elst./Hog.	q	1780	152	q	254	114	25		0,025	0,025	0,012	0,012	29,5	2,4	31,4	25	321			0,467	0,41	17,5
189	19-9	I A-7b	Elst./Hog.	q	1780	152	q	254	114	25		0,025	0,025	0,012	0,012	22,0	1,9	28,5	25	321			0,512	0,45	10,0
190	19-10	I A-3a	Elst./Hog.	q	1780	152	q	254	114	25		0,037	0,037	0,012	0,012	10,1	0,6	22,0	25	321			0,356	0,21	-1,9
191	19-11	I A-3b	Elst./Hog.	q	1780	152	q	254	114	25		0,037	0,037	0,012	0,012	17,9	1,5	26,5	25	321			0,445	0,26	5,9
192	19-12	I A-3c	Elst./Hog.	q	1780	152	q	254	114	25		0,037	0,037	0,012	0,012	20,9	1,8	28,0	25	321			0,534	0,32	8,9
193	19-13	I A-3d	Elst./Hog.	q	1780	152	q	254	114	25		0,037	0,037	0,012	0,012	27,3	2,3	30,6	25	321			0,547	0,32	15,3
194	19-14	II A-4	Elst./Hog.	q	1780	152	q	356	118	19		0,012	0,012	0,006	0,006	20,6	1,7	27,8	25	332			0,400	0,60	8,6
195	19-15	II A-5	Elst./Hog.	q	1780	152	q	356	114	25		0,025	0,025	0,012	0,012	22,0	1,8	28,4	25	321			0,534	0,43	10,0
197	19-16	V A-9	Elst./Hog.	q	1780	152	q	254	114	25		0,069	0,017	0,012	0,012	23,6	2,0	29,1	25	321			0,445	0,14	11,6
198	19-17	V A-10	Elst./Hog.	q	1780	152	q	356	114	25		0,069	0,017	0,012	0,012	23,5	2,0	29,1	25	321			0,489	0,14	11,5
199	19-18	VII A-13	Elst./Hog.	q	1780	152	q	356	121	13		0,005	0,005	0,003	0,003	20,7	1,7	27,9	25	294			0,236	0,92	8,7
200	19-19	VIII B-1	Elst./Hog.	q	1780	152	q	254	114	13		0,005	0,005			11,2	0,7	22,7	38	324			0,178	0,77	-0,8
201	19-20	VIII B-2	Elst./Hog.	q	1780	152	q	254	114	13		0,005	0,005			37,6	3,0	34,0	38	321			0,200	0,88	25,6
202	19-21	VIII B-4	Elst./Hog.	q	1780	152	q	254	114	16		0,009	0,009			37,7	3,0	34,0	38	303			0,334	0,86	25,7
203	19-22	VIII B-9	Elst./Hog.	q	1780	152	q	254	114	22		0,020	0,020			34,7	2,8	33,1	38	341			0,505	0,52	22,7
204	19-23	VIII B-11	Elst./Hog.	q	1780	152	q	254	114	25		0,030	0,030			10,7	0,7	22,3	38	409			0,329	0,19	-1,3
205	19-24	VIII B-14	Elst./Hog.	q	1780	152	q	254	114	25		0,030	0,030			39,9	3,2	34,7	38	325			0,578	0,42	27,9

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II - 1a Databank on slabs with shear reinforcement – Collected data

Graf 1938

no.	geometry of slab and column					flexural reinforcement					concrete	flexural rein- forcement	failure	
	specimen no.	constraint conditions	side length of the slab	diameter between the load or bearing points	total slab heights	column diameter	mean effective depth	Ø tension reinforcement	clear spacing of the reinforcement in x and y-direction	Ø compression reinforcement	clear spacing of the tension reinforcement in x and y-direction	yield strength	tensile strength	type of failure
	notation	-	a	a ₁	h	a ₂	d	d _s	e _x /e _y	d _s	e _x '/e _y '	f _{cy}	P _u	-
	units	-	mm	mm	mm	mm	mm	mm	mm/mm	mm	mm/mm	[MPa]	[MN]	-
*	1362	qa	1700	1500	302	300	271		+2818		keine	270	1,157	P
*	1375	qa	1700	1500	504	300	271	16	72/72		keine	270	1,647	P
1-1	1355	qa	1700	1500	302	200	273		+1911		keine	278	1,206	P
1-2	1356	qa	1700	1500	303	200	274		+1911		keine	278	1,284	P
1-3	1361	qa	1700	1500	303	300	272		+2842		keine	270	1,726	P
1-4	1363	qa	1700	1500	502	300	469	16	72/72		keine	270	3,020	P
1-5	1376	qa	1700	1500	503	200	474	14	84/77		keine	278	2,255	P
1-6	1377	qa	1700	1500	505	200	474	14	84/77		keine	278	2,206	P

*reverence test without shear reinforcement

+ flexural reinforcement cross section area in mm²/m

cylinder geometry Ø/h = 150/ 300 mm

II - 1a Databank on slabs with shear reinforcement – Collected data

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Keefe 1954

no.	geometry of slab and column					flexural reinforcement					concrete		flexural reinforcement	failure					
	specimen no.	constraint conditions	side length of the slab	diameter between the load or bearing points	total slab heights	column diameter	mean effective depth	$\emptyset$ tension reinforcement	clear spacing of the tension reinforcement in x and y-direction	e'_x/e'_y	d_s	$\emptyset$ compression reinforcement		clear spacing of the tension reinforcement in x and y-direction	compressive cylinder strength	compressive cube strength	yield strength	failure load	type of failure
notation	units	-	a	c _I	h	c	d	d _s	e _x /e _y	d _s	e'x/e'y	d _s	$\emptyset$ compression reinforcement	clear spacing of the tension reinforcement in x and y-direction	f _{c,cyl}	f _{c,cube}	f _y	P _u	-
	*	P-I	k	mm	mm	mm	mm	mm	mm/mm	mm	mm/mm	mm	mm	mm/mm	MPa	MPa	[MPa]	[MN]	-
*	P-II	k	1090	940	127	153	113	19,1	101/101	101/101	keine	26,1		keine	27,0	34,4	288	0,315	
		k	1090	940	127	153	113	19,1	101/101	101/101	keine	26,1		keine	26,3	33,2	288	0,370	
2-1	S-I	k	1090	940	127	153	113	19,1	101/101	101/101	keine	26,3		keine	25	33,5	288	0,492	
2-2	S-II	k	1090	940	127	153	113	19,1	101/101	101/101	keine	25		keine		31,9	288	0,492	

* reference specimen without shear reinforcement, cylinder geometry $\emptyset/h = 150/300$ mm, cube side length $l = 200$ mm

II - 1a Databank on slabs with shear reinforcement – Collected data

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Elstner, Hognestad 1956

no.		geometry of slab and column				flexural reinforcement					concrete		flexural reinforcement		failure		
serial no.	specimen no.	constraint conditions	side length of the slab	diameter between the load or bearing points	total slab heights	column side length	mean effective depth	Ø tension reinforcement	clear spacing of the tension reinforcement in x and y-direction	Ø compression reinforcement	clear spacing of the tension reinforcement in x and y-direction	compressive cylinder strength	maximum aggregate size	yield strength	tensile strength	failure load	type of failure
		-	a	a ₁	h	a ₂	d	d _s	e _x /e _y	d _s	e _x /e _y	f _{c,cyl}	d _g	f _y	f _t	P _u	-
notation		-	mm	mm	mm	mm	mm	mm	mm/mm	mm	mm/mm	MPa	mm	MPa	MPa	MN	-
3-1	B3	qa	1930	1829	152	254	114	15,9	187/ 162		keine	13,8	36	299	482	0,286	P
3-2	B6	qa	1930	1829	152	254	114	22,2	187/ 152		keine	13,8	36	380	482	0,468	P
3-3	B10	qa	1930	1829	152	254	114	22,2	187/ 152		keine	44,8	36	335	482	0,533	P
3-4	B17	qa	1930	1829	152	254	114	19,1	178/ 152		keine	13,8	36	308	482	0,357	P
3-5	B5	qa	1930	1829	152	254	114	22,2	187/ 152		keine	13,8	36	420	620	0,378	P
3-6	B12	qa	1930	1829	152	254	114	25,4	165/ 133		keine	44,8	36	332	551	0,787	P
3-7	B13	qa	1930	1829	152	254	114	25,4	165/ 133		keine	44,8	36	328	551	0,778	P
3-8	B15	qa	1930	1829	152	254	114	25,4	165/ 133		keine	44,8	36	332	551	0,689	FP
3-8	B16	qa	1930	1829	152	254	114	25,4	165/ 133		keine	44,8	36	339	551	0,747	P

cylinder geometry Ø/h = 150/ 300 mm

II - 1a Databank on slabs with shear reinforcement – Collected data

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Moe 1961

no.	geometry of slab and column					flexural reinforcement					concrete		Flexural reinforcement		failure					
	serial no.	specimen no.	constraint conditions	side length of the slab	diameter between the load or bearing points	total slab heights	column side length	mean effective depth	$\varnothing$ tension reinforcement	clear spacing of the tension reinforcement in x and y-direction	d_s	e_x/e_y	d_s	e'_x/e'_y	clear spacing of the tension reinforcement in x and y-direction	compressive cylinder strength	maximum aggregate size	yield strength	tensile strength	failure load
notation			units	-	a	a ₁	h	a ₂	d	d _s	e _x /e _y	d _s	e _x /e _y	f _{c,cyl}	d _g	f _y	f _t	P _u	-	
4-1	S8-60	qa		1829	1829	154	203	98	15,9	165/ 165		keine	23,0	38	399	705	0,367	P		

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II - 1a Databank on slabs with shear reinforcement – Collected data

Andersson 1963

no.		geometry of slab and column				flexural reinforcement						concrete		flexural reinforcement	failure		
serial no.	specimen no.	constraint conditions		side length of the slab	diameter between the load or bearing points	total slab heights	column diameter	mean effective depth	Ø tension reinforcement	clear spacing of the tension reinforcement in x and y-direction	Ø compression reinforcement	clear spacing of the tension reinforcement in x and y-direction	compressive cylinder strength	compressive cube strength	yield strength	failure load	type of failure
		-	-	a	c _l	h	c	d	d _s	e _x /e _y	d _s	e _x /e _y	f _{c,cyl}	f _{c,cube}	f _y	P _u	-
notation		-	-	mm	mm	mm	mm	mm	mm	mm/mm	mm	mm/mm	MPa	MPa	MPa	MN	-
5-1	A-S1-62	k		1840	1710	150	150	120	13	142/ 142		142/ 142		32,9	448	0,367	P
5-2	A-S1-63	k		1840	1710	150	150	120	13	142/ 142		142/ 142		33,0	436	0,3531	P
5-3	A-S1-64	k		1840	1710	150	150	120	13	142/ 142		142/ 142		33,0	432	0,3707	P
5-4	A-S1-65	k		1840	1710	150	150	121	13	142/ 142		142/ 142		32,8	437	0,3727	P
5-5	A-S2-66	k		1840	1710	150	150	119	13	142/ 142		142/ 142		33,4	438	0,2922	P
5-6	A-S2-67	k		1840	1710	150	150	121	13	142/ 142		142/ 142		34,2	435	0,2942	P
5-7	A-S3-68	k		1840	1710	150	150	121	13	142/ 142		142/ 142		32,1	439	0,2746	P
5-8	A-S3-69	k		1840	1710	150	150	121	13	142/ 142		142/ 142		31,0	448	0,2491	P
5-9	A-S4-70	k		1840	1710	150	150	121	13	142/ 142		142/ 142		29,6	435	0,2726	P
5-10	A-S4-71	k		1840	1710	150	150	123	13	142/ 142		142/ 142		31,5	440	0,2922	P
5-11	A-S1-76	k		1840	1710	150	300	122	12	90/ 90		90/ 90	27,0		457	0,5345	P
5-12	A-S1-77	k		1840	1710	150	300	125	12	90/ 90		90/ 90	28,0		453	0,5492	P
5-13	A-S1-78	k		1840	1710	150	300	120	12	90/ 90		90/ 90	28,2		461	0,6061	P
5-14	A-S1-79	k		1840	1710	150	300	119	12	90/ 90		90/ 90	28,4		469	0,612	P
5-15	A-S1-80	k		1840	1710	150	300	121	13	131/ 131		131/ 131	28,0		436	0,4531	P
5-16	A-S1-81	k		1840	1710	150	300	120	13	131/ 131		131/ 131	25,9		440	0,4707	P
5-17	A-S2-82	k		1840	1710	150	300	120	12	90/ 90		90/ 90	27,2		442	0,459	P
5-18	A-S2-83	k		1840	1710	150	300	119	12	90/ 90		90/ 90	23,4		442	0,459	P
5-19	A-S3-84	k		1840	1710	150	300	123	12	90/ 90		90/ 90	24,7		447	0,4119	P
5-20	A-S3-85	k		1840	1710	150	300	122	12	90/ 90		90/ 90	24,9		436	0,3923	P

II - 1a Databank on slabs with shear reinforcement – Collected data

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Franz 1963/ 1964

no.		geometry of slab and column					flexural reinforcement					concrete					flexural reinforcement			failure	
serial no.	specimen no.	constraint conditions	side length of the slab	diameter between the load or bearing points	total slab heights	column diameter	mean effective depth	Ø tension reinforcement	clear spacing of the tension reinforcement in x and y-direction	Ø compression reinforcement	clear spacing of the tension reinforcement in x and y-direction	compressive cylinder strength	compressive cube strength	tensile flexural strength	maximum aggregate size	modulus of elasticity	yield strength	tensile strength	modulus of elasticity	failure load	type of failure
		-	a	c _I	h	c	d	d _s	e _x /e _y	d _s	e _x '/e _y '	f _{c,cyl}	f _{c,cube}	f _{ct,fl}	d _g	E _c	f _y	f _t	E _s	P _u	-
*	1	k	1960	1680	140	210	129	12	80/80		keine	21,4	29,1	3,04	30	26,9	457	554	204,3	0,343	P
6-1	2	k	1960	1680	140	210	129	12	80/80		keine	22,3	29,3	3,39	30	25,5	440	554	204,3	0,384	P
6-2	3	k	1960	1680	140	210	129	12	80/80		keine	23,7	28,9	3,53	30	32,1	440	554	204,3	0,392	P
6-3	4	k	1960	1680	140	210	129	12	80/80		keine	23,9	30,4	4,03	30	33,1	440	554	204,3	0,387	P
6-4	5	k	1960	1680	140	210	129	12	80/80		keine	26,9	32,6	3,56	30	28,4	440	554	204,3	0,431	P
6-5	6	k	1960	1680	140	210	129	12	80/80		keine	21,0	29,7	4,56	30	26,5	440	554	204,3	0,426	P
6-6	7	k	1960	1680	140	210	129	12	80/80		keine	21,8	27,5	4,47	30	26,0	440	554	204,3	0,353	P
6-7	8	k	1960	1680	143	210	125	12	80/80		keine	26,8	32,0	4,05	30	27,4	434	530	212,0	0,431	P
6-8	9	k	1960	1680	143	210	125	12	95/95		keine	20,3	26,5	4,24	30	22,7	434	530	212,0	0,431	FP
6-9	10	k	1960	1680	143	210	125	12	113/113		keine	21,7	27,9	3,6	30	26,3	434	530	212,0	0,362	B
6-10	11	k	1960	1680	143	210	125	12	130/130		keine	23,7	30,6	3,9	30	28,3	434	530	212,0	0,343	B
6-11	12	k	1960	1680	143	210	125	12	160/160		keine	22,5	28,6	2,6	30	27,9	434	530	212,0	0,323	B
6-12	13	k	1960	1680	143	210	125	12	190/190		keine	24,7	31,1	3,56	30	28,8	434	530	212,0	0,279	B
6-13	14		1960	1680	143	210	125	12	75/80		keine	19,8	26,0	3,5	30	22,0	434	530	212,0	0,411	P

* reverence test without shear reinforcement

cylinder geometry Ø/h 200/400 mm, cube side length = 200mm, geometry of the concrete tensile strength test specimen: l/b/h = 700/ 150/ 100

II - 1a Databank on slabs with shear reinforcement – Collected data

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Narasiham 1971

no.		geometry of slab and column					flexural reinforcement					concrete				flexural reinforcement		failure			
serial no.	specimen no.	constraint conditions	side length of the slab	diameter between the load or bearing points	total slab heights	column side length	mean effective depth	d_s	e_x/e_y	clear spacing of the tension reinforcement in x and y-direction	d_s	e'_x/e'_y	clear spacing of the tension reinforcement in x and y-direction	$f_{c,cyl}$	$f_{c,cube}$	$f_{ct,sp}$	d_g	yield strength	tensile strength	failure load	type of failure
		-	a	a_1	h	a_2	d	mm	mm/mm	mm/mm	mm	mm/mm	mm/mm	mm/mm	MPa	MPa	MPa	MPa	MPa	MPa	P_u
*	L7	-	2280	2000	178	305	143	16	127/127	16	127/127	127/127	127/127	33,0	44,4	2,97	19	398	685	0,588	P
*	L9	-	2280	2000	178	305	143	16	127/127	16	127/127	127/127	127/127	30,4	41,4	2,78	19	398	685	0,687	P
7-1	L2	qa	2280	2000	178	305	143	16	127/127	16	127/127	127/127	127/127	33,4	34,2	3,10	19	398	685	0,868	FP

* reverence test without shear reinforcement
cylinder geometry $\emptyset/h = 150/300$, cube side length = 150 mm

II - 1a Databank on slabs with shear reinforcement – Collected data

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Petcu, Stanculescu 1973]

no.	geometry of slab and column					flexural reinforcement					concrete	flexural reinforcement			failure	
	constraint conditions	side length of the slab	diameter between the load or bearing points	total slab heights	column side length	mean effective depth	d_s	a_{sx}/a_{sy}	d_s	clear spacing of the tension reinforcement in x and y-direction	$f_{c,cyl}$	yield strength	tensile strength	modulus of elasticity	failure load	type of failure
serial no.	specimen no.	notation	units									f_y	f_t	E_s	P_u	-
												MPa	MPa	GPa	MN	-
8-1	B6		qa	2700	2400	193	200	171		872/872	31,4	404			0,570	
8-2	B7		qa	2700	2400	192	200	167		868/868	32,1	404			0,629	
8-3	B8		qa	2700	2400	192	200	167		868/868	29,4	404			0,610	
8-4	B9		qa	2700	2400	230	200	203		873/873	22,1	404			0,635	
8-5	B10		qa	2700	2400	237	200	212		911/911	20,5	404			0,786	
8-6	A7		qa	2700	2400	142	200	120		876/876	22,4	420			0,323	
8-7	A8		qa	2700	2400	142	200	121		871/871	20,4	420			0,411	
8-8	A5		qa	2700	2400	141	200	113		881/881	23,6	420			0,274	
8-9	A6		qa	2700	2400	138	200	110		880/880	25,9	420			0,274	

II - 1a Databank on slabs with shear reinforcement – Collected data

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Marti, Pranlog, Thürlimann 1977

no.		geometry of slab and column					flexural reinforcement					concrete						flexural reinforcement			failure	
serial no.	specimen no.	constraint conditions	diameter of the slab	diameter between the load or bearing points	total slab heights	column diameter	mean effective depth	Ø tension reinforcement	clear spacing of the tension reinforcement in x and y-direction	Ø compression reinforcement	clear spacing of the tension reinforcement in x and y-direction	compressive cylinder strength	compressive cube strength	tensile splitting strength	tensile flexural strength	modulus of elasticity	maximum aggregate size	yield strength	tensile strength	modulus of elasticity	failure load	type of failure
		-	c ₂	c ₁	h	c	d	d _s	e _x /e _y	d _s	e _x 'e _y '	f _{c,cyl}	f _{c,cube}	f _{ct,sp}	f _{ct,fl}	E _c	d _g	f _y	f _t	E _s	P _u	-
*	P2	-	mm	mm	mm	mm	mm	mm/mm	mm/mm	mm	mm/mm	MPa	MPa	MPa	MPa	GPa	mm	MPa	MPa	GPa	MN	-
9-1	P3	k	2750	2600	180	300	152	18	120/ 120	10	120/ 120	35,4	43,2	3,4	6,3	35,7	16	558	712	210	0,627	P
		k	2750	2600	180	300	152	18	120/ 120	10	120/ 120	34		3,3	6,1	34,8	16	558	712	210	0,875	FP

* reference test without shear reinforcement, prism geometry $a/b/h = 120/120/300$, cube side length = 150 mm

II - 1a Databank on slabs with shear reinforcement – Collected data

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Sunquist 1977

no.		geometry of slab and column				flexural reinforcement				concrete	flexural reinforcement	failure			
serial no.	specimen no.	constraint conditions	side length of the slab	diameter between the load or bearing points	total slab heights	column diameter	mean effective depth	Ø tension reinforcement	clear spacing of the tension reinforcement in x and y-direction	Ø compression reinforcement	clear spacing of the tension reinforcement in x and y-direction	compressive cylinder strength	yield strength	failure load	type of failure
notation	-	-	c_2	c_1	h	c	d	d_s	e_x/e_y	d_s	e'_x/e'_y	$f_{c,cyl}$	f_y	P_u	-
units	-	-	mm	mm	mm	mm	mm	mm	mm/mm	mm	mm/mm	MPa	MPa	MN	-
10-1	A	k	1900	1710	200	40	170	12	100/ 100	8	140/ 140	24,0	457	0,370	P
10-2	C	k	1900	1710	200	40	170	12	120/ 120	8	140/ 140	24,5	455	0,365	P
10-3	D	k	1900	1710	200	140	170	12	120/ 120	8	140/ 140	26,8	454	0,580	P
10-4	E	k	1900	1710	200	250	170	12	120/ 120	8	140/ 140	20,6	457	0,560	P

II - 1a Databank on slabs with shear reinforcement – Collected data

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Seible, Ghali, Dilger 1980

no.	geometry of slab and column					flexural reinforcement					concrete	flexural reinforcement	failure					
	serial no.	specimen no.	constraint conditions	side length of the slab	diameter between the load or bearing points	total slab heights	column side length	mean effective depth	$\emptyset$ tension reinforcement	clear spacing of the tension reinforcement in x and y-direction	d_s	e_x/e_y	d_s	e'_x/e'_y	compressive cylinder strength	yield strength	failure load	type of failure
	notation	-	-	a	a_1	h	a_2	d	d_s	e_x/e_y	d_s	e'_x/e'_y	d_s	e'_x/e'_y	$f_{c,cyl}$	f_y	P_u	-
	units	-	-	mm	mm	mm	mm	mm	mm	mm/mm	mm	mm/mm	mm	mm/mm	MPa	MPa	MN	-
11-1	SC8	qa	1800	1698	157	305	305	121	12,7	90/ 90	121	available		available	33,6	448	0,591	P
11-2	SC9	qa	1800	1698	157	305	305	121	12,7	90/ 90	121	available		available	33,6	448	0,596	P
11-3	SC10	qa	1800	1698	157	305	305	121	12,7	90/ 90	121	available		available	33,6	448	0,538	P

II - 1a Databank on slabs with shear reinforcement – Collected data

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Müller, Muttoni, Thürlimann 1984, Pralong, Brändli, Thürlimann 1979

no.	geometry of slab and column					flexural reinforcement				concrete						flexural reinforcement			failure						
	specimen no.	constraint conditions	diameter of the slab	diameter between the load or bearing points	total slab heights	column diameter	mean effective depth	$\emptyset$ tension reinforcement	clear spacing of the tension reinforcement in x and y-direction	e_x/e_y	d_s	$\emptyset$ compression reinforcement	clear spacing of the tension reinforcement in x and y-direction	e'_x/e'_y	compressive cylinder strength	compressive cube strength	tensile splitting strength	tensile flexural strength	modulus of elasticity	maximum aggregate size	yield strength	tensile strength	modulus of elasticity	failure load	type of failure
notation	units	-	c	c_I	h	c	d	d_s	e_x/e_y	e_y/e_x	d_s	$\emptyset$ compression reinforcement	clear spacing of the tension reinforcement in x and y-direction	e'_x/e'_y	$f_{c,cyl}$	$f_{c,cube}$	$f_{ct,sp}$	$f_{ct,fl}$	E_c	d_g	f_y	f_t	E_s	P_u	-
*	P4	k	2750	1300	180	300	162	16	100+	100+	8	8	100/100	100/100	30,8	37,4	2,90	4,72	36,4	16	515	683	204	0,366	P
13-1	P6	k	2750	1300	180	300	154	16	100+	100+	8	8	100/100	100/100	30,0	33,5	2,46	4,65	38,1	16	515	683	204	0,594	P
13-2	P22	k	2750	1300	185	300	153.5	16	100/100	100/100	8	8	100/100	100/100	32.9	39.5	2.65		31.6	16	507	637	199	1,040	FP

* reference test without shear reinforcement

+ the tension reinforcement is built as with tangential loops, cylinder geometry $\emptyset/h = 150/300$ mm, cube side length = 150 mm

II - 1a Databank on slabs with shear reinforcement – Collected data

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Hallgren, Kinnunen 1990

no.	geometry of slab and column					flexural reinforcement					concrete							flexural rein- forcement	failure		
	specimen no.	constraint conditions	diameter of the slab	diameter between the load or bearing points	total slab heights	column diameter	mean effective depth	Ø tension reinforcement	clear spacing of the tension reinforcement in x and y-direction	Ø compression reinforcement	clear spacing of the tension reinforcement in x and y-direction	compressive cylinder strength	compressive cube strength	tensile splitting strength	tensile flexural strength	modulus of elasticity	maximum aggregate size	fracture energie	yield strength	failure load	type of failure
P_u																				MPa	
notation	-	-	c_2	c_1	h	c	d	d_s	e_x/e_y	d_s	e'_x/e'_y	$f_{c,cyl}$	$f_{c,cube}$	$f_{ct,sp}$	$f_{ct,fl}$	E_c	d_g	G_f	f_y	P_u	-
	units	-	mm	mm	mm	mm	mm	mm	mm/mm	mm	mm/mm	MPa	MPa	MPa	MPa	GPa	mm	N/m	MPa	MN	-
14-1	HSC3S	k	2540	2400	240	250	200	16	160/160		160/160	92,4	97,5	6,5	7,7	37,4	18	158	632	1,329	P
14-2	HSC7S	k	2540	2400	240	250	200	16	210/210		210/210	85,0	99,4	7,4	10,6	39,4	18	156	630	1,106	FP

cylinder geometry $\varnothing/h = 150/300$ mm, cube side length 150 mm, geometry of the concrete tensile strength test specimen $l/b/h = 800/150/100$

II - 1a Databank on slabs with shear reinforcement – Collected data

Broms 1990

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no.		geometry of slab and column				flexural reinforcement					concrete		flexural reinforcement		failure				
serial no.	specimen no.	constraint conditions	side length of the slab	diameter between the load or bearing points	total slab heights	column side length	mean effective depth	$\emptyset$ tension reinforcement	clear spacing of the tension reinforcement in x and y-direction	e_x/e_y	$\emptyset$ compression reinforcement	clear spacing of the tension reinforcement in x and y-direction	compressive cube strength	tensile splitting strength	maximum aggregate size	yield strength	tensile strength	failure load	type of failure
		-	a	c _l	h	a ₂	d	d _s	e _x 'e _y	d _s	e _x 'e _y	d _s	e _x 'e _y	f _{c,cube}	f _{ct,sp}	d _g	f _y	f _u	P _u
			mm	mm	mm	mm	mm	mm	mm/mm	mm/mm	mm	mm/mm	MPa	MPa	mm	MPa	MPa	MN	
*	1	k	2600	2000	180	250	150	12	80/80				29,3			681	830	0,435	P
15-1	2	k	2600	2000	180	250	150	12	120/120				33,6			681	830	0,523	P
15-2	3	k	2600	2000	180	250	150	12	80/80				32,1			681	830	0,558	P
15-3	4	k	2600	2000	180	250	150	10	90/90				20,7			656	783	0,415	P
15-4	5	k	2600	2000	180	250	150	12	180/180				23,6			684	827	0,434	P
								10	180/180							565	783		
15-5	6	k	2600	2000	180	250	150	10	180/180		6	90/90	27,9			665	793	0,608	B
								8	180/180							678	817		
15-6	7	k	2600	2000	180	250	150	12	90/90		8	90/90	23,6			691	969	1 006	B

* reference test without shear reinforcement; cube side length = 150 mm

II - 1a Databank on slabs with shear reinforcement – Collected data

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Lovrovich, Mc Lean 1990

no.		geometry of slab and column				flexural reinforcement					concrete			flexural reinforcement		failure			
serial no.	specimen no.	constraint conditions	diameter of the slab	diameter between the load or bearing points	total slab heights	column diameter	mean effective depth	d_s	clear spacing of the tension reinforcement	e_x/e_y	d_s	clear spacing of the tension reinforcement in x and y-direction	compressive cylinder strength	tensile splitting strength	maximum aggregate size	yield strength	tensile strength	failure load	type of failure
		-	c_2	c_1	h	c	d	mm	mm/mm	mm/mm	mm	mm/mm	mm	MPa	MPa	mm	MPa	MPa	P_u
16-1	FV1	k	507,8	203	100	101	82,5	9,52	49/ 49	49/ 49	9,52	49/ 49	38,9	MPa	mm	530,9	620,6	0,2619	P
16-2	FV2	k	711,2	406,4	100	101	82,5	9,52	49/ 49	49/ 49	9,52	49/ 49	38,9	MPa	mm	530,9	620,6	0,2753	P
16-3	FV3	k	914,4	609,6	100	101	82,5	9,52	49/ 49	49/ 49	9,52	49/ 49	38,9	MPa	mm	530,9	620,6	0,2793	P
16-4	FV4	k	1117,6	812,8	100	101	82,5	9,52	49/ 49	49/ 49	9,52	49/ 49	38,9	MPa	mm	530,9	620,6	0,3416	P
16-5	FV5	k	1524	1219,2	100	101	82,5	9,52	49/ 49	49/ 49	9,52	49/ 49	38,9	MPa	mm	530,9	620,6	0,5497	P

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II - 1a Databank on slabs with shear reinforcement – Collected data

Regan 1990

serial no.	specimen no.	geometry of slab and column					flexural reinforcement					concrete			flexural reinforcement		failure	
		constraint conditions	side length of the slab	diameter between the load or bearing points	total slab heights	column diameter	mean effective depth	d_s	e_x/e_y	d_s	clear spacing of the tension reinforcement in x and y-direction	e'_x/e'_y	compressive cube strength	tensile splitting strength	maximum aggregate size	yield strength	tensile strength	failure load
notation		-	a	c_I	h	c	d	d_s	e_x/e_y	d_s	clear spacing of the tension reinforcement in x and y-direction	$f_{c,cube}$	$f_{ct,sp}$	d_g	f_y	f_u	P_u	-
units		-	mm	mm	mm	mm	mm	mm	mm/mm	mm	mm/mm	MPa	MPa	mm	MPa	MPa	MN	-
17-1	1	qa	3000	2742	300	600	259	16	125/ 125	16	187,5	50,3	3,54	20	520		1,8	P

cylinder geometry $\emptyset/h = 150/300$, cube side length = 150 mm

II - 1a Databank on slabs with shear reinforcement – Collected data

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Tolf 1991; Kinnunen, Nylander, Tolf 1980

no.		geometry of slab and column					flexural reinforcement					concrete		flexural reinforcement	failure		
serial no.	specimen no.	constraint conditions	diameter of the slab	diameter between the load or bearing points	total slab heights	column diameter	mean effective depth	Ø tension reinforcement	clear spacing of the tension reinforcement in x and y-direction	Ø compression reinforcement	clear spacing of the tension reinforcement in x and y-direction	compressive cylinder strength	compressive cube strength	maximum aggregate size	yield strength	failure load	type of failure
		-	C_2 mm	C_1 mm	h mm	c mm	d mm	d_s mm	e_x/e_y mm/mm	d_s mm	e'_x/e'_y mm/mm	$f_{c,cyl}$ MPa	$f_{c,cube}$ MPa	d_g mm	f_y MPa	P_u MN	-
18-1	S2	qe	5820	4680	730	800	669	16	120*/160**	16	220/330	26,7	31,1	32	619,6	8,215	B
18-2	S3	qe	5820	4680	730	800	669	16	120*/160**	16	220/330	26,3	24,9	32	614,2	8,15	B
19-1	S1.1s	k	1270	1190	120	125	100	8	60/65			*24,1	30,9	16	706	0,261	P
19-2	S1.2s	k	1270	1190	120	125	100	8	60/65			*24,8	29,9	16	711	0,259	P
19-3	S1.3s	k	1270	1190	120	125	100	8	150/150				29,0	16	709	0,144	P
19-4	S1.4s	k	1270	1190	120	125	100	8	150/150				28,9	16	710	0,147	P
19-5	S2.1s	k	2540	2400	240	250	195	16	120/130			#19,3	31,3	32	669	0,894	P
19-6	S2.2s	k	2540	2400	240	250	195	16	120/130			#19,7	29,5	32	672	0,851	P
19-7	S2.3s	k	2540	2400	240	250	200	16	300/300				30,9	32	671	0,562	P
19-8	S2.4s	k	2540	2400	240	250	200	16	300/300				31,6	32	669	0,542	P

* double wires, ** three wires, cube side length = 150 mm, *prism geometry l/b/h = 240/120/120 mm, #prism geometry l/b/h: 240/120/120 mm

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II - 1a Databank on slabs with shear reinforcement – Collected data

Chana, Desai 1991, 1992

no.		geometry of slab and column				flexural reinforcement						concrete			flexural reinforcement	failure	
serial no.	specimen no.	constraint conditions	side length of the slab	diameter between the load or bearing points	total slab heights	column side length	mean effective depth	Ø tension reinforcement	clear spacing of the tension reinforcement in x and y-direction	Ø compression reinforcement	clear spacing of the tension reinforcement in x and y-direction	compressive cube strength	tensile splitting strength	maximum aggregate size	yield strength	failure load	type of failure
		-	a	c _l	h	a ₂	d	d _s	e _x /e _y	d _s	e' _x /e' _y	f _{c,cube}	f _{ct,sp}	d _g			
	notation	-	mm	mm	mm	mm	mm	mm	mm/mm	mm	mm/mm	MPa	MPa	mm	MPa	MN	-
*	1	qs	3000	2400	240	300	200	20	200/ 200		available	40,3		20	500	0,805	P
20-1	2	qs	3000	2400	240	300	200	20	200/ 200		available	44,4		20	500	1,057	P
20-2	3	qs	3000	2400	240	300	200	20	200/ 200		available	41,1		20	500	1,139	P
20-3	4	qs	3000	2400	240	300	200	20	200/ 200		available	45,4		20	500	1,302	P
20-4	5	qs	3000	2400	250	300	210	20	175/ 175		available	38,3		20	500	1,382	P
20-5	6	qs	3000	2400	250	300	210	20	175/ 175		available	43,4		20	500	1,283	P
20-6	7	qs	3000	2400	250	300	210	20	175/ 175		available	40,4		20	500	1,429	P
20-7	8	qs	3000	2400	250	300	210	20	175/ 175		available	39,7		20	500	1,327	P
20-8	9	qs	3000	2400	228	300	188	20	200/ 200		available	42,5		20	500	1,135	P
*	FPS1	qa	4500	2400	250	400	210	20	175/ 175	20	175/ 175	26,8	2,31	20	500	1,225	P
21-1	FPS2	qa	4500	2400	250	400	210	20	175/ 175	20	175/ 175	34,3	2,64	20	500	1,51	P
21-2	FPS3	qa	4500	2400	250	400	210	20	175/ 175	20	175/ 175	34	2,74	20	500	1,773	P
21-3	FPS4	qa	4500	2400	250	400	210	20	175/ 175	20	175/ 175	38,4	2,66	20	500	2,028	P
21-4	FPS5	qa	4500	2400	250	400	210	20	175/ 175	20	175/ 175	32,2	2,81	20	500	1,94	P

cube side length = 100 mm

II - 1a Databank on slabs with shear reinforcement – Collected data

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Yamada, Nanni, Katsushiko 1992

no.		geometry of slab and column					flexural reinforcement					concrete			flexural reinforcement		failure	
serial no.	specimen no.	constraint conditions	side length of the slab	diameter between the load or bearing points	total slab heights	column side length	mean effective depth	Ø tension reinforcement	clear spacing of the tension reinforcement in x and y-direction	Ø compression reinforcement	clear spacing of the tension reinforcement in x and y-direction	compressive cube strength	tensile splitting strength	maximum aggregate size	yield strength	tensile strength	failure load	type of failure
		-	a	a ₁	h	a ₂	d	d _s	e _x /e _y	d _s	e _x '/e _y '	f _{c,cube}	f _{ct,sp}	d _g	f _y	f _u	P _u	-
*	T1	-	mm	mm	mm	mm	mm	mm	mm/mm	mm	mm/mm	MPa	MPa	mm	MPa	MPa	MN	-
22-1	T2	qa	2000	1500	200	300	167	13	60/60	13	120/120	21,6			811		0,441	P
22-2	T3	qa	2000	1500	200	300	167	13	60/60	13	120/120	23,35			811		0,600	P
22-3	T4	qa	2000	1500	200	300	167	13	60/60	13	120/120	23,74			811		0,727	P
22-4	T5	qa	2000	1500	200	300	167	13	60/60	13	120/120	24,43			811		0,697	P
22-5	T6	qa	2000	1500	200	300	167	13	60/60	13	120/120	22,66			811		0,762	P
*	K1	qa	2000	1500	200	300	167	13	80/80	13	80/80	26,0			568		0,735	P
22-6	K2	qa	2000	1500	200	300	167	13	80/80	13	80/80	27,17			568		0,685	P
22-7	K3	qa	2000	1500	200	300	167	13	80/80	13	80/80	25,9			568		0,950	P
22-8	K4	qa	2000	1500	200	300	167	13	80/80	13	80/80	27,37			568		1,183	P
22-9	K5	qa	2000	1500	200	300	167	13	80/80	13	80/80	26,0			568		1,153	P
22-10	K6	qa	2000	1500	200	300	167	13	80/80	13	80/80	26,39			568		1,440	P
22-11	K7	qa	2000	1500	200	300	167	13	80/80	13	80/80	27,76			568		1,274	P
																	1,498	P

* reverence test without shear reinforcement; cube side length =150 mm

II - 1a Databank on slabs with shear reinforcement – Collected data

Chana 1993

no.		geometry of slab and column					flexural reinforcement					concrete		flexural reinforcement		failure		
serial no.	specimen no.	constraint conditions	side length of the slab	diameter between the load or bearing points	total slab heights	column side length	mean effective depth	$\varnothing$ tension reinforcement	clear spacing of the tension reinforcement in x and y-direction	$\varnothing$ compression reinforcement	clear spacing of the tension reinforcement in x and y-direction	compressive cube strength	tensile splitting strength	maximum aggregate size	yield strength	tensile strength	failure load	type of failure
		-	a	a ₁	h	a ₂	d	d _s	e _x /e _y	d _s	e' _x /e' _y	f _{c,cube} MPa	f _{ct,sp} MPa	d _g mm	f _y MPa	f _t MPa	P _u	-
23-1	2	k	3000	2400	228	300	188	20	200/200	8	200/200	41,2			520		0,981	P
23-2	3	k	3000	2400	228	300	188	20	200/200	8	200/200	44,1			520		1,286	P
23-3	4	k	3000	2400	228	300	188	20	200/200	8	200/200	42,5			520		1,135	P

cube side length = 100 mm

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II - 1a Databank on slabs with shear reinforcement – Collected data

Marzouk, Jiang 1993

no.		geometry of slab and column				flexural reinforcement					concrete			flexural reinforcement		failure				
serial no.	specimen no.	constraint conditions	side length of the slab	diameter between the load or bearing points	total slab heights	column side length	mean effective depth	d_s	e_x/e_y	d_s	clear spacing of the tension reinforcement in x and y-direction	$\emptyset$ compression reinforcement	clear spacing of the tension reinforcement in x and y-direction	compressive cylinder strength	tensile splitting strength	maximum aggregate size	yield strength	tensile strength	failure load	type of failure
		-	a	a_1	h	a_2	d	mm	mm/mm	mm	mm/mm	mm	$\emptyset$ tension reinforcement	mm/mm	MPa	MPa	mm	MPa	f_t	P_u
*	HS 17	-	mm	mm	mm	mm	mm	mm	mm/mm	mm	mm/mm	mm	mm/mm	MPa	MPa	mm	MPa	MPa	MN	-
	24-1	qa	1950	1830	150	250	120	15	166/ 166	10	305/ 305	67	305/ 305	67		20	490	690	0,511	P
24-2	HS20	qa	1950	1830	150	250	120	15	166/ 166	10	305/ 305	74	305/ 305	74		20	490	690	0,418	P
24-3	HS21	qa	1950	1830	150	250	120	15	166/ 166	10	305/ 305	72	305/ 305	72		20	490	690	0,609	P
24-4	HS18	qa	1950	1830	150	250	120	15	166/ 166	10	305/ 305	68	305/ 305	68		20	490	690	0,511	P

* reverence test without shear reinforcement

II - 1a Databank on slabs with shear reinforcement – Collected data

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Beutel, Hegger 1998

no.		geometry of slab and column					flexural reinforcement					concrete						flexural reinforcement			failure		
serial no.	specimen no.	constraint conditions	diameter of the slab	diameter between the load or bearing points	total slab heights	column side length	mean effective depth	Ø tension reinforcement	clear spacing of the tension reinforcement in x and y-direction	Ø compression reinforcement	clear spacing of the tension reinforcement in x and y-direction	compressive cylinder strength	compressive cube strength	tensile splitting strength	tensile flexural strength	modulus of elasticity	maximum aggregate size	yield strength	tensile strength	tensile strength	modulus of elasticity	failure load	type of failure
		-	c	c _l	h	a ₂	d	d _s	e _x /e _y	d _s	e' _x /e' _y	f _{c,cyl} MPa	f _{c,cube} MPa	f _{ct,sp} MPa	f _{ct,fl} MPa	E _c GPa	d _g mm	f _y MPa	f _t MPa	E _s GPa	P _u MN	-	
*		-	mm	mm	mm	mm	mm	mm	mm/mm	mm	mm/mm	MPa	MPa	MPa	MPa	GPa	mm	MPa	MPa	GPa	MN		
25-1	P1-I	k	2750	2400	230	400	190	14	100/100	10	175/175	21,9	27,5	1,7	3,6	21,7	16	572	671	218,5	0,615	P	
25-2	P1-II	k	2750	2400	230	400	190	14	100/100	10	175/175	27,3	37,1	1,9	3,7	23,9	16	572	671	218,5	1,151	P	
25-3	P2-I	k	2750	2400	230	400	190	14	100/100	10	175/175	26,2	36,2	1,5	3,4	25,5	16	572	671	218,5	1,055	P	
25-4	P2-II	k	2750	2400	230	400	190	14	100/100	10	175/175	37,9	45,0	2,0	4,1	28,8	16	549	621	203,2	1,326	P	
25-5	P2-III	k	2750	2400	230	400	190	14	100/100	10	175/175	29,8	34,8	2,1	3,7	24,9	16	549	621	203,2	1,109	P	
25-6	P3-I	k	2750	2400	270	320	220	20	125/125	12	125/125	23,2	29,1	2,0	3,7	22,2	16	557	667	212	1,624	P	
25-7	P4-III	k	2750	2400	267	320	222	20	125/125	12	125/125	27,8	33,0	2,1	3,9	25,7	16	557	667	212	1,522	P	
25-8	P5-I	k	2750	2400	270	320	220	20	105/105	12	125/125	45,3	57,5	3,3	4,0	28,5	16	557	667	212	1,936	P	
25-9	P6-I	k	2750	2400	273	320	223	25	125/125	14	125/125	46,3	56,8	3,3	5,0	27,3	16	532	641	204,5	2,349	P	
25-10	P7-I	k	2750	2400	275	320	229	20	105/105	12	125/125	40	47,9	2,7	5,0	28,6	16	557	667	212	2,117	P	

* reverence test without shear reinforcement

cylinder geometry $\emptyset/h = 150/300$ mm, cube side length = 200 mm, geometry of the concrete flexure tensile test specimen $l/b/h = 700/ 150/ 100$

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II - 1a Databank on slabs with shear reinforcement – Collected data

Lee, Lee, Teng, Morley 1999

no.	geometry of slab and column					flexural reinforcement							concrete		flexural reinforcement		failure		
	serial no.	specimen no.	constraint conditions	side length of the slab	diameter between the load or bearing points	total slab heights	column side length	mean effective depth	Ø tension reinforcement	clear spacing of the tension reinforcement in x and y-direction	Ø compression reinforcement	clear spacing of the tension reinforcement in x and y-direction	compressive cube strength	tensile splitting strength	maximum aggregate size	yield strength	tensile strength	failure load	type of failure
notation			-	a	a ₁	h	a ₂	d	d _s	e _x /e _y	d _s	e' x/e' y	f _{c,cube} MPa	f _{ct,sp} MPa	d _g mm	f _y MPa	f _t MPa	P _u MN	-
*	2F22	k	2000	1500	200	290	160	20	100/100			available	92,7			500		1,100	P
*	3F22	k	2750	2250	275	400	230	20	90/90			available	87,7			500		1,640	P
26-1	2V10	k	2000	1500	200	290	155	20	100/100			available	90,4			500		1,190	P
26-2	2V20	k	2000	1500	200	290	160	20	100/100			available	92,9			500		1,450	P
26-3	2V21	k	2000	1500	200	290	160	20	100/100			available	94,1			500		1,480	P
26-4	2V22	k	2000	1500	200	290	160	20	100/100			available	93,7			500		1,690	FP
26-5	3V20	k	2750	2250	275	400	230	20	90/90			available	85,6			500		1,900	P

* reverence test specimen without shear reinforcement

cube side length = 150 mm

II - 1a Databank on slabs with shear reinforcement – Collected data

Olivera, Denio, Melo, Guilherme 1999

no.		geometry of slab and column				flexural reinforcement					concrete			flexural reinforcement		failure			
serial no.	specimen no.	constraint conditions	side length of the slab	diameter between the load or bearing points	total slab heights	column diameter	mean effective depth	d_s	clear spacing of the tension reinforcement	e_x/e_y	d_s	clear spacing of the tension reinforcement in x and y-direction	compressive cylinder strength	tensile splitting strength	maximum aggregate size	yield strength	tensile strength	failure load	type of failure
		-	a	a_1	h	a_2	d	mm	mm/mm	mm/mm	mm	mm/mm	in x and y-direction	MPa	MPa	mm	MPa	MPa	P_u
*	1	-	mm	1800	1572	130	93	12,5	88/88	6,3	143/143	60,8	695	838		695	838	0,270	P
*	2	qs	mm	1800	1572	130	97	12,5	90/90	6,3	143/143	62,9	695	838		695	838	0,335	P
27-1	3V	qs	mm	1800	1572	130	105	12,5	97/97	6,3	143/143	63,0	575	1000		575	1000	0,386	P
27-2	4V	qs	mm	1800	1572	130	103	12,5	92/92	6,3	143/143	66,3	575	643		575	643	0,377	P
27-3	5V	qs	mm	1800	1572	130	103	12,5	92/92	6,3	143/143	61,4	575	1000		575	1000	0,423	P
27-4	6I	qs	mm	1800	1572	130	100	12,5	88/88	6,3	143/143	62,3	575	1000		575	1000	0,41	P
27-5	7I	qs	mm	1800	1572	130	100	12,5	88/88	6,3	143/143	60,0	575	1000		575	1000	0,49	P
27-6	8I	qs	mm	1800	1572	130	100	12,5	88/88	6,3	143/143	62,6	575	1000		575	1000	0,54	FP
27-7	9I	qs	mm	1800	1572	130	100	12,5	88/88	6,3	143/143	60,0	575	1000		575	1000	0,56	FP
27-8	10I	qs	mm	1800	1572	130	105	12,5	90/90	6,3	143/143	61,0	575	1000		575	1000	0,52	P
27-9	11I	qs	mm	1800	1572	130	102	12,5	90/90	6,3	143/143	62,0	575	1000		575	1000	0,551	FP

* reverence test without shear reinforcement

cylinder geometry $\phi/h = 100/200$

II - 1b Databank on slabs with shear reinforcement – Additional data of the shear reinforcement (1 of 9)

no.	shape and distribution		row one			row two			row three			row four			row five			material properties			failure
	specimen no.	type of shear reinforcement	distance to the column face	shear reinforcement cross section area	diameter/ inclination	distance to the column face	shear reinforcement cross section area	diameter/ inclination	distance to the column face	shear reinforcement cross section area	diameter/ inclination	distance to the column face	shear reinforcement cross section area	diameter/ inclination	yield strength	tensile strength	modulus of elasticity				
serial no.	notation	-	$l_{sw,1}$ mm	$A_{sw,1}$ mm ²	d_s/α mm/ ^o	$l_{sw,2}$ mm	$A_{sw,2}$ mm ²	d_s/α mm/ ^o	$l_{sw,3}$ mm	$A_{sw,3}$ mm ²	d_s/α mm/ ^o	$l_{sw,4}$ mm	$A_{sw,4}$ mm ²	d_s/α mm/ ^o	f_y MPa	f_t MPa	E_s GPa	-			
	unit	-																-			
1-1	1355	SE	HA	3660											280			Ps			
1-2	1356	SE	HA	3660											280			Ps			
1-3	1361	SE	HA	5190											280			Ps			
1-4	1363	SE	HA	0	3041	22/ 60	140	5630							280			Ps			
1-5	1376	SE	HA	0	3041	22/ 60	210	3694							280			Ps			
1-6	1377	SE	HA	0	3041	22/ 60	210	3694							280			Ps			
2-1	S-I	SZ	ST	43,7	362	6,2/ 90	43,7	362	6,2/ 45	134	158	158	158	4,1/ 90	398						
2-2	S-II	SZ	ST	43,7	362	6,2/ 90	43,7	362	6,2/ 45	134	158	158	158	4,1/ 90	398						
3-1	B3	SE	HA	0	568	9,52/ 45	0	0							323	551		Ps			
3-2	B6	SE	HA	0	1032	12,7/ 45	0	0							330	551		Po			
3-3	B10	SE	HA	0	1032	12,7/ 45	0	0							330	551		Po			
3-4	B17	SE	HA	0	568	9,52/ 45	0	0							330	551		Po			
3-5	B5	SE	HA	0	516	12,7/ 45	0	0							330	551		Po			
3-6	B12	BÜ	U	50	1420	9,52/ 90	150	852	9,52/ 90						323	551		Po			
3-7	B13	SE	HA	0	4000	15,88/ 20	0	0							373	551		Po			
3-8	B15	SE	HA	0	1548	12,7/ 45	0	0							330	551		Po			
3-9	B16	SE	HA	0	2064	12,7/ 45	100	1032	12,7/ 45						330	551		Po			
4-1	S8-60	SZ	U	0	510	6,3/ 45									350	705		Po			

II - 1b Databank on slabs with shear reinforcement – Additional data of the shear reinforcement

(2 of 9)

no.	specimen no.	shape and distribution		row one			row two			row three			row four			row five			material properties			failure
		type of shear reinforcement	shear reinforcement distribution	distance to the column face	shear reinforcement area	diameter/ inclination	distance to the column face	shear reinforcement area	diameter/ inclination	distance to the column face	shear reinforcement area	diameter/ inclination	distance to the column face	shear reinforcement area	diameter/ inclination	yield strength	tensile strength	modulus of elasticity				
notation		-	-	$l_{sw,1}$	$A_{sw,1}$	d_s/α	$l_{sw,2}$	$A_{sw,2}$	d_s/α	$l_{sw,3}$	$A_{sw,3}$	d_s/α	$l_{sw,4}$	$A_{sw,4}$	d_s/α	$l_{sw,5}$	$A_{sw,5}$	d_s/α	f_y	f_t	E_s	place of the mayor shear crack
unit		-	-	mm	mm ²	mm/°	mm	mm ²	mm/°	mm	mm ²	mm/°	mm	mm ²	mm/°	mm	mm ²	mm/°	MPa	MPa	GPa	-
5-1	A-S1-62	SE	ST	50	1356	12/30	0	0											448			Ps
5-2	A-S1-63	SE	ST	50	1356	12/30	0	0											436			Ps
5-3	A-S1-64	SE	ST	50	2260	12/30	0	0											436			Ps
5-4	A-S1-65	SE	ST	50	2260	12/30	0	0											437			Ps
5-5	A-S2-66	BÜ	U	60	340	6/90	120	565	6/90										294			
5-6	A-S2-67	BÜ	U	60	340	6/90	120	565	6/90										294			
5-7	A-S3-68	SE	ST	50	680	6/45	0	0											294			
5-8	A-S3-69	SE	ST	50	680	6/45	0	0											294			
5-9	A-S4-70	SE	ST	0	200	8/45	110	200	6/45										532			
5-10	A-S4-71	SE	ST	0	200	8/45	110	200	6/45										532			
5-11	A-S1-76	SE	ST	0	1357	12/30	0	0											457			
5-12	A-S1-77	SE	ST	0	1357	12/30	0	0											453			
5-13	A-S1-78	SE	ST	0	2712	12/30	0	0											461			
5-14	A-S1-79	SE	ST	0	2712	12/30	0	0											469			
5-15	A-S1-80	SE	ST	0	1808	12/30	0	0											437			
5-16	A-S1-81	SE	ST	0	1808	12/30	0	0											440			
5-17	A-S2-82	BÜ	U	60	452	6/90	110	838	90										294			
5-18	A-S2-83	BÜ	U	60	452	6/90	110	838	90										294			
5-19	A-S3-84	BÜ	U	40	905	6/45	0	0											294			
5-20	A-S3-85	BÜ	U	40	905	6/45	0	0											294			

II - 1b Databank on slabs with shear reinforcement – Additional data of the shear reinforcement (3 of 9)

no.	shape and distribution		row one			row two			row three			row four			row five			material properties		failure		
	specimen no.	type of shear reinforcement	shear reinforcement distribution	distance to the column face	shear reinforcement area	diameter/ inclination	distance to the column face	shear reinforcement area	diameter/ inclination	distance to the column face	shear reinforcement area	diameter/ inclination	distance to the column face	shear reinforcement area	diameter/ inclination	cross section area	shear reinforcement area	cross section area	diameter/ inclination		yield strength	tensile strength
notation	unit	-	-	$l_{sw,1}$	$A_{sw,1}$	d_s/α	$l_{sw,2}$	$A_{sw,2}$	d_s/α	$l_{sw,3}$	$A_{sw,3}$	d_s/α	$l_{sw,4}$	$A_{sw,4}$	d_s/α	$l_{sw,5}$	$A_{sw,5}$	d_s/α	f_y	f_t	E_s	-
6-1	2	SE	HA	70	904	12/60			-										440	554	204,3	Pc
6-2	3	SE	HA	70	904	12/60	170	452	12/60										440	554	204,3	Pc
6-3	4	BÜ	ST	80	300	8/90	210	300	8/90										300	410		Pc
6-4	5	BÜ	ST	95	600	8/90	175	600	8/90										300	410		Pc
6-5	6	BÜ	ST	95	900	8/90	175	900	8/90										300	410		Pc
6-6	7	BÜ	ST	95	714	7,1/90	175	714	7,1/90										285	416		Pc
6-7	8	BÜ	U	40	600	8/90	200	600	8/90										305	399	207,0	Ps
6-8	9	BÜ	U	40	400	8/90	200	600	8/90										305	399	207,0	Ps
6-9	10	BÜ	U	40	400	8/90	200	600	8/90										305	399	207,0	Ps
6-10	11	BÜ	U	100	600	8/90	230	200	8/90										305	399	207,0	Ps
6-11	12	BÜ	U	120	600	8/90	300	200	8/90										305	399	207,0	Ps
6-12	13	BÜ	U	0	200	8/90	190	400	8/90										305	399	207,0	Ps
6-13	14	BÜ	U	75	500	8/90	225	700	8/90										305	399	207,0	Ps
7-1	L2	BÜ	U	0	1710	9,5/45													309	474		
8-1	B6	SE	HA	0	620	/45	150	620	/45										404			
8-2	B7	SE	HA	0	620	/45	150	620	/45										404			
8-3	B8	SE	HA	0	700	/45	150	1600	/45										354			
8-4	B9	SE	HA	0	620	/45	150	620	/45										404			
8-5	B10	SE	HA	0	700	/45	100	1600	/45										354			
8-6	A7	SE	HA	0	620	/45	150	620	/45										420			
8-7	A8	SE	HA	0	1240	/45	150	1240	/45										395			
8-8	A5	BÜ	U	50	740	/90	175	740	/90										208			
8-9	A6	BÜ	U	50	1165	/90	175	1240	/90										234			

II - 1b Databank on slabs with shear reinforcement – Additional data of the shear reinforcement (4 of 9)

no.	shape and distribution		row one			row two			row three			row four			row five			material properties			failure
	specimen no.	type of shear reinforcement	shear reinforcement distribution	distance to the column face	shear reinforcement area	diameter/ inclination	distance to the column face	shear reinforcement area	diameter/ inclination	distance to the column face	shear reinforcement area	diameter/ inclination	distance to the column face	shear reinforcement area	diameter/ inclination	distance to the column face	shear reinforcement area	yield strength	tensile strength	modulus of elasticity	place of the mayor shear crack
notation	unit	-	-	$l_{sw,1}$ mm	$A_{sw,1}$ mm ²	d_s/α mm/°	$l_{sw,2}$ mm	$A_{sw,2}$ mm ²	d_s/α mm/°	$l_{sw,3}$ mm	$A_{sw,3}$ mm ²	d_s/α mm/°	$l_{sw,4}$ mm	$A_{sw,4}$ mm ²	d_s/α mm/°	$l_{sw,5}$ mm	$A_{sw,5}$ mm ²	f_y MPa	f_t MPa	E_s GPa	-
6-1	2	SE	HA	70	904	12/60	0	452	-									440	554	204,3	Pc
6-2	3	SE	HA	70	904	12/60	170	452	12/60									440	554	204,3	Pc
6-3	4	BÜ	ST	80	300	8/90	210	300	8/90									300	410		Pc
6-4	5	BÜ	ST	95	600	8/90	175	600	8/90									300	410		Pc
6-5	6	BÜ	ST	95	900	8/90	175	900	8/90									300	410		Pc
6-6	7	BÜ	ST	95	714	7,1/90	175	714	7,1/90									285	416		Pc
6-7	8	BÜ	U	40	600	8/90	200	600	8/90									305	399	207,0	Ps
6-8	9	BÜ	U	40	400	8/90	200	600	8/90									305	399	207,0	Ps
6-9	10	BÜ	U	40	400	8/90	200	600	8/90									305	399	207,0	Ps
6-10	11	BÜ	U	100	600	8/90	230	200	8/90									305	399	207,0	Ps
6-11	12	BÜ	U	120	600	8/90	300	200	8/90									305	399	207,0	Ps
6-12	13	BÜ	U	0	200	8/90	190	400	8/90									305	399	207,0	Ps
6-13	14	BÜ	U	75	500	8/90	225	700	8/90									305	399	207,0	Ps
7-1	L2	BÜ	U	0	1710	9,5/45												309	474		
8-1	B6	SE	HA	0	620	/45	150	620	/45									404			
8-2	B7	SE	HA	0	620	/45	150	620	/45									404			
8-3	B8	SE	HA	0	700	/45	150	1600	/45									354			
8-4	B9	SE	HA	0	620	/45	150	620	/45									404			
8-5	B10	SE	HA	0	700	/45	100	1600	/45									354			
8-6	A7	SE	HA	0	620	/45	150	620	/45									420			
8-7	A8	SE	HA	0	1240	/45	150	1240	/45									395			
8-8	A5	BÜ	U	50	740	/90	175	740	/90									208			
8-9	A6	BÜ	U	50	1165	/90	175	1240	/90									234			

II - 1b Databank on slabs with shear reinforcement – Additional data of the shear reinforcement (5 of 9)

no.	shape and distribution		row one			row two			row three			row four			row five			material properties			failure
	type of shear reinforcement	shear reinforcement distribution	distance to the column face	shear reinforcement area	diameter/ inclination	distance to the column face	shear reinforcement area	diameter/ inclination	distance to the column face	shear reinforcement area	diameter/ inclination	distance to the column face	shear reinforcement area	diameter/ inclination	yield strength	tensile strength	modulus of elasticity				
serial no.	specimen no.	notation	$l_{sw,1}$	$A_{sw,1}$	d_s/α	$l_{sw,2}$	$A_{sw,2}$	d_s/α	$l_{sw,3}$	$A_{sw,3}$	d_s/α	$l_{sw,4}$	$A_{sw,4}$	d_s/α	$l_{sw,5}$	$A_{sw,5}$	d_s/α	f_y	f_t	E_s	place of the mayor shear crack
		unit	mm	mm ²	mm/°	mm	mm ²	mm/°	mm	mm ²	mm/°	mm	mm ²	mm/°	mm	mm ²	mm/°	MPa	MPa	GPa	-
9-1	P3	BÜ	120	400	8/90	240	800	8/90	360	800	8/90	480	1000	8/90				543	692	210	Pc
10-1	A	SE	80	1608	16/45													435			
10-2	C	SE	80	1810	12/45													455			
10-3	D	SE	60	1810	12/45													455			
10-4	E	SE	60	1810	12/45													455			
11-1	SC8	BÜ	60	900	5,7/90	180	900	5,7/90	0	0		0	0					490			Pc/Po
11-2	SC9	BÜ	60	1020	5,7/90	120	1020	5,7/90	180	460	5,7/90	0	0					490			Po
11-3	SC10	BÜ	60	560	5,7/90	100	560	5,7/90	120	560	5,7/90	180	560	5,7/90				490			Pc
12-1	S14	SE	82,5	502,6	8/60	187,5	502,6	8/60	292,5	402,2	8/60	373,5	301,6	8/60	0	0		462	619		Ps
12-2	S17	SE	82,5	628,3	10/60	187,5	628,3	10/60	292,5	471,2	10/60	373,5	314	10/60	453,5	157	10/60	462	619		
13-1	P6	BÜ	50	2400	8/90	150	2400	8/90	250	2400	5/90	350	2400	8/90	450	1200	8/90	534	728	202	Po
13-2	P22	BÜ	75	1200	8/90	230	2400	8/90	385	2400	8/90	540	2400	8/90	695	1200	8/90	562	643	200	Pc
14-1	HSC3S	SE	90	4021,3	16/45													503			Pc
14-2	HSC7S	SE	90	3217	16/45													501			Pc
15-1	2	BÜ	150	1876,5	8/90	300	1876,5	8/90	450	1876,5	8/90	600	1876,5	8/90	750	1876,5	8/90	471	575		Pc
15-2	3	BÜ	150	2412,7	8/90	300	2412,7	8/90	450	2412,7	8/90	600	2412,7	8/90	750	3619	8/90	471	575		Po
15-3	4	BÜ	150	2447,6	6/90	300	3263,8	6/90	0	0	0	0	0	0	0	0	0	442	655		Pc
15-4	5	BÜ	150	2447,6	6/90	300	3263,8	6/90	0	0	0	0	0	0	0	0	0	442	655		Pc
15-5	6	SE/ BÜ	150	1131	6/90	300	2035,7	6/90	450	2940,5	6/90	600	3845,3	6/90	0	1809,6	12/31	671/454	807/658		
15-6	7	SE/ BÜ	150	1131	6/90	300	2035,7	6/90	450	2940,5	6/90	600	3845,3	6/90	0	3217	16/31	656/496	950/658		

II - 1b Databank on slabs with shear reinforcement – Additional data of the shear reinforcement (6 of 9)

no.	shape and distribution		row one			row two			row three			row four			row five			material properties			failure		
	serial no.	specimen no.	type of shear reinforcement	shear reinforcement distribution	distance to the column face	shear reinforcement area	diameter/ inclination	distance to the column face	shear reinforcement area	diameter/ inclination	distance to the column face	shear reinforcement area	diameter/ inclination	distance to the column face	shear reinforcement area	diameter/ inclination	distance to the column face	shear reinforcement area	diameter/ inclination	yield strength		tensile strength	modulus of elasticity
notation			-	-	$l_{sw,1}$ mm	$A_{sw,1}$ mm ²	d_s/α mm/°	$l_{sw,2}$ mm	$A_{sw,2}$ mm ²	d_s/α mm/°	$l_{sw,3}$ mm	$A_{sw,3}$ mm ²	d_s/α mm/°	$l_{sw,4}$ mm	$A_{sw,4}$ mm ²	d_s/α mm/°	$l_{sw,5}$ mm	$A_{sw,5}$ mm ²	d_s/α mm/°	f_y MPa	f_t MPa	E_s GPa	-
unit			-	-	mm	mm ²	mm/°	mm	mm ²	mm/°	mm	mm ²	mm/°	mm	mm ²	mm/°	mm	mm ²	mm/°	MPa	MPa	GPa	-
16-1	FV1	HA	U	U		193	3,73/90													280			
16-2	FV2	HA	U	U		774	3,73/90													280			
16-3	FV3	HA	U	U		1743	3,73/90													280			
16-4	FV4	HA	U	U		3098	3,73/90													280			
16-5	FV5	HA	U	U		6984	3,73/90													280			
17-1	1	SZ	U	U	145	1809,5	12/90	337,5	2261,9	12/90	530	2261,9	12/90							520			Ps
18-1	S2	Bü	U	U	200	8143	12/90	700	12667	12/90	1350	27143	12/90							428,4			Ps
18-2	S3	Bü	U	U	200	8143	12/90	700	12667	12/90	1350	27143	12/90							425,5			Ps
19-1	S1.1s	BÜ	U	U	45	314	5/90	120	314	5/90										706			Ps
19-2	S1.2s	BÜ	U	U	45	314	5/90	120	314	5/90										711			Ps
19-3	S1.3s	BÜ	U	U	45	157	5/90	120	157	5/90										709			Ps
19-4	S1.4s	BÜ	U	U	45	157	5/90	120	157	5/90										710			Ps
19-5	S2.1s	BÜ	U	U	90	1256	10/90	240	1256	10/90										503			Po
19-6	S2.2s	BÜ	U	U	90	1260	10/90	240	1260	10/90										501			Po
19-7	S2.3s	BÜ	U	U	90	628	10/90	240	628	10/90										671			Ps
19-8	S2.4s	BÜ	U	U	90	628	10/90	240	628	10/90										669			Po
20-1	2	BÜ	U	U	100	603	8/90	250	603	8/90	400	missing	8/90	550	missing	8/90				520			
20-2	3	BÜ	U	U	100	603	8/90	250	603	8/90	400	missing	8/90	550	missing	8/90				520			
20-3	4	BÜ	U	U	100	1206	8/90	250	1206	8/90	400	missing	8/90	550	missing	8/90				520			
20-4	5	BÜ	U	U	105	943	10/90	262,5	1570	10/90	420	missing	10/90	577,5	missing	10/90				520			
20-5	6	BÜ	U	U	105	1570	10/90	262,5	942,5	10/90	420	missing	10/90	577,5	missing	10/90				520			
20-6	7	BÜ	U	U	105	2513	10/90	262,5	0	10/90	420	missing	10/90	577,5	missing	10/90				520			
20-7	8	BÜ	U	U	105	603	8/90	262,5	1005	8/90	420	missing	8/90	577,5	missing	8/90				520			
20-8	9	BÜ	U	U	94	1005	8/90	235	1407	8/90	376	missing	8/90	517	missing	8/90				520			

II - 1b Databank on slabs with shear reinforcement – Additional data of the shear reinforcement (7 of 9)

no.	serial no.	specimen no.	shape and distribution		row one			row two			row three			row four			row five			material properties			failure
			type of shear reinforcement	shear reinforcement distribution	distance to the column face	shear reinforcement cross section area	diameter/ inclination	distance to the column face	shear reinforcement cross section area	diameter/ inclination	distance to the column face	shear reinforcement cross section area	diameter/ inclination	distance to the column face	shear reinforcement cross section area	diameter/ inclination	distance to the column face	shear reinforcement cross section area	diameter/ inclination	yield strength	tensile strength	modulus of elasticity	
notation			-	-	$l_{sw,1}$	$A_{sw,1}$	d_s/α	$l_{sw,2}$	$A_{sw,2}$	d_s/α	$l_{sw,3}$	$A_{sw,3}$	d_s/α	$l_{sw,4}$	$A_{sw,4}$	d_s/α	$l_{sw,5}$	$A_{sw,5}$	d_s/α	f_y	f_t	E_s	-
unit			-	-	mm	mm ²	mm/°	mm	mm ²	mm/°	mm	mm ²	mm/°	mm	mm ²	mm/°	mm	mm ²	mm/°	MPa	MPa	GPa	-
21-1		FPS2	BÜ	U	105	942	10/90	262,5	943	10/90	420	missing	10/90	577,5	missing	10/90				500			
21-2		FPS3	BÜ	U	105	402	8/90	262,5	603	8/90	420	missing	8/90	577,5	missing	8/90				500			
21-3		FPS4	BÜ	U	105	1257	10/90	262,5	1885	10/90	420	missing	10/90	577,5	missing	10/90				500			
21-4		FPS5	BÜ	U	105	1571	10/90	262,5	2199	10/90	420	missing	10/90	577,5	missing	10/90				500			
22-1		T2	BÜ	U	40	628	10/90	140	1099,6	10/90	240	1570,8	10/90	340	2042	10/90				361			Ps
22-2		T3	BÜ	U	40	785,4	10/90	140	1727,9	10/90	240	2356,2	10/90	340	2984,5	10/90				361			Ps
22-3		T4	BÜ	U	40	1327	13/90	140	2123,7	13/90	240	2654,6	13/90	340	3716,5	13/90				331			Ps
22-4		T5	BÜ	U	40	1592,8	13/90	140	2654,6	13/90	240	3716,5	13/90	340	4778,4	13/90				331			Ps
22-5		T6	BÜ	U	40	2010,6	16/90	140	3217	16/90	240	4021,2	16/90	340	5629,7	16/90				367			Ps
22-6		K2	HA	U	167	424	6/90	334	650	6/90	501	876,5	6/90	668	1102,7	6/90				347			Pc
22-7		K3	HA	U	167	848	6/90	334	1300	6/90	501	1753	6/90	668	2205,4	6/90				347			Pc
22-8		K4	HA	U	167	1178,1	10/90	334	1806,4	10/90	501	2434,7	10/90	668	3063	10/90				317			Pc
22-9		K5	HA	U	167	2356,2	10/90	334	3612,8	10/90	501	4869,4	10/90	668	6126	10/90				317			Pc
22-10		K6	HA	U	167	1991	13/90	334	3052,8	13/90	501	4114,7	13/90	668	5176,6	13/90				330			Pc
22-11		K7	HA	U	167	3982	13/90	334	6105,6	13/90	501	8229,4	13/90	668	10353,2	13/90				330			Pc
23-1		2	BÜ	U	100	1005,3	8/90	200	1407,4	90	0	0	8/90	0	0	8/90				520			Po
23-2		3	BÜ	U	100	1005,3	8/90	200	1407,4	90	300	1809,6	8/90	400	2211,7	8/90				520			Po
23-3		4	BÜ	U	100	1005,3	8/90	200	1407,4	90	300	1809,6	8/90	400	2211,7	8/90				520			Ps
24-1		HS20	BÜ	HA	60	1570	10/90	170	1570	10/90	490									690			
24-2		HS21	SE	HA	0	1570	10/45	90	1570	10/45	490									690			
24-3		HS18	SE	HA	0	1570	10/45	0	0		490									690			

II - 1b Databank on slabs with shear reinforcement – Additional data of the shear reinforcement (8 of 9)

no.	shape and distribution		row one			row two			row three			row four			row five			material properties			failure	
	serial no.	specimen no.	distance to the column face	shear reinforcement area	diameter/ inclination	distance to the column face	shear reinforcement area	diameter/ inclination	distance to the column face	shear reinforcement area	diameter/ inclination	distance to the column face	shear reinforcement area	diameter/ inclination	distance to the column face	shear reinforcement area	diameter/ inclination	yield strength	tensile strength	modulus of elasticity		
$l_{sw,1}$																					$A_{sw,1}$	d_s/α
notation		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	
unit																						
25-1	P1-I	SZ	U	100	1099,5	10/90	200	1099,5	10/90	0	0	0	0	0	0	0	0	0	570	649	221,4	Po
25-2	P1-II	Bü	U	105	1256,6	10/90	195	942,5	10/90	0	0	0	0	0	0	0	0	0	570	649	221,4	Po
25-3	P2-I	ÜS	U	50	703,7	8/90	150	703,7	8/90	250	904,8	8/90	350	904,8	8/90	375	904,8	8/90	592	679	226,5	Ps
25-4	P2-II	Bü	U	75	603,2	8/90	175	1005,3	8/90	275	703,7	8/90	375	904,8	8/90	0	0	0	592	679	226,5	Ps
25-5	P2-III	ÜS	U	65	703,7	8/90	165	703,7	8/90	265	703,7	8/90	365	1105,8	8/90	0	0	0	592	679	226,5	Ps
25-6	P3-I	ÜS	U	110	603,2	8/90	274	904,8	8/90	374	804,2	8/90	538	1507,9	8/90	688	502,6	8/90	597	666	219,5	Ps
25-7	P4-III	ÜS	U	110	603,2	8/90	210	1005,3	8/90	360	804,2	8/90	460	1608,5	8/90	620	402,1	8/90	597	666	219,5	Ps
25-8	P5-I	ÜS	U	120	1570,8	10/90	260	1884,9	10/90	390	1099,6	10/90	530	2513,3	10/90	690	1884,9	10/90	596	665	228,4	Ps
25-9	P6-I	ÜS	U	110	2261,9	12/90	278	3166,7	12/90	408	2261,9	12/90	576	3166,7	12/60	716	2261,9	12/90	563	643	229,0	Pc
25-10	P7-I	ÜS	U	110	1570,8	10/90	250	2199,1	10/60	400	1413,7	10/90	540	2356,2	10/60	710	1570,8	10/90	596	665	228,4	Ps
26-1	2V10	BÜ	U	77,5	804,2	8/90	194	1005,3	8/90									360				
26-2	2V20	BÜ	U	80	1256,6	10/90	200	1570,8	10/90									360				
26-3	2V21	BÜ	U	80	1256,6	10/90	200	1570,8	10/90	320	1570,8	10/90						360				
26-4	2V22	BÜ	U	80	1256,6	10/90	200	1570,8	10/90	320	2199,1	10/90						480				
26-5	3V20	BÜ	U	115	942,5	10/90	288	1256,6	10/90									360				

II - 1b Databank on slabs with shear reinforcement – Additional data of the shear reinforcement

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no.	specimen no.	shape and distribution		row one			row two			row three			row four			row five			material properties			failure
		type of shear reinforcement	shear reinforcement distribution	distance to the column face	shear reinforcement area	diameter/ inclination	distance to the column face	shear reinforcement area	diameter/ inclination	distance to the column face	shear reinforcement area	diameter/ inclination	distance to the column face	shear reinforcement area	diameter/ inclination	distance to the column face	shear reinforcement area	diameter/ inclination	yield strength	tensile strength	modulus of elasticity	
serial no.	notation	-	-	$l_{sw,1}$ mm	$A_{sw,1}$ mm ²	d_s/α mm/°	$l_{sw,2}$ mm	$A_{sw,2}$ mm ²	d_s/α mm/°	$l_{sw,3}$ mm	$A_{sw,3}$ mm ²	d_s/α mm/°	$l_{sw,4}$ mm	$A_{sw,4}$ mm ²	d_s/α mm/°	$l_{sw,5}$ mm	$A_{sw,5}$ mm ²	d_s/α mm/°	f_y MPa	f_t MPa	E_s GPa	-
	unit	-	-	mm	mm ²	mm/°	mm	mm ²	mm/°	mm	mm ²	mm/°	mm	mm ²	mm/°	mm	mm ²	mm/°	MPa	MPa	GPa	-
27-1	3V	Bü	HA	52,5	157,1	5/90	131	157,1	5/90	0	0		0	0					900	948		Po
27-2	4V	Bü	HA	51,5	249,4	6,3/90	128	249,4	6,3/90	0	0		0	0					1000	916		Po
27-3	5V	Bü	HA	51,5	249,4	6,3/90	128	249,4	6,3/90	206	249,4	6,3/90	0	0					643	916		Po
27-4	6I	SE	HA	0	157,1	6,3/57	75	157,1	6,3/57	0	0	6,3/57	0	0					900	948		Po
27-5	7I	SE	HA	0	249,4	6,3/57	75	249,4	6,3/57	0	0	6,3/57	0	0					643	916		Po
27-6	8I	SE	HA	0	249,4	6,3/57	75	249,4	6,3/57	150	249,4	6,3/57	0	0					643	916		Po
27-7	9I	SE	HA	0	249,4	6,3/57	75	249,4	6,3/57	150	249,4	6,3/57	225	249,4	6,3/57				643	916		Po
27-8	10I	SE	U	0	249,4	6,3/57	78,5	498,8	6,3/57	157,2	748,1	6,3/57	0	0					643	916		Pc
27-9	11I	SE	U	0	249,4	6,3/57	76,5	498,8	6,3/57	153	748,1	6,3/57	229,5	997,5	6,3/57				643	916		Pc

II - 2 Databank on slabs with shear reinforcement – Input data for calculation

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running number	notations		geometry		reinforcement		material properties					failure																	
	serial no.	slab no.	author	slab	column	dimensions and ratio	concrete	reinforcing steel	failure																				
no.	notat.	n	A	BC	c_1/a_1	h	SC	c_2/a_2	shape of column	diameter of circular / side length of quadratic column	boundary condition	diameter of circle along the load was distributed / span of quadratic slab	total height of slab	reinf. ratio (inner section)	Type of the distribution of the shear reinforcement	inclination of the first shear reinforcement row	Sum of the shear reinforcement	compressive uniaxial strength (prism strength)	tensile uniaxial strength of concrete	modulus of elasticity of concrete	Maximum aggregate size	yield strength of the flexural reinforcement	yield strength of the shear reinforcement	modulus of elasticity of reinforcing steel	failure load	P_u / P_{flex}	failure load over flexure load (plasticity)	Type of failure	
unit	unit	-	-	-	mm	mm	-	mm	mm	mm	mm	mm	mm	mm	-	-	cm ²	MPa	MPa	MPa	GPa	mm	MPa	MPa	GPa	MN	-	-	-
1	01-01	1355	Graf	qa	1500	302	q	200	273	0,700 HA	45	25,9	14,4	1,1	24,7	278	280	1,2063	1,131	PS									
2	01-02	1356	Graf	qa	1500	303	q	200	274	0,697 HA	45	25,9	14,4	1,1	24,7	278	280	1,2847	1,200	PS									
3	01-03	1361	Graf	qa	1500	303	q	300	272	1,045 HA	45	36,7	13,7	1,0	24,3	270	280	1,726	1,063	PS									
4	01-04	1363	Graf	qa	1500	502	q	300	469	0,596 HA	60	75,1	15,6	1,2	25,4	270	280	3,0206	1,041	PS									
5	01-05	1376	Graf	qa	1500	503	q	200	474	0,404 HA	60	58,3	15,5	1,2	25,3	278	280	2,2556	1,179	PS									
6	01-06	1377	Graf	qa	1500	505	q	200	474	0,404 HA	60	58,3	14,4	1,1	24,7	278	280	2,2066	1,156	PS									
7	02-01	S-I	Keefe	k	940	127	k	153	113	2,535 ST	90	7,8	25,0	2,1	29,7	288	398	0,4923	0,817										
8	02-02	S-II	Keefe	k	940	127	k	153	113	2,535 ST	90	7,8	23,8	2,0	29,2	288	398	0,4923	0,824										
9	03-01	B3	Elstn.,Hog.	qa	1829	152	q	254	114	1,008 HA	45	4,0	13,1	1,0	23,9	299	323	0,2869	1,039	PS									
10	03-02	B6	Elstn.,Hog.	qa	1829	152	q	254	114	1,002 HA	45	7,3	13,1	1,0	23,9	36	330	0,4684	0,686	PO									
11	03-03	B10	Elstn.,Hog.	qa	1829	152	q	254	114	2,014 HA	45	7,3	42,6	3,3	35,4	36	335	0,5338	0,832	PO									
12	03-04	B17	Elstn.,Hog.	qa	1829	152	q	254	114	1,515 HA	45	4,0	13,1	1,0	23,9	36	308	0,3572	0,896	PO									
13	03-05	B5	Elstn.,Hog.	qa	1829	152	q	254	114	1,314 HA	45	3,6	13,1	1,0	23,9	36	420	0,3781	0,627	PO									
14	03-06	B12	Elstn.,Hog.	qa	1829	152	q	254	114	3,020 U	90	22,7	42,6	3,3	35,4	36	332	0,7873	0,860	PO									
15	03-07	B13	Elstn.,Hog.	qa	1829	152	q	254	114	3,020 HA	20	13,7	42,6	3,3	35,4	36	328	0,7784	0,859	PO									
16	03-08	B15	Elstn.,Hog.	qa	1829	152	q	254	114	3,020 HA	45	10,9	42,6	3,3	35,4	36	332	0,6894	0,753	PO									
17	03-09	B16	Elstn.,Hog.	qa	1829	152	q	254	114	3,020 HA	45	21,9	42,6	3,3	35,4	36	339	0,7473	0,801	PO									
18	04-01	S8-60	Moe	qa	1829	152	q	203	98	1,224 U	45	3,5	21,9	1,8	28,4	38,1	399	0,367	1,148	PO									
19	05-01	A-S1-62	Andersson	k	1710	150	k	150	120	0,779 ST	30	6,8	27,3	2,3	30,6	448	448	0,367	1,129	PS									
20	05-02	A-S1-63	Andersson	k	1710	150	k	150	120	0,779 ST	30	6,8	27,4	2,3	30,6	436	436	0,3531	1,114	PS									

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II - 2 Databank on slabs with shear reinforcement – Input data for calculation

no.	notat.	n	A	BC	c_1/a_1	h	SC	c_2/a_2	d	d_s	d_{s2}	ρ_u	α	ΣA_{sw}	f_{tc}	f_{td}	E_c	d_o	f_y	$f_{y,sw}$	E_s	P_u	P_u/P_{uex}	-
unit	unit	-	-	-	mm	mm	-	mm	mm	mm	mm	-	°	cm ²	MPa	MPa	GPa	mm	MPa	MPa	GPa	MN	-	-
21	05-03	A-S1-64	Andersson	k	1710	150	k	150	120	13		0,779 ST	30	11,3	27,4	2,3	30,6		432	436		0,3707	1,180	PS
22	05-04	A-S1-65	Andersson	k	1710	150	k	150	121	13		0,772	30	11,3	27,2	2,3	30,5		437	437		0,3727	1,163	PS
23	05-05	A-S2-66	Andersson	k	1710	150	k	150	119	13		0,785 U	90	9,1	27,7	2,3	30,7		438	294		0,2922	0,925	
24	05-06	A-S2-67	Andersson	k	1710	150	k	150	121	13		0,772 U	90	9,1	28,4	2,4	31,0		435	294		0,2942	0,920	
25	05-07	A-S3-68	Andersson	k	1710	150	k	150	121	13		0,772 ST	45	4,8	26,6	2,2	30,3		439	294		0,2746	0,855	
26	05-08	A-S3-69	Andersson	k	1710	150	k	150	121	13		0,772 ST	45	4,8	25,7	2,2	30,0		448	294		0,2491	0,763	
27	05-09	A-S4-70	Andersson	k	1710	150	k	150	121	13		0,772 ST	45	2,8	24,6	2,1	29,5		435	532		0,2726	0,860	
28	05-10	A-S4-71	Andersson	k	1710	150	k	150	123	13		0,760 ST	45	2,8	26,1	2,2	30,1		440	532		0,2922	0,893	
29	05-11	A-S1-76	Andersson	k	1710	150	k	300	122	12		1,030 ST	30	6,8	25,7	2,1	29,9		457	457		0,5345	1,097	
30	05-12	A-S1-77	Andersson	k	1710	150	k	300	125	12		1,005 ST	30	6,8	26,6	2,2	30,3		453	453		0,5492	1,103	
31	05-13	A-S1-78	Andersson	k	1710	150	k	300	120	12		1,047 ST	30	13,6	26,8	2,2	30,4		461	461		0,6061	1,252	
32	05-14	A-S1-79	Andersson	k	1710	150	k	300	119	12		1,055 ST	30	13,6	27,0	2,3	30,4		469	469		0,612	1,255	
33	05-15	A-S1-80	Andersson	k	1710	150	k	300	121	13		0,837 ST	30	9,0	26,6	2,2	30,3		436	437		0,4531	1,190	
34	05-16	A-S1-81	Andersson	k	1710	150	k	300	120	13		0,844 ST	30	9,0	24,6	2,1	29,5		440	440		0,4707	1,244	
35	05-17	A-S2-82	Andersson	k	1710	150	k	300	120	12		1,047 U	90	12,9	25,8	2,2	30,0		442	294		0,459	0,988	
36	05-18	A-S2-83	Andersson	k	1710	150	k	300	119	12		1,055 U	90	12,9	22,2	1,9	28,5		442	294		0,459	1,013	
37	05-19	A-S3-84	Andersson	k	1710	150	k	300	123	12		1,021 U	45	6,4	23,5	2,0	29,1		447	294		0,4119	0,862	
38	05-20	A-S3-85	Andersson	k	1710	150	k	300	122	12		1,030 U	45	6,4	23,7	2,0	29,1		436	294		0,3923	0,847	
39	06-01	2	Franz	k	1680	140	k	210	129	12		1,095 HA	60	7,8	21,2	2,0	25,5		30	440	204,3	0,3844	0,749	PC
40	06-02	3	Franz	k	1680	140	k	210	129	12		1,095 HA	60	7,8	22,5	2,1	32,1		30	440	204,3	0,3923	0,759	PC
41	06-03	4	Franz	k	1680	140	k	210	129	12		1,095 ST	90	3,0	22,7	2,4	33,1		30	440	204,3	0,3874	0,748	PC
42	06-04	5	Franz	k	1680	140	k	210	129	12		1,095 ST	90	12,0	25,6	2,1	28,5		30	440	204,3	0,4315	0,823	PC
43	06-05	6	Franz	k	1680	140	k	210	129	12		1,095 ST	90	18,0	20,0	2,7	26,5		30	440	204,3	0,4266	0,837	PC
44	06-06	7	Franz	k	1680	140	k	210	129	12		1,095 ST	90	14,3	20,7	2,7	26,0		30	440	285	0,3531	0,690	PC
45	06-07	8	Franz	k	1680	143	k	210	125	12		1,130 U	90	6,0	25,5	2,4	27,4		30	434	305	0,4315	0,863	PS
46	06-08	9	Franz	k	1680	143	k	210	125	12		0,952 U	90	4,0	19,3	2,5	22,8		30	434	305	0,4315	1,037	PS
47	06-09	10	Franz	k	1680	143	k	210	125	12		0,800 U	90	4,0	20,6	2,2	26,3		30	434	305	0,3629	1,013	PS
48	06-10	11	Franz	k	1680	143	k	210	125	12		0,695 U	90	6,0	22,5	2,3	28,3		30	434	305	0,3432	1,082	PS
49	06-11	12	Franz	k	1680	143	k	210	125	12		0,565 U	90	6,0	21,4	1,6	27,9		30	434	305	0,3236	1,244	PS
50	06-12	13	Franz	k	1680	143	k	210	125	12		0,476 U	90	2,0	23,5	2,1	28,8		30	434	305	0,2795	1,259	PS
51	06-13	14	Franz.	k	1680	143	k	210	125	12		1,205 U	90	5,0	18,8	2,1	22,0		30	434	305	0,4119	0,809	PS
52	07-01	L2	Narasiham	qa	2000	178	q	305	143	16	16	1,107 U	45	12,1	31,7	2,8	32,1		398	309		0,868	1,279	
53	08-01	B6	Petcu, St.	qa	2400	193	q	200	171			0,510 Ha	45	8,8	29,8	2,5	31,5		404	404		0,5708	1,333	
54	08-02	B7	Petcu, St.	qa	2400	192	q	200	167			0,520 HA	45	8,8	30,5	2,5	31,7		404	404		0,6296	1,512	
55	08-03	B8	Petcu, St.	qa	2400	192	q	200	167			0,520 HA	45	16,3	27,9	2,3	30,8		404	354		0,61	1,469	
56	08-04	B9	Petcu, St.	qa	2400	230	q	200	203			0,430 HA	45	8,8	21,0	1,8	28,0		404	404		0,6355	1,257	

II - 2 Databank on slabs with shear reinforcement – Input data for calculation (3 of 5)

no.	notat.	n	A	BC	c_1/a_1	h	SC	c_2/a_2	d	d_c	d_{s2}	ρ_{s1}	α	ΣA_{sw}	f_{ic}	f_{ist}	E_c	d_a	f_y	f_{ysw}	E_s	P_u	P_u/P_{flex}
unit	unit	-	-	-	mm	mm	-	mm	mm	mm	mm	-	-	cm ²	MPa	MPa	MPa	mm	MPa	MPa	MPa	MN	-
57	08-05	B10	Petcu, St.	qa	2400	237	q	200	212			0,430 HA	45	16,3	19,5	1,6	27,3		404	354		0,7865	1,432
58	08-06	A7	Petcu, St.	qa	2400	142	q	200	120			0,730 HA	45	4,4	21,3	1,8	28,1		420	420		0,3236	1,070
59	08-07	A8	Petcu, St.	qa	2400	142	q	200	121			0,720 HA	45	8,8	19,4	1,6	27,3		420	395		0,4119	1,367
60	08-08	A5	Petcu, St.	qa	2400	141	q	200	113			0,780 U	90	7,4	22,4	1,9	28,6		420	208		0,2746	0,960
61	08-09	A6	Petcu, St.	qa	2400	138	q	200	110			0,800 U	90	11,7	24,6	2,1	29,5		420	234		0,2746	0,983
62	09-01	P3	Marl et al	k	2600	180	k	300	152	18	10	1,395 U	90	4,0	32,3	3,0	34,8	16	558	543	210	0,875	0,774
63	10-01	A	Sunquist	k	1710	200	k	40	170	12	8	0,665 HA	45	11,4	22,8	1,9	28,8		457	435		0,37	0,699
64	10-02	C	Sunquist	k	1710	200	k	40	170	12	8	0,554 HA	45	12,8	23,3	2,0	29,0		455	455		0,365	0,821
65	10-03	D	Sunquist	k	1710	200	k	140	170	12	8	0,554 HA	45	12,8	25,5	2,1	29,9		454	455		0,58	1,223
66	10-04	E	Sunquist	k	1710	200	k	250	170	12	8	0,554 HA	45	12,8	19,6	1,6	27,4		457	455		0,56	1,108
67	11-01	SC8	Seib., Gha.	qa	1698	157	q	305	121	12,7		1,170 HA	90	18,0	31,9	2,6	32,2		448	490		0,5916	0,998
68	11-02	SC9	Seib., Gha.	qa	1698	157	q	305	121	12,7		1,170 HA	90	25,0	31,9	2,6	32,2		448	490		0,596	1,006
69	11-03	SC10	Seib., Gha.	qa	1698	157	q	305	121	12,7		1,170 HA	90	22,4	31,9	2,6	32,2		448	490		0,5382	0,908
70	12-01	S14	Swamy, Ali	qa	1690	125	q	150	100	10	8	0,748 HA	60	4,4	35,6	2,3	33,4	10	462	462		0,334	1,369
71	12-02	S17	Swamy, Ali	qa	1690	125	q	150	100	10	8	0,748 HA	60	5,4	35,6	2,3	33,4	10	462	485		0,357	1,463
72	13-01	P6	Pral. et al	k	1300	180	k	300	154	16	8	1,305 U	90	48,0	28,5	2,8	38,1	16	515	534	204	0,594	0,514
73	13-02	P22	Müller et al	k	1300	185	k	300	153,5	16	8	1,309 U	90	36,0	31,3	2,6	31,6	16	507,3	562,9	199	1,04	0,905
74	14-01	HSC3S	Hallig., Kin.	k	2400	240	k	250	200	16		0,628 HA	45	28,4	87,8	5,9	37,4	18	632	503		1,329	1,219
75	14-02	HSC7S	Hallig., Kin.	k	2400	240	k	250	200	16		0,479 HA	45	22,7	80,8	6,7	39,4	18	630	501		1,106	1,331
76	15-01	2	Broms	k	2000	180	q	250	150	12	0	0,628 HA	90	18,8	26,5	2,2	30,3		681	471		0,523	0,819
77	15-02	3	Broms	k	2000	180	q	250	150	12	0	0,942 HA	90	24,1	25,4	2,1	29,8		681	471		0,558	0,612
78	15-03	4	Broms	k	2000	180	q	250	150	10	0	0,582 U	90	24,5	16,4	1,3	25,8		656	442		0,415	0,757
79	15-04	5	Broms	k	2000	180	q	250	150	12	0	0,710 U	90	24,5	18,6	1,5	26,9		684	442		0,434	0,631
80	15-05	6	Broms	k	2000	180	q	250	150	10	6	0,477 U	90	11,3	22,0	1,9	28,5		665	671		0,608	1,273
81	15-06	7	Broms	k	2000	180	q	250	150	12	8	0,838 U	90	11,3	18,6	1,5	26,9		691	656		1,006	1,262
82	16-01	FV1	Lovro. et al	k	203	100	k	101	82,5	9,52	9,52	1,750 U	90	1,9	37,0	3,0	33,8	19	530,9	280		0,2619	0,376
83	16-02	FV2	Lovro. et al	k	406,4	100	k	101	82,5	9,52	9,52	1,750 U	90	7,7	37,0	3,0	33,8	19	530,9	280		0,2753	0,591
84	16-03	FV3	Lovro. et al	k	609,6	100	k	101	82,5	9,52	9,52	1,750 U	90	17,4	37,0	3,0	33,8	19	530,9	280		0,2793	0,666
85	16-04	FV4	Lovro. et al	k	812,8	100	k	101	82,5	9,52	9,52	1,750 U	90	31,0	37,0	3,0	33,8	19	530,9	280		0,3416	0,855
86	16-05	FV5	Lovro. et al	k	1219,2	100	k	101	82,5	9,52	9,52	1,750 U	90	69,8	37,0	3,0	33,8	19	530,9	280		0,5497	1,441
87	17-01	1	Regan	qa	2742	300	k	600	259	16	16	0,621 U	90	40,7	39,7	3,2	34,6	20	520	520		1,8	0,975
88	18-01	S2	Kinn. et al	rs	4680	730	k	800	669	16	16	0,531 U	90	208,1	26,6	2,4	30,3	32	619,6	428,4		8,215	0,280
89	18-02	S3	Kinn. et al	rs	4680	730	k	800	669	16	16	0,531 U	90	208,1	26,2	2,3	30,2	32	614,2	425,5		8,15	0,280
90	19-01	S1.1s	Tolf	k	1190	120	k	125	100	8		0,838 U	90	6,3	22,9	1,9	28,8	16	706	706		0,261	0,717
91	19-02	S1.2s	Tolf	k	1190	120	k	125	100	8		0,838 U	90	6,3	23,6	2,0	29,1	16	711	711		0,259	0,704
92	19-03	S1.3s	Tolf	k	1190	120	k	125	100	8		0,335 U	90	3,1	23,6	2,0	29,1	16	709	709		0,144	0,907

II - 2 Databank on slabs with shear reinforcement – Input data for calculation

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no.	notat.	n	A	BC	c_1/a_1	h	SC	c_2/a_2	d	d_s	d_{12}	ρ_{L1}	α	ΣA_{sw}	f_{ic}	f_{1a}	E_c	d_o	f_y	$f_{y,sw}$	E_s	P_u	P_u/P_{fda}
unit	unit	-	-	-	mm	mm	-	mm	mm	mm	mm	-	°	cm ²	MPa	MPa	GPa	mm	MPa	MPa	GPa	MN	-
93	19-04	S1.4s	Tolf	k	1190	120	k	125	100	8		0,335 U	90	3,1	23,6	2,0	29,1	16	710	710		0,147	0,924
94	19-05	S2.1s	Tolf	k	2400	240	k	250	195	16		0,859 U	90	25,1	18,3	1,5	26,8	32	669	503		0,894	0,885
95	19-06	S2.2s	Tolf	k	2400	240	k	250	195	16		0,859 U	90	25,2	18,7	1,6	27,0	32	672	501		0,851	0,648
96	19-07	S2.3s	Tolf	k	2400	240	k	250	200	16		0,335 U	90	12,6	18,7	1,6	27,0	32	671	671		0,562	0,945
97	19-08	S2.4s	Tolf	k	2400	240	k	250	200	16		0,335 U	90	12,6	18,7	1,6	27,0	32	669	669		0,542	0,914
98	20-01	2	Chana, Des.	qs	2400	240	q	300	200	20	0	0,785	90	12,1	31,6	2,6	32,1	20	500	520		1,057	0,783
99	20-02	3	Chana, Des.	qs	2400	240	q	300	200	20	0	0,785	90	12,1	29,2	2,4	31,3	20	500	520		1,139	0,848
100	20-03	4	Chana, Des.	qs	2400	240	q	300	200	20	0	0,785	90	24,1	32,3	2,6	32,3	20	500	520		1,302	0,963
101	20-04	5	Chana, Des.	qs	2400	250	q	300	210	20	0	0,854	90	25,1	27,2	2,3	30,5	20	500	520		1,382	0,867
102	20-05	7	Chana, Des.	qs	2400	250	q	300	210	20	0	0,854	90	25,1	28,7	2,4	31,1	20	500	520		1,429	0,893
103	20-05	6	Chana, Des.	qs	2400	250	q	300	210	20	0	0,854	90	25,1	30,9	2,5	31,8	20	500	520		1,283	0,797
104	20-07	8	Chana, Des.	qs	2400	250	q	300	210	20	0	0,854	90	16,1	28,2	2,3	30,9	20	500	520		1,327	0,830
105	20-08	9	Chana, Des.	qs	2400	228	q	300	188	20	0	0,835	90	24,1	30,2	2,5	31,6	20	500	520		1,135	0,900
106	21-01	FPS2	Chana, Des.	qa	2400	250	q	400	210	20	20	0,854	90	18,9	24,4	2,4	29,4	20	500	500		1,51	1,063
107	21-02	FPS3	Chana, Des.	qa	2400	250	q	400	210	20	20	0,854	90	10,1	24,2	2,5	29,4	20	500	500		1,773	1,249
108	21-03	FPS4	Chana, Des.	qa	2400	250	q	400	210	20	20	0,854	90	31,4	27,3	2,4	30,6	20	500	500		2,028	1,414
109	21-04	FPS5	Chana, Des.	qa	2400	250	q	400	210	20	20	0,854	90	37,7	22,9	2,5	28,8	20	500	500		1,94	1,374
110	22-01	T2	Yam. et al	qa	1500	200	q	300	167	13	13	0,958 U	90	33,0	18,4	1,5	26,8		811	361		0,6	0,321
111	22-02	T3	Yam. et al	qa	1500	200	q	300	167	13	13	0,974 U	90	48,7	18,8	1,6	27,0		811	361	0	0,727	0,387
112	22-03	T4	Yam. et al	qa	1500	200	q	300	167	13	13	1,002 U	90	61,1	19,3	1,6	27,2		811	331	0	0,697	0,367
113	22-04	T5	Yam. et al	qa	1500	200	q	300	167	13	13	0,929 U	90	79,6	17,9	1,5	26,6		811	331	0	0,762	0,412
114	22-05	T6	Yam. et al	qa	1500	200	q	300	167	13	13	0,998 U	90	92,5	19,2	1,6	27,2		811	367	0	0,735	0,387
115	22-06	K2	Yam. et al	qa	1500	200	q	300	167	13	13	0,994 U	90	4,2	21,5	1,8	28,2		568	347	0	0,95	0,800
116	22-07	K3	Yam. et al	qa	1500	200	q	300	167	13	13	0,994 U	90	8,5	20,5	1,7	27,8		568	347	0	1,183	1,003
117	22-08	K4	Yam. et al	qa	1500	200	q	300	167	13	13	0,994 U	90	11,8	21,6	1,8	28,3		568	317	0	1,153	0,970
118	22-09	K5	Yam. et al	qa	1500	200	q	300	167	13	13	0,994 U	90	23,6	20,5	1,7	27,8		568	317	0	1,44	1,220
119	22-10	K6	Yam. et al	qa	1500	200	q	300	167	13	13	0,994 U	90	19,9	20,8	1,7	27,9		568	330	0	1,274	1,077
120	22-11	K7	Yam. et al	qa	1500	200	q	300	167	13	13	0,994 U	90	39,8	21,9	1,8	28,4		568	330	0	1,498	1,257
121	23-01	2	Chana	k	2400	228	q	300	188	20	8	0,836 U	90	24,1	29,3	2,4	31,3		520	520	0	0,981	0,957
122	23-02	3	Chana	k	2400	228	q	300	188	20	8	0,836 U	90	24,1	31,4	2,6	32,0		520	520	0	1,286	1,248
123	23-03	4	Chana	k	2400	228	q	300	188	20	8	0,836 U	90	24,1	30,2	2,5	31,6		520	520	0	1,135	1,105
124	24-01	HS20	Marz. et al	qa	1830	150	q	250	120	15	10	0,887 HA	90	31,4	70,3	4,5	39,2	20	490	490	0	0,4188	0,874
125	24-02	HS21	Marz. et al	qa	1830	150	q	250	120	15	10	0,887 HA	45	22,2	68,4	4,5	38,9	20	490	490	0	0,6097	1,273
126	24-03	HS18	Marz. et al	qa	1830	150	q	250	120	15	10	0,887 HA	45	11,1	64,6	4,4	38,2	20	490	490	0	0,5117	1,070
127	25-01	P1-I	Beu, Heg	k	2400	230	q	400	190	14	10	0,810 U	90	22,0	25,9	1,7	23,9	16	572	570	218,5	1,151	0,997
128	25-02	P1-II	Beu, Heg	k	2400	230	q	400	190	14	10	0,810 U	90	22,0	24,9	1,4	25,5	16	572	570	218,5	1,055	0,918

II - 2 Databank on slabs with shear reinforcement – Input data for calculation

(5 of 5)

no.	notat.	n	A	BC	c_1/a_1	h	SC	c_2/a_2	d	d_s	d_{Δ}	ρ_{li}	α	ΣA_{sw}	f_{ic}	f_{icr}	E_c	d_u	f_y	$f_{y,sw}$	E_s	P_s	P_s/P_{flex}	-
unit	unit	-	-	-	mm	mm	-	mm	mm	mm	mm	-	-	cm ²	MPa	MPa	GPa	mm	MPa	MPa	GPa	MN	-	-
129	25-03	P2-I	Beu, Heg	k	2400	230	q	400	190	14	10	0,810 U	90	23,1	36,0	1,8	28,8	16	549	592	203,2	1,326	1,164	PO
130	25-04	P2-II	Beu, Heg	k	2400	230	q	400	190	14	10	0,810 U	90	23,1	28,3	1,9	24,9	16	549	592	203,2	1,109	0,990	PS
131	25-05	P2-III	Beu, Heg	k	2400	230	q	400	190	14	10	0,810 U	90	21,1	35,6	1,9	26,9	16	549	592	203,2	1,276	1,121	PC/PS
132	25-06	P3-I	Beu, Heg	k	2400	270	q	320	220	20	12	1,142 U	90	15,1	22,0	1,8	22,2	16	557	597	212	1,624	0,843	PS
133	25-07	P4-III	Beu, Heg	k	2400	267	q	320	222	20	12	1,132 U	90	16,1	26,4	1,9	25,7	16	557	597	212	1,522	0,762	PS
134	25-08	P5-I	Beu, Heg	k	2400	270	q	320	220	20	12	1,360 U	90	34,6	43,0	3,0	28,5	16	557	596	212	1,936	0,795	PS
135	25-09	P6-I	Beu, Heg	k	2400	273	q	320	223	25	14	1,761 U	90	54,3	44,0	3,0	27,3	16	532	563	204,5	2,349	0,774	PC
136	25-10	P7-I	Beu, Heg	k	2400	275	q	320	229	20	12	1,001 U	90	34,8	38,0	2,4	28,6	16	557	596	212	2,117	1,074	PS
137	26-01	2V10	Lee et al	qs	1500	200	q	290	155	20	0	2,030 U	90	18,1	71,4	4,5	39,4		500	360	0	1,19	0,528	
138	26-02	2V20	Lee et al	qs	1500	200	q	290	160	20	0	1,960 U	90	28,3	73,4	4,6	39,7		500	360	0	1,45	0,622	
139	26-03	2V21	Lee et al	qa	1500	200	q	290	160	20	0	1,960 U	90	28,3	74,3	4,6	39,9		500	360	0	1,48	0,737	
140	26-04	2V22	Lee et al	qa	1500	200	q	290	160	20	0	1,960 U	90	28,3	74,0	4,6	39,8		500	480	0	1,69	0,842	
141	26-05	3V20	Lee et al	qs	2250	275	q	400	230	20	0	1,520 U	90	22,0	67,6	4,4	38,7		500	360	0	1,9	0,513	
142	27-01	3V	Oliv. et al	qs	1572	130	q	120	105	12,5	6,3	1,200 HA	90	3,1	53,9	4,0	36,2		575	900	0	0,386	0,624	PO
143	27-02	4V	Oliv. et al	qs	1572	130	q	120	103	12,5	6,3	1,300 HA	90	5,0	56,7	4,1	36,8		575	1000	0	0,377	0,586	PO
144	27-03	5V	Oliv. et al	qs	1572	130	q	120	103	12,5	6,3	1,300 HA	90	5,0	52,5	4,0	35,9		575	643	0	0,423	0,660	PO
145	27-04	6I	Oliv. et al	qs	1572	130	q	120	100	12,5	6,3	1,400 HA	57	2,6	53,3	4,0	36,1		575	900	0	0,41	0,633	PO
146	27-05	7I	Oliv. et al	qs	1572	130	q	120	100	12,5	6,3	1,400 HA	57	4,2	51,3	3,9	35,7		575	643	0	0,49	0,759	PO
147	27-06	8I	Oliv. et al	qs	1572	130	q	120	100	12,5	6,3	1,400 HA	57	4,2	53,5	4,0	36,1		575	643	0	0,54	0,834	PO
148	27-07	9I	Oliv. et al	qs	1572	130	q	120	100	12,5	6,3	1,400 HA	57	4,2	51,3	3,9	35,7		575	643	0	0,56	0,868	PO
149	27-08	10I	Oliv. et al.	qs	1572	130	q	120	105	12,5	6,3	1,300 U	57	6,3	52,2	4,0	35,8		575	643	0	0,52	0,782	PC
150	27-09	11I	Oliv. et al	qs	1572	130	q	120	102	12,5	6,3	1,338 U	57	6,3	53,1	4,0	36,0		575	643	0	0,551	0,853	PC

Appendix III

Table of comparison of test data with codes

(1 of 6)

a) Punching tests without shear reinforcement

running number	serial no.	slab no.	author	effective depth	reinforcement ratio (inner section)	compressive uniaxial strength (prism strength)	failure load	DIN 1045 (1988)	Model Code 90	FIP Recommendations	BS 8110-97 (1997)	Eurocode 2, Part 1 (1992)	ACI 318-95 (1995)	DIN1045-1 (2000)
no.	notat.	n	A	d	$\rho_{l,i}$	f_{ic}	V_{exp}	$V_{exp}/V_{cal,u}$						
unit	unit	-	-	mm	-	MPa	MN	-						
1	1-1	HSC 0	Hallgren	200	0,008	89,2	0,965	-	1,317	1,276	-	0,986	1,171	
2	1-2	HSC 1	Hallgren	200	0,008	86,7	1,021	-	1,407	1,363	-	1,058	1,251	
3	1-3	HSC 2	Hallgren	194	0,008	81,4	0,889	-	1,296	1,254	-	0,994	1,160	
4	1-4	HSC 4	Hallgren	200	0,012	87,0	1,041	-	1,251	1,213	-	1,077	1,113	
5	1-5	HSC 6	Hallgren	201	0,006	103,4	0,960	-	1,365	1,323	-	0,905	1,214	
6	1-6	N/HSC8	Hallgren	198	0,008	90,2	0,944	-	-	-	-	0,974	1,158	
7	1-7	HSC 9	Hallgren	202	0,003	79,9	0,565	-	1,299	0,930	-	-	-	
8	2-1	ND65-1-1	Tomas.	275	0,015	64,1	2,050	-	1,569	1,632	-	1,702	1,418	
9	2-2	ND65-2-1	Tomas.	200	0,017	70,0	1,200	-	1,473	1,527	-	1,780	1,329	
10	2-3	ND95-1-1	Tomas.	275	0,015	83,5	2,250	-	1,578	1,641	-	1,638	1,425	
11	2-4	ND95-1-3	Tomas.	275	0,025	89,7	2,400	-	1,373	1,428	-	1,685	1,240	
12	2-5	ND95-2-1	Tomas.	200	0,017	88,0	1,100	-	1,251	1,297	-	1,455	1,129	
13	2-6	ND95-2-1D	Tomas.	200	0,017	86,5	1,300	-	1,487	1,542	-	1,735	1,342	
14	2-7	ND95-2-3	Tomas.	200	0,026	89,3	1,450	-	1,434	1,486	-	1,904	1,293	
15	2-8	ND95-2-3D	Tomas.	200	0,026	80,1	1,250	-	1,303	1,350	-	1,776	1,175	
16	2-9	ND95-2-3D+	Tomas.	200	0,026	97,8	1,450	-	1,391	1,442	-	1,820	1,255	
17	2-10	ND95-3-1	Tomas.	88	0,018	84,9	0,330	-	1,400	1,477	-	1,881	1,548	
18	2-11	ND115-1-1	Tomas.	275	0,015	111,7	2,450	-	1,559	1,621	-	1,541	1,408	
19	2-12	ND115-2-1	Tomas.	200	0,017	118,7	1,400	-	1,441	1,494	-	1,595	1,300	
20	2-13	ND115-2-3	Tomas.	200	0,026	107,8	1,550	-	1,439	1,492	-	1,852	1,298	
21	3-1	1	Ramdane	98	0,006	87,1	0,224	-	1,152	1,121	-	0,879	1,230	
22	3-2	2	Ramdane	98	0,006	55,5	0,212	-	1,268	1,233	-	1,043	1,353	
23	3-3	3	Ramdane	98	0,006	26,5	0,169	1,327	1,292	1,257	1,228	1,201	1,379	
24	3-4	4	Ramdane	98	0,006	58,0	0,233	-	1,373	1,336	1,006	1,121	1,465	
25	3-5	6	Ramdane	98	0,006	100,3	0,233	-	-	-	-	-	-	
26	3-6	12	Ramdane	98	0,013	59,6	0,319	-	1,143	1,112	-	0,851	1,219	
27	3-7	13	Ramdane	98	0,013	43,1	0,297	-	1,434	1,395	-	1,513	1,530	
28	3-8	14	Ramdane	98	0,013	60,0	0,341	1,141	1,488	1,448	1,310	1,658	1,588	
29	3-9	16	Ramdane	98	0,013	97,2	0,362	-	1,529	1,488	-	1,612	1,632	
30	3-10	21	Ramdane	98	0,013	41,4	0,286	-	-	-	-	-	-	
31	3-11	22	Ramdane	98	0,013	83,2	0,405	-	1,383	1,345	-	1,345	1,476	
32	3-12	23	Ramdane	100	0,009	55,7	0,341	1,030	1,452	1,412	1,295	1,628	1,549	
33	3-13	25	Ramdane	100	0,012	32,5	0,244	-	1,629	1,585	-	1,626	1,739	
34	3-14	26	Ramdane	100	0,012	37,1	0,294	-	1,727	1,681	1,354	1,627	1,834	
35	3-15	27	Ramdane	102	0,010	33,3	0,227	-	-	-	1,032	-	-	

Table of comparison of test data with codes

(2 of 6)

a) Punching tests without shear reinforcement

running number	serial no.	slab no.	author	effective depth	reinforcement ratio (inner section)	compressive uniaxial strength (prism strength)	failure load	DIN 1045 (1988)	Model Code 90	FIP Recommendations	BS 8110-97 (1997)	Eurocode 2, Part 1 (1992)	ACI 318-95 (1995)	DIN 1045-1 (2000)
no.	notat.	n	A	d	$\rho_{l,i}$	f_{lc}	V_{exp}	$V_{exp}/V_{cal,u}$						
unit	unit	-	-	mm	-	MPa	MN	-						
36	4-1	I. HS1	Marz./Hus.	95	0,004	63,7	0,178	-	-	-	-	-	-	-
37	4-2	I. HS2	Marz./Hus.	95	0,007	66,5	0,249	-	1,281	1,342	-	1,140	1,356	
38	4-3	I. HS3	Marz./Hus.	95	0,012	65,6	0,356	-	1,251	1,311	-	1,641	1,324	
39	4-4	I. HS4	Marz./Hus.	90	0,021	62,7	0,418	-	1,520	1,593	-	1,970	1,609	
40	4-5	I. HS7	Marz./Hus.	95	0,009	70,3	0,356	-	1,601	1,677	-	1,585	1,694	
41	4-6	II. HS5	Marz./Hus.	125	0,005	64,6	0,365	-	-	-	-	-	-	-
42	4-7	II. HS6	Marz./Hus.	120	0,005	66,5	0,489	-	-	-	-	-	-	-
43	4-8	II. HS8	Marz./Hus.	120	0,010	65,6	0,436	-	1,424	1,482	-	1,444	1,430	
44	4-9	II. HS9	Marz./Hus.	120	0,015	70,3	0,543	-	1,514	1,575	-	1,736	1,520	
45	4-10	II. HS10	Marz./Hus.	120	0,021	76,0	0,645	-	1,566	1,629	-	1,984	1,573	
46	4-11	III. HS11	Marz./Hus.	70	0,007	66,5	0,196	-	-	-	-	-	-	-
47	4-12	III. HS12	Marz./Hus.	70	0,012	71,3	0,258	-	-	-	-	-	-	-
48	4-13	III. HS13	Marz./Hus.	70	0,016	64,6	0,267	-	-	-	-	-	-	-
49	4-14	IV. HS14	Marz./Hus.	95	0,012	68,4	0,498	-	1,493	1,557	-	1,748	1,538	
50	4-15	IV. HS15	Marz./Hus.	95	0,012	67,5	0,560	-	1,461	1,519	-	1,579	1,472	
51	4-16	I. NS1	Marz./Hus.	95	0,012	39,9	0,320	0,999	1,327	1,390	1,155	1,891	1,405	
52	4-17	II. NS2	Marz./Hus.	120	0,005	28,5	0,396	-	-	-	-	-	-	-
53	5-1	F1	Lov./McL.	83	0,017	37,3	0,480	-	-	-	-	-	-	-
54	5-2	F2	Lov./McL.	83	0,017	37,3	0,204	-	-	-	-	-	-	-
55	5-3	F3	Lov./McL.	83	0,017	37,3	0,149	-	-	-	-	-	-	-
56	5-4	F4	Lov./McL.	83	0,017	37,3	0,129	0,727	0,846	0,841	0,807	1,201	0,963	
57	5-5	F5	Lov./McL.	83	0,017	37,3	0,139	0,782	0,909	0,904	0,867	1,291	1,035	
58	6-1	S2.1	Tolf	200	0,008	19,4	0,603	-	1,369	1,326	1,339	1,321	1,217	
59	6-2	S2.2	Tolf	199	0,008	18,9	0,600	-	1,374	1,331	1,356	1,332	1,222	
60	6-3	S2.3	Tolf	200	0,005	19,3	0,489	-	1,415	1,371	1,228	1,074	1,258	
61	6-4	S2.4	Tolf	197	0,005	21,6	0,444	-	1,237	1,199	1,034	0,922	1,100	
62	6-5	S1.1	Tolf	100	0,008	30,1	0,216	1,212	1,403	1,380	1,337	1,520	1,506	
63	6-6	S1.2	Tolf	99	0,008	23,8	0,194	1,279	1,363	1,341	1,404	1,535	1,463	
64	6-7	S1.3	Tolf	98	0,004	26,0	0,145	-	1,158	1,139	1,078	1,098	1,243	
65	6-8	S1.4	Tolf	99	0,004	25,5	0,148	-	1,190	1,170	1,114	1,132	1,277	

Table of comparison of test data with codes

(3 of 6)

a) Punching tests without shear reinforcement

running number	serial no.	slab no.	author	effective depth	reinforcement ratio (inner section)	compressive uniaxial strength (prism strength)	failure load	DIN 1045 (1988)	Model Code 90	FTF Recommendations	BS 8110-97 (1997)	Eurocode 2, Part 1 (1992)	ACI 318-95 (1995)	DIN1045-1 (2000)
no.	notat.	n	A	d	$\rho_{l,i}$	f_{lc}	V_{exp}	$V_{exp}/V_{cal,u}$						
unit	unit	-	-	mm	-	MPa	MN	-						
66	7-1	I/1	Regan	77	0,024	25,4	0,194	1,095	1,023	1,078	1,030	1,567	1,115	
67	7-2	I/2	Regan	77	0,012	23,1	0,176	1,082	1,215	1,280	1,304	1,490	1,324	
68	7-3	I/3	Regan	77	0,014	27,1	0,194	1,200	1,071	1,128	1,078	1,518	1,167	
69	7-4	I/4	Regan	77	0,012	31,9	0,194	1,104	1,319	1,389	1,249	1,399	1,437	
70	7-5	I/5	Regan	79	0,015	27,8	0,165	1,098	0,962	1,012	0,969	1,233	1,042	
71	7-6	I/6	Regan	79	0,008	21,6	0,165	-	-	-	-	-	-	
72	7-7	I/7	Regan	79	0,008	30,0	0,186	1,161	1,253	1,319	1,154	1,187	1,357	
73	7-8	II/1	Regan	200	0,010	34,4	0,825	1,180	1,461	1,511	1,199	1,526	1,298	
74	7-9	II/2	Regan	128	0,010	32,9	0,390	1,279	1,436	1,491	1,263	1,603	1,416	
75	7-10	II/3	Regan	128	0,010	33,9	0,365	1,189	1,330	1,381	1,158	1,477	1,312	
76	7-11	II/4	Regan	64	0,010	32,9	0,117	1,564	1,401	1,505	1,453	1,924	1,700	
77	7-12	II/5	Regan	64	0,010	33,9	0,105	1,375	1,244	1,337	1,277	1,700	1,510	
78	7-13	II/6	Regan	64	0,010	35,7	0,105	1,328	1,223	1,314	1,234	1,656	1,484	
79	7-14	III/1	Regan	95	0,008	22,9	0,197	1,527	1,446	1,406	1,503	1,536	1,554	
80	7-15	III/2	Regan	95	0,008	9,4	0,123	-	1,215	1,181	1,700	1,497	1,306	
81	7-16	III/3	Regan	95	0,008	37,4	0,214	1,197	1,334	1,298	1,178	1,307	1,435	
82	7-17	III/4	Regan	93	0,015	11,8	0,154	-	1,187	1,154	1,604	1,725	1,283	
83	7-18	III/5	Regan	93	0,015	26,5	0,214	1,180	1,259	1,224	1,299	1,599	1,361	
84	7-19	III/6	Regan	93	0,015	42,1	0,248	1,003	1,250	1,215	1,104	1,469	1,351	
85	7-20	V/1	Regan	118	0,008	33,9	0,170	1,043	0,957	0,976	0,877	1,250	1,022	
86	7-21	V/2	Regan	118	0,008	31,8	0,280	1,072	1,319	1,279	1,175	1,270	1,340	
87	7-22	V/3	Regan	118	0,008	32,0	0,265	1,275	1,374	1,364	1,250	1,513	1,428	
88	7-23	V/4	Regan	118	0,008	35,8	0,285	1,244	1,377	1,438	1,196	1,594	1,419	

Table of comparison of test data with codes

(4 of 6)

a) Punching tests without shear reinforcement

running number	serial no.	slab no.	author	effective depth	reinforcement ratio (inner section)	compressive uniaxial strength (prism strength)	failure load	DIN 1045 (1988)	Model Code 90	FIP Recommendations	BS 8110-97 (1997)	Eurocode 2, Part 1 (1992)	ACI 318-95 (1995)	DIN1045-1 (2000)
no.	notat.	n	A	d	$\rho_{l,i}$	f_{lc}	V_{exp}	$V_{exp}/V_{cal,u}$						
unit	unit	-	-	mm	-	MPa	MN	-						
89	8-1	S1	Swamy/Ali	100	0,006	40,1	0,198	1,243	1,237	1,293	1,225	1,152	1,294	
90	8-2	S7	Swamy/Ali	100	0,007	37,4	0,222	-	-	-	-	-	-	
91	9-1	P2	ETH	143	0,015	35,4	0,628	0,922	1,417	1,338	1,205	1,367	1,315	
92	9-2	P5	ETH	171	0,012	27,1	0,626	-	-	-	-	-	-	
93	10-1	0	Schaeffers	113	0,008	21,9	0,280	1,410	1,467	1,402	1,471	1,416	1,484	
94	10-2	3	Schaeffers	170	0,006	22,1	0,460	1,658	1,531	1,486	1,382	1,320	1,420	
95	11-1	DA 6	Ladner	80	0,018	29,6	0,183	1,479	1,405	1,397	1,517	2,140	1,612	
96	11-2	DA 7	Ladner	80	0,018	33,1	0,288	1,313	1,718	1,635	1,710	2,046	1,887	
97	11-3	DA 10	Ladner	80	0,018	31,6	0,281	1,183	1,581	1,485	1,578	1,787	1,714	
98	11-4	DA 11	Ladner	80	0,018	30,0	0,324	1,159	1,623	1,494	1,614	1,692	1,724	
99	12-1	P1	Ladner	240	0,013	27,6	1,662	1,597	2,283	2,145	-	1,548	1,967	
100	12-2	M1	Ladner	109	0,012	31,4	0,362	1,149	1,535	1,458	-	1,538	1,557	
101	13-1	AN-1	Corl./Hawk.	111	0,015	42,2	0,334	0,835	1,011	1,048	1,833	1,103	0,996	
102	13-2	AN-2	Corl./Hawk.	111	0,010	42,2	0,266	0,833	0,921	0,955	1,412	0,878	0,908	
103	14-1	A1/M1	Base	114	0,011	15,5	0,322	-	-	-	-	-	-	
104	14-2	A1/M2	Base	117	0,015	14,7	0,346	-	1,518	1,576	1,773	2,092	1,503	
105	14-3	A1/M3	Base	121	0,019	13,5	0,307	-	1,224	1,270	1,456	1,852	1,204	
106	14-4	A1/M4	Base	124	0,010	13,3	0,259	-	1,243	1,288	1,464	1,522	1,215	
107	14-5	A1/M5	Base	117	0,012	20,0	0,346	-	1,478	1,534	1,551	1,797	1,463	
108	14-6	A2/M1	Base	124	0,010	33,6	0,409	-	-	-	-	-	-	
109	14-7	A2/M2	Base	117	0,015	31,2	0,419	-	1,432	1,486	1,302	1,742	1,418	
110	14-8	A2/M3	Base	121	0,019	30,9	0,430	-	1,301	1,350	1,174	1,715	1,279	
111	14-9	A2/T1	Base	124	0,010	37,3	0,419	-	1,425	1,478	1,190	1,469	1,394	
112	14-10	A2/T2	Base	124	0,017	39,3	0,439	-	1,230	1,275	1,025	1,500	1,203	
113	14-11	A3/M1	Base	124	0,010	17,9	0,247	1,415	1,074	1,114	1,147	1,252	1,051	
114	14-12	A3/M2	Base	102	0,017	18,3	0,336	1,573	1,580	1,646	1,746	2,191	1,612	
115	14-13	A3/M3	Base	117	0,019	25,9	0,298	-	1,001	1,039	0,961	1,358	0,991	
116	14-14	A3/T1	Base	121	0,010	19,6	0,328	-	1,431	1,484	1,487	1,643	1,407	
117	14-15	A3/T2	Base	119	0,012	15,2	0,298	-	1,362	1,413	1,562	1,733	1,344	
118	14-16	A4/M1	Base	114	0,011	36,4	0,259	-	0,966	1,003	0,828	1,032	0,961	
119	14-17	A4/M2	Base	119	0,015	27,7	0,341	-	1,184	1,228	1,117	1,468	1,168	
120	14-18	A4/M3	Base	117	0,019	30,6	0,541	-	-	-	-	-	-	
121	14-19	A4/T1	Base	114	0,011	31,2	0,384	-	1,508	1,566	1,362	1,654	1,501	
122	14-20	A4/T2	Base	117	0,012	27,8	0,402	-	1,537	1,595	1,444	1,768	1,522	

Table of comparison of test data with codes

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a) Punching tests without shear reinforcement

running number	serial no.	slab no.	author	effective depth	reinforcement ratio (inner section)	compressive uniaxial strength (prism strength)	failure load	DIN 1045 (1988)	Model Code 90	FIP Recommendations	BS 8110-97 (1997)	Eurocode 2, Part 1 (1992)	ACI 318-95 (1995)	DIN1045-1 (2000)
no.	notat.	n	A	d	$\rho_{t,i}$	f_{tc}	V_{exp}	$V_{exp} / V_{cal,u}$						
unit	unit	-	-	mm	-	MPa	MN	-						
123	15-1	P1-S1	Manterola	107	0,011	24,3	0,216	1,738	1,255	1,315	1,296	1,716	1,325	
124	15-2	P2-S1	Manterola	107	0,011	32,1	0,257	-	-	-	-	-	-	
125	15-3	P3-S1	Manterola	107	0,011	28,2	0,301	1,081	0,927	0,971	0,827	1,178	0,978	
126	15-4	P1-S2	Manterola	107	0,011	23,0	0,196	1,600	1,162	1,217	1,223	1,604	1,227	
127	15-5	P2-S2	Manterola	107	0,011	31,4	0,283	1,056	1,122	1,164	1,002	1,145	1,116	
128	15-6	P3-S2	Manterola	107	0,011	30,3	0,397	0,971	1,013	1,052	0,899	1,031	1,008	
129	15-7	P1-S3	Manterola	107	0,011	37,7	0,184	-	-	-	-	-	-	
130	15-8	P2-S3	Manterola	107	0,014	34,0	0,211	1,147	0,997	1,044	0,932	1,418	1,052	
131	15-9	P3-S3	Manterola	107	0,005	37,2	0,165	-	-	-	-	-	-	
132	15-10	P1-S4	Manterola	107	0,005	25,1	0,175	-	-	-	-	-	-	
133	15-11	P2-S4	Manterola	107	0,005	29,7	0,246	-	-	-	-	-	-	
134	15-12	P3-S4	Manterola	107	0,005	32,5	0,294	-	-	-	-	-	-	
135	16-1	II-5	Yitzhaki	82	0,005	16,0	0,152	-	-	-	-	-	-	
136	16-2	II-8	Yitzhaki	82	0,006	17,4	0,218	-	-	-	-	-	-	
137	16-3	IIS20-1	Yitzhaki	78	0,007	10,2	0,128	-	-	-	-	-	-	
138	16-4	II-1	Yitzhaki	82	0,012	9,7	0,181	-	1,636	1,547	-	1,967	1,774	
139	16-5	II-4a	Yitzhaki	82	0,009	16,6	0,245	-	2,039	1,927	-	2,036	2,210	
140	16-6	II-4b	Yitzhaki	82	0,009	9,1	0,162	-	1,548	1,626	-	1,945	1,661	
141	16-7	II-4c	Yitzhaki	82	0,009	12,9	0,215	-	1,830	1,922	-	2,171	1,963	
142	16-8	IIR20-2	Yitzhaki	83	0,009	13,9	0,307	-	-	-	-	-	-	
143	16-9	IIR30-1	Yitzhaki	80	0,020	16,3	0,239	-	1,381	1,277	-	1,638	1,473	
144	16-10	II-2	Yitzhaki	82	0,013	9,1	0,152	-	1,370	1,295	-	1,712	1,485	
145	16-11	II-3	Yitzhaki	82	0,013	12,5	0,244	-	1,855	1,948	-	2,500	1,990	
146	16-12	II-6	Yitzhaki	82	0,013	20,1	0,240	-	1,660	1,569	-	1,817	1,800	
147	16-13	II-9	Yitzhaki	79	0,085	8,6	0,157	-	0,905	0,952	-	2,037	0,979	
148	16-14	III-3	Yitzhaki	82	0,012	16,8	0,201	-	1,515	1,432	-	1,663	1,643	
149	16-15	7	Yitzhaki	82	0,007	9,3	0,117	-	1,579	1,555	-	1,962	1,784	
150	16-16	II-10	Yitzhaki	82	0,010	10,8	0,098	-	1,115	1,098	-	1,521	1,260	

Table of comparison of test data with codes

(6 of 6)

a) Punching tests without shear reinforcement

running number	serial no.	slab no.	author	effective depth	reinforcement ratio (inner section)	compressive uniaxial strength (prism strength)	failure load	DIN 1045 (1988)	Model Code 90	FIP Recommendations	BS 8110-97 (1997)	Eurocode 2, Part 1 (1992)	ACI 318-95 (1995)	DIN1045-1 (2000)
no.	nc tat.	n	A	d	ρ_{li}	f_{lc}	V_{exp}	$V_{exp}/V_{cal,u}$						
unit	unit	-	-	mm	-	MPa	MN	-						
151	17-1	S1-60	Moe	114	0,011	22,1	0,389	1,538	1,592	1,649	1,582	1,713	1,560	
152	17-2	S2-60	Moe	114	0,015	21,0	0,356	1,445	1,313	1,360	1,346	1,608	1,287	
153	17-3	S3-60	Moe	114	0,020	21,5	0,364	1,423	1,155	1,197	1,150	1,626	1,133	
154	17-4	S4-60	Moe	114	0,026	22,6	0,334	1,073	-	-	-	-	-	
155	17-5	S1-70	Moe	114	0,011	23,3	0,393	1,371	1,578	1,634	1,542	1,684	1,546	
156	17-6	S3-70	Moe	114	0,020	24,1	0,378	1,342	1,208	1,252	1,209	1,705	1,184	
157	17-7	S4-70	Moe	114	0,026	33,4	0,374	0,748	1,241	1,118	1,087	1,339	1,217	
158	17-8	S4-70A	Moe	114	0,026	19,5	0,312	-	1,240	1,117	1,300	1,464	1,215	
159	17-9	S5-60	Moe	114	0,011	21,1	0,343	1,460	1,534	1,594	1,556	1,761	1,528	
160	17-10	S5-70	Moe	114	0,011	21,9	0,378	-	1,614	1,676	1,559	1,808	1,607	
161	17-11	R1	Moe	114	0,014	25,3	0,312	1,649	1,335	1,391	1,332	1,778	1,355	
162	17-12	R2	Moe	114	0,014	26,2	0,394	1,494	1,316	1,371	1,297	1,742	1,336	
163	17-13	H1	Moe	114	0,011	24,8	0,372	1,366	1,423	1,474	1,369	1,544	1,395	
164	17-14	M1A	Moe	114	0,015	19,8	0,433	1,057	1,501	1,552	1,550	1,771	1,453	
165	18-1	IA15a-5	Kinn./Nyl.	117	0,008	27,6	0,255	1,703	1,294	1,264	1,226	1,350	1,326	
166	18-2	IA15a-6	Kinn./Nyl.	118	0,008	25,4	0,275	1,437	1,418	1,385	1,376	1,496	1,450	
169	18-3	IA15c-11	Kinn./Nyl.	121	0,018	31,0	0,334	-	-	-	-	-	-	
170	18-4	IA15c-12	Kinn./Nyl.	122	0,017	28,4	0,332	-	-	-	-	-	-	
171	18-5	IA30a-24	Kinn./Nyl.	128	0,010	25,6	0,430	1,290	1,240	1,211	1,163	1,587	1,261	
172	18-6	IA30a-25	Kinn./Nyl.	124	0,011	24,3	0,408	1,113	1,256	1,228	1,211	1,627	1,275	
175	18-7	IA30c-30	Kinn./Nyl.	120	0,021	29,2	0,491	1,123	1,474	1,381	1,383	1,346	1,417	
176	18-8	IA30c-31	Kinn./Nyl.	119	0,021	29,2	0,540	1,179	1,470	1,376	1,412	1,367	1,423	
177	18-9	IA30d-32	Kinn./Nyl.	123	0,005	25,5	0,258	-	-	-	-	-	-	
178	18-10	IA30d-33	Kinn./Nyl.	125	0,005	25,8	0,258	-	-	-	-	-	-	
179	18-11	IA30e-34	Kinn./Nyl.	120	0,010	26,5	0,332	1,268	1,547	1,419	1,419	1,565	1,508	
180	18-12	IA30e-35	Kinn./Nyl.	122	0,010	24,3	0,332	1,228	1,716	1,573	1,576	1,741	1,676	

Remark:

- neglected by the application rules according to the procedure in Chapter 8.4

Appendix III

Table of comparison of test data with codes

(1 of 4)

b) punching with shear reinforcement

running number	serial no.	slab no.	author	effective depth	reinforcement ratio (inner section)	Sum of shear reinforcement	failure load	DIN 1045 (1988)	Model Code 90	BS 8110-97 (1997)	Eurocode 2, Part 1 (1992)	ACI 318-95 (1995)	DIN1045-1 (2000)
no.	notat.	n	A	d	ρ_{li}	ΣA_{sw}	V_{exp}	$\max V_{exp}/V_{cal,u}$					
unit	unit	-	-	mm	-	cm ²	MN	-					
1	01-01	1355	Graf	273	0,700	25,9	1,206	-	1,29	1,29	1,70	1,82	1,28
2	01-02	1356	Graf	274	0,697	25,9	1,284	-	1,47	1,37	1,81	1,92	1,35
3	01-03	1361	Graf	272	1,045	36,7	1,726	-	1,67	1,27	2,15	1,51	1,20
4	01-04	1363	Graf	469	0,596	75,1	3,020	-	1,42	1,21	1,75	1,09	1,31
5	01-05	1376	Graf	474	0,404	58,3	2,255	-	1,18	1,34	1,36	0,98	1,10
6	01-06	1377	Graf	474	0,404	58,3	2,206	-	1,19	1,36	1,39	0,96	1,10
7	02-01	S-I	Keefe	113	2,535	7,8	0,492	-	1,06	1,61	1,15	2,55	2,06
8	02-02	S-II	Keefe	113	2,535	7,8	0,492	-	1,16	1,65	1,19	2,62	2,08
9	03-01	B3	Elstn.,Hog.	114	1,008	4,0	0,286	-	1,08	0,83	1,53	1,28	1,11
10	03-02	B6	Elstn.,Hog.	114	1,002	7,3	0,468	-	1,76	1,28	2,51	1,75	1,34
11	03-03	B10	Elstn.,Hog.	114	2,014	7,3	0,533	1,66	1,08	0,83	1,04	1,30	1,11
12	03-04	B17	Elstn.,Hog.	114	1,515	4,0	0,357	-	1,17	0,90	1,70	1,57	1,21
13	03-05	B5	Elstn.,Hog.	114	1,314	3,6	0,378	-	1,30	0,94	1,88	1,76	1,34
14	03-06	B12	Elstn.,Hog.	114	3,020	22,7	0,787	0,95	1,26	1,09	1,15	1,40	1,31
15	03-07	B13	Elstn.,Hog.	114	3,020	13,7	0,778	-	1,02	1,08	0,90	3,21	1,85
16	03-08	B15	Elstn.,Hog.	114	3,020	10,9	0,689	1,43	1,21	0,95	1,12	1,43	1,25
17	03-09	B16	Elstn.,Hog.	114	3,020	21,9	0,747	0,90	1,09	1,03	0,99	1,33	1,15
18	04-01	S8-60	Moe	98	1,224	3,5	0,367	-	1,45	1,12	1,82	1,68	1,56
19	05-01	A-S1-62	Andersson	120	0,779	6,8	0,367	1,46	1,04	1,10	1,08	1,87	2,10
20	05-02	A-S1-63	Andersson	120	0,779	6,8	0,353	1,42	1,00	1,06	1,04	1,79	2,01
21	05-03	A-S1-64	Andersson	120	0,779	11,3	0,370	1,50	1,05	1,11	1,09	1,88	2,11
22	05-04	A-S1-65	Andersson	121	0,772	11,3	0,372	1,49	1,05	1,11	1,09	1,87	2,11
23	05-05	A-S2-66	Andersson	119	0,785	9,1	0,292	1,17	1,03	0,88	1,13	1,35	1,44
24	05-06	A-S2-67	Andersson	121	0,772	9,1	0,294	1,15	1,01	0,86	1,09	1,34	1,43
25	05-07	A-S3-68	Andersson	121	0,772	4,8	0,274	1,46	0,88	0,83	0,97	1,73	1,94
26	05-08	A-S3-69	Andersson	121	0,772	4,8	0,249	1,32	0,81	0,77	0,90	1,60	1,78
27	05-09	A-S4-70	Andersson	121	0,772	2,8	0,272	1,17	0,91	0,86	0,96	1,57	1,37
28	05-10	A-S4-71	Andersson	123	0,760	2,8	0,292	1,18	0,95	0,88	0,98	1,62	1,44
29	05-11	A-S1-76	Andersson	122	1,030	6,8	0,534	1,29	1,28	0,89	1,42	2,47	2,50
30	05-12	A-S1-77	Andersson	125	1,005	6,8	0,549	1,34	1,26	0,88	1,38	2,41	2,47
31	05-13	A-S1-78	Andersson	120	1,047	13,6	0,606	1,34	1,46	1,02	1,60	2,80	1,29
32	05-14	A-S1-79	Andersson	119	1,055	13,6	0,612	1,34	1,48	1,04	1,63	2,85	1,31
33	05-15	A-S1-80	Andersson	121	0,837	9,0	0,453	1,14	1,16	0,81	1,25	2,08	2,27
34	05-16	A-S1-81	Andersson	120	0,844	9,0	0,470	1,25	1,25	0,87	1,39	2,27	2,44
35	05-17	A-S2-82	Andersson	120	1,047	12,9	0,459	1,06	1,29	0,92	1,46	1,51	1,50

Table of comparison of test data with codes

(2 of 4)

b) punching tests with shear reinforcement

no.	notat.	n	A	d	ρ_{li}	Sum of shear reinforcement ΣA_{sw}	V_{exp}	DIN 1045 (1988)	Model Code 90	BS 8110-97 (1997)	Eurocode 2, Part 1 (1992)	ACI 318-95 (1995)	DIN1045-1 (2000)
unit	unit	-	-	mm	-	cm ²	MN	max $V_{exp}/V_{cal,u}$					
36	05-18	A-S2-83	Andersson	119	1,055	12,9	0,459	1,18	1,37	0,97	1,63	1,58	1,54
37	05-19	A-S3-84	Andersson	123	1,021	6,4	0,411	1,64	1,08	0,76	1,25	2,01	2,01
38	05-20	A-S3-85	Andersson	122	1,030	6,4	0,392	1,56	1,04	0,73	1,19	1,92	1,92
39	06-01	2	Franz	129	1,095	7,8	0,384	1,14	1,08	0,87	1,30	2,61	2,14
40	06-02	3	Franz	129	1,095	7,8	0,392	1,12	0,90	0,86	1,04	2,59	2,14
41	06-03	4	Franz	129	1,095	3,0	0,387	1,61	1,09	0,89	1,13	1,58	1,55
42	06-04	5	Franz	129	1,095	12,0	0,431	1,13	1,06	0,89	1,19	1,20	1,20
43	06-05	6	Franz	129	1,095	18,0	0,466	1,32	1,14	1,00	1,39	1,06	1,16
44	06-06	7	Franz	129	1,095	14,3	0,353	1,07	0,93	0,81	1,12	0,97	0,95
45	06-07	8	Franz	125	1,130	6,0	0,415	1,17	1,04	0,92	1,17	1,32	1,31
46	06-08	9	Franz	125	0,952	4,0	0,431	1,54	1,23	1,06	1,47	1,74	1,73
47	06-09	10	Franz	125	0,800	4,0	0,362	1,35	1,06	0,89	1,23	1,44	1,48
48	06-10	11	Franz	125	0,695	6,0	0,343	1,29	0,96	0,78	1,06	2,06	2,33
49	06-11	12	Franz	125	0,565	6,0	0,323	1,40	0,96	0,75	1,00	1,82	2,18
50	06-12	13	Franz	125	0,476	2,0	0,279	1,24	-	-	-	-	-
51	06-13	14	Franz.	125	1,205	5,0	0,411	1,33	1,04	1,02	1,28	1,44	1,38
52	07-01	L2	Narasiham	143	1,107	12,1	0,868	1,74	1,62	1,24	1,66	1,40	1,57
53	08-01	B6	Petcu, St.	171	0,510	8,8	0,570	1,33	0,97	0,75	0,88	1,49	1,30
54	08-02	B7	Petcu, St.	167	0,520	8,8	0,629	1,48	1,09	0,84	0,99	1,66	1,45
55	08-03	B8	Petcu, St.	167	0,520	16,3	0,61	1,52	1,09	0,79	1,02	1,66	1,43
56	08-04	B9	Petcu, St.	203	0,430	8,8	0,635	1,58	1,01	0,78	0,99	1,60	1,41
57	08-05	B10	Petcu, St.	212	0,430	16,3	0,786	1,93	1,28	0,94	1,29	1,97	1,73
58	08-06	A7	Petcu, St.	120	0,730	4,4	0,323	-	-	-	-	-	-
59	08-07	A8	Petcu, St.	121	0,720	8,8	0,411	1,71	1,14	0,86	1,33	1,17	1,21
60	08-08	A5	Petcu, St.	113	0,780	7,4	0,274	1,09	0,88	0,69	1,01	1,02	0,99
61	08-09	A6	Petcu, St.	110	0,800	11,7	0,274	-	-	-	-	-	-
62	09-01	P3	Marti et al	152	1,395	4,0	0,875	0,98	1,48	1,12	1,40	2,74	2,60
63	10-01	A	Sunquist	170	0,665	11,4	0,37	-	-	-	-	-	-
64	10-02	C	Sunquist	170	0,554	12,8	0,365	-	-	-	-	-	-
65	10-03	D	Sunquist	170	0,554	12,8	0,58	-	-	-	-	-	-
66	10-04	E	Sunquist	170	0,554	12,8	0,56	-	-	-	-	-	-
67	11-01	SC8	Seib., Gha.	121	1,170	18,0	0,591	1,00	1,20	0,90	1,25	1,08	1,23
68	11-02	SC9	Seib., Gha.	121	1,170	25,0	0,596	1,00	1,21	0,91	1,26	1,02	1,24
69	11-03	SC10	Seib., Gha.	121	1,170	22,4	0,538	0,91	1,10	0,82	1,13	1,95	1,25
70	12-01	S14	Swamy, Ali	100	0,748	4,4	0,334	1,35	1,07	0,89	1,02	2,66	2,35
71	12-02	S17	Swamy, Ali	100	0,748	5,4	0,357	1,44	1,14	0,95	1,10	2,84	1,25
72	13-01	P6	Pral. et al	154	1,305	48,0	0,594	-	-	-	-	-	-
73	13-02	P22	Müller et al	153,5	1,309	36,0	1,04	1,28	1,37	1,15	1,32	1,63	1,32
74	14-01	HSC3S	Hallg., Kin.	200	0,628	28,4	1,329	-	1,27	1,35	0,84	2,00	2,41

Table of comparison of test data with codes

(3 of 4)

b) punching tests with shear reinforcement

no.	notat.	n	A	d	ρ_{ti}	Sum of shear reinforcement ΣA_{sw}	V_{exp}	DIN 1045 (1988)	Model Code 90	BS 8110-97 (1997)	Eurocode 2, Part 1 (1992)	ACI 318-95 (1995)	DIN1045-1 (2000)
unit	unit	-	-	mm	-	cm ²	MN	$\max V_{exp}/V_{calc}$					
75	14-02	HSC7S	Hallg., Kin.	200	0,479	22,7	1,106	-	1,19	1,13	0,77	1,73	2,26
76	15-01	2	Broms	150	0,628	18,8	0,523	-	-	-	-	-	-
77	15-02	3	Broms	150	0,942	24,1	0,558	-	-	-	-	-	-
78	15-03	4	Broms	150	0,582	24,5	0,415	1,10	0,93	0,67	1,06	1,82	1,84
79	15-04	5	Broms	150	0,710	24,5	0,434	0,93	0,87	0,63	0,98	1,78	1,72
80	15-05	6	Broms	150	0,477	11,3	0,608	1,45	1,27	0,89	1,22	2,29	1,23
81	15-06	7	Broms	150	0,838	11,3	1,006	1,98	1,85	1,38	2,04	4,12	1,79
82	16-01	FV1	Lovro. et al	82,5	1,750	1,9	0,261	-	1,73	1,60	1,36	3,65	2,11
83	16-02	FV2	Lovro. et al	82,5	1,750	7,7	0,275	1,18	1,26	1,68	1,43	3,83	1,22
84	16-03	FV3	Lovro. et al	82,5	1,750	17,4	0,279	1,19	1,28	1,71	1,45	3,89	1,24
85	16-04	FV4	Lovro. et al	82,5	1,750	31,0	0,341	1,46	1,57	2,09	1,77	4,76	1,51
86	16-05	FV5	Lovro. et al	82,5	1,750	69,8	0,549	-	-	-	-	-	-
87	17-01	1	Regan	259	0,621	40,7	1,8	-	1,09	0,75	0,89	1,01	1,30
88	18-01	S2	Kinn. et al	669	0,531	208,1	8,215	1,12	1,29	0,90	1,27	1,29	1,20
89	18-02	S3	Kinn. et al	669	0,531	208,1	8,15	1,13	1,28	0,89	1,27	1,29	1,19
90	19-01	S1.1s	Tolf	100	0,838	6,3	0,261	1,28	1,22	1,23	1,49	1,18	1,35
91	19-02	S1.2s	Tolf	100	0,838	6,3	0,259	1,24	1,20	1,21	1,45	1,16	1,33
92	19-03	S1.3s	Tolf	100	0,335	3,1	0,144	1,09	0,91	0,67	0,93	0,86	1,11
93	19-04	S1.4s	Tolf	100	0,335	3,1	0,147	1,11	0,93	0,69	0,95	0,88	1,13
94	19-05	S2.1s	Tolf	195	0,859	25,1	0,894	1,34	1,39	1,21	1,63	1,17	1,29
95	19-06	S2.2s	Tolf	195	0,859	25,2	0,851	1,25	1,32	1,14	1,53	1,10	1,23
96	19-07	S2.3s	Tolf	200	0,335	12,6	0,562	1,28	1,15	0,82	1,13	0,91	1,16
97	19-08	S2.4s	Tolf	200	0,335	12,6	0,542	1,24	1,11	0,80	1,09	0,88	1,12
98	20-01	2	Chana,Des.	200	0,785	12,1	1,057	1,26	1,05	0,87	0,99	1,64	1,73
99	20-02	3	Chana,Des.	200	0,785	12,1	1,139	1,36	1,16	0,96	1,10	1,81	1,90
100	20-03	4	Chana,Des.	200	0,785	24,1	1,302	1,32	1,28	1,06	1,13	1,44	1,56
101	20-04	5	Chana,Des.	210	0,854	25,1	1,382	1,40	1,30	1,08	1,23	1,75	1,18
102	20-05	7	Chana,Des.	210	0,854	25,1	1,429	1,40	1,32	1,37	1,23	1,93	1,20
103	20-05	6	Chana,Des.	210	0,854	25,1	1,283	1,20	1,16	0,96	1,05	1,33	1,30
104	20-07	8	Chana,Des.	210	0,854	16,1	1,327	1,32	1,24	1,02	1,15	2,04	2,11
105	20-08	9	Chana,Des.	188	0,835	24,1	1,135	1,27	1,22	1,02	1,11	1,46	1,56
106	21-01	FPS2	Chana,Des.	210	0,854	18,9	1,51		1,34	1,15	1,28	1,83	1,19
107	21-02	FPS3	Chana,Des.	210	0,854	10,1	1,773		1,76	1,41	1,72	2,96	1,40
108	21-03	FPS4	Chana,Des.	210	0,854	31,4	2,028		1,73	1,48	1,60	2,07	1,54
109	21-04	FPS5	Chana,Des.	210	0,854	37,7	1,94		1,75	1,50	1,72	2,16	1,56
110	22-01	T2	Yam. et al	167	0,958	33,0	0,6	0,90	0,88	0,62	1,02	1,32	1,38
111	22-02	T3	Yam. et al	167	0,974	48,7	0,727	1,06	1,06	0,74	1,22	1,41	1,53
112	22-03	T4	Yam. et al	167	1,002	61,1	0,697	0,99	0,99	0,70	1,14	1,03	0,96
113	22-04	T5	Yam. et al	167	0,929	79,6	0,762	1,18	1,14	0,80	1,33	1,13	1,10

Table of comparison of test data with codes

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b) punching tests with shear reinforcement

no.	notat.	n	A	d	$\rho_{l,i}$	Sum of shear reinforcement ΣA_{sw}	V_{exp}	DIN 1045 (1988)	Model Code 90	BS 8110-97 (1997)	Eurocode 2, Part 1 (1992)	ACI 318-95 (1995)	DIN1045-1 (2000)
unit	unit	-	-	mm	-	cm ²	MN	max $V_{exp}/V_{cal,u}$					
114	22-05	T6	Yam. et al	167	0,998	92,5	0,735	1,05	1,05	0,74	1,20	1,05	1,01
115	22-06	K2	Yam. et al	167	0,994	4,2	0,95	1,34	1,62	1,40	1,66	2,83	1,19
116	22-07	K3	Yam. et al	167	0,994	8,5	1,183	1,72	1,63	1,46	1,73	3,60	1,50
117	22-08	K4	Yam. et al	167	0,994	11,8	1,153	1,62	1,54	1,29	1,61	3,42	1,44
118	22-09	K5	Yam. et al	167	0,994	23,6	1,44	2,09	1,96	1,41	2,08	4,38	1,83
119	22-10	K6	Yam. et al	167	0,994	19,9	1,274	1,83	1,72	1,24	1,82	3,85	1,61
120	22-11	K7	Yam. et al	167	0,994	39,8	1,498	2,08	1,99	1,42	2,07	4,41	1,86
121	23-01	2	Chana	188	0,836	24,1	0,981	1,09	1,34	1,01	1,32	1,25	1,34
122	23-02	3	Chana	188	0,836	24,1	1,286	1,37	1,36	0,98	1,28	1,62	1,24
123	23-03	4	Chana	188	0,836	24,1	1,135	1,24	1,22	0,88	1,16	1,44	1,54
124	24-01	HS20	Marz. et al	120	0,887	31,4	0,418	-	-	-	-	-	-
125	24-02	HS21	Marz. et al	120	0,887	22,2	0,609	-	1,10	0,81	0,86	0,90	1,14
126	24-03	HS18	Marz. et al	120	0,887	11,1	0,511	-	1,10	0,85	0,90	0,80	1,12
127	25-01	P1-I	Beu, Heg	190	0,810	22,0	1,151	1,03	1,50	1,14	1,53	1,25	1,36
128	25-02	P1-II	Beu, Heg	190	0,810	22,0	1,055		1,41	1,07	1,45	1,06	1,27
129	25-03	P2-I	Beu, Heg	190	0,810	23,1	1,326	1,02	1,32	0,96	1,18	1,64	1,74
130	25-04	P2-II	Beu, Heg	190	0,810	23,1	1,109		1,16	0,85	1,12	1,57	1,62
131	25-05	P2-III	Beu, Heg	190	0,810	21,1	1,276		1,25	0,92	1,12	1,59	1,68
132	25-06	P3-I	Beu, Heg	220	1,142	15,1	1,624		1,37	1,09	1,41	2,38	1,24
133	25-07	P4-III	Beu, Heg	222	1,132	16,1	1,522		1,19	0,94	1,17	2,11	2,06
134	25-08	P5-I	Beu, Heg	220	1,360	34,6	1,936		1,23	1,10	1,02	1,45	1,52
135	25-09	P6-I	Beu, Heg	223	1,761	54,3	2,349		1,33	1,32	1,10	1,46	1,21
136	25-10	P7-I	Beu, Heg	229	1,001	34,8	2,117		1,46	1,16	1,16	1,62	1,21
137	26-01	2V10	Lee et al	155	2,030	18,1	1,19		1,15	1,06	0,88	1,73	1,72
138	26-02	2V20	Lee et al	160	1,960	28,3	1,45		1,34	1,25	1,02	1,66	1,75
139	26-03	2V21	Lee et al	160	1,960	28,3	1,48		1,17	1,28	0,86	1,69	1,78
140	26-04	2V22	Lee et al	160	1,960	28,3	1,69		1,33	1,46	0,99	1,83	1,21
141	26-05	3V20	Lee et al	230	1,520	22,0	1,9		1,05	0,83	0,80	1,64	1,69
142	27-01	3V	Oliv. et al	105	1,200	3,1	0,386		1,08	1,23	0,99	1,75	1,72
143	27-02	4V	Oliv. et al	103	1,300	5,0	0,377		1,03	1,22	0,94	1,22	1,25
144	27-03	5V	Oliv. et al	103	1,300	5,0	0,423		1,01	1,37	0,96	1,83	1,79
145	27-04	6I	Oliv. et al	100	1,400	2,6	0,41		1,23	1,37	1,11	2,11	1,48
146	27-05	7I	Oliv. et al	100	1,400	4,2	0,49		1,38	1,63	1,31	2,39	1,33
147	27-06	8I	Oliv. et al	100	1,400	4,2	0,54		1,40	1,80	1,30	2,61	1,45
148	27-07	9I	Oliv. et al	100	1,400	4,2	0,56		1,68	1,87	1,63	2,73	1,52
149	27-08	10I	Oliv. et al	105	1,300	6,3	0,52		1,21	1,65	1,08	2,28	1,33
150	27-09	11I	Oliv. et al	102	1,338	6,3	0,551		1,32	1,80	1,17	2,45	1,45

IV Table of comparison of test data with selected models

(1 of 6)

n.p. running number	nbat. serial no.	slab no.	author	effective depth reinforcement ratio (inner section) compressive uniaxial strength (prism strength)			failure load	Model by Kinnunen/Nylander	Model by Moe	Model by Braestrup/Nielsen et al.	Model by Georgopoulos	Model by Menétrey	Model by Hallgren	Model by Staller		
				d	$\rho_{l,i}$	f_{lc}									V_{exp}	$V_{exp}/V_{cal,u}$
unit	-	-	-	mm	%	MPa	MN	-								
1	1-1	HSC 0	Hallgren	200	0,800	89,2	0,965	0,845	0,793	-	1,263	1,116	0,754	0,969		
2	1-2	HSC 1	Hallgren	200	0,800	86,7	1,021	0,909	0,854	-	1,358	1,029	1,379	0,848		
3	1-3	HSC 2	Hallgren	194	0,800	81,4	0,889	0,867	0,796	-	1,233	1,073	1,211	0,884		
4	1-4	HSC 4	Hallgren	200	1,200	87,0	1,041	0,750	0,796	-	1,025	0,843	1,240	0,824		
5	1-5	HSC 6	Hallgren	201	0,600	103,4	0,960	0,968	0,820	-	1,717	1,122	1,257	0,883		
6	1-6	N/HSC8	Hallgren	198	0,800	90,2	0,944	0,832	0,788	-	1,279	0,950	1,229	0,842		
7	1-7	HSC 9	Hallgren	202	0,330	79,9	0,565	0,920	0,662	-	1,511	0,994	1,039	0,611		
8	2-1	ND65-1-1	Tomas.	275	1,500	64,1	2,050	0,788	1,167	-	1,179	1,207	1,142	1,084		
9	2-2	ND65-2-1	Tomas.	200	1,700	70,0	1,200	0,837	1,193	-	1,184	1,120	1,105	1,101		
10	2-3	ND95-1-1	Tomas.	275	1,500	83,5	2,250	0,773	1,144	-	1,338	1,174	1,185	1,106		
11	2-4	ND95-1-3	Tomas.	275	2,500	89,7	2,400	0,622	1,106	-	0,972	0,840	0,986	1,021		
12	2-5	ND95-2-1	Tomas.	200	1,700	88,0	1,100	0,696	0,990	-	1,112	0,919	0,989	0,939		
13	2-6	ND95-2-1D	Tomas.	200	1,700	86,5	1,300	0,829	1,179	-	1,312	1,102	1,137	1,128		
14	2-7	ND95-2-3	Tomas.	200	2,600	89,3	1,450	0,740	1,229	-	1,069	0,886	1,147	1,075		
15	2-8	ND95-2-3D	Tomas.	200	2,600	80,1	1,250	0,670	1,113	-	0,922	0,807	0,977	0,964		
16	2-9	ND95-2-3D+	Tomas.	200	2,600	97,8	1,450	0,710	1,180	-	1,072	0,855	1,100	1,062		
17	2-10	ND95-3-1	Tomas.	88	1,800	84,9	0,330	0,983	1,134	-	1,382	1,094	0,856	1,035		
18	2-11	ND115-1-1	Tomas.	275	1,500	111,7	2,450	0,744	1,103	-	1,545	1,147	1,113	1,158		
19	2-12	ND115-2-1	Tomas.	200	1,700	118,7	1,400	0,780	1,110	-	1,494	1,053	1,057	1,158		
20	2-13	ND115-2-3	Tomas.	200	2,600	107,8	1,550	0,726	1,207	-	1,153	0,874	1,140	1,106		
21	3-1	1	Ramdane	98	0,600	87,1	0,224	1,091	0,806	0,250	1,623	1,208	1,018	0,856		
22	3-2	2	Ramdane	98	0,600	55,5	0,212	1,161	0,877	0,371	1,383	1,331	0,975	0,881		
23	3-3	3	Ramdane	98	0,600	26,5	0,169	1,158	0,903	0,618	1,017	1,471	0,928	0,850		
24	3-4	4	Ramdane	98	0,600	58,0	0,233	1,261	0,950	0,390	1,534	1,413	1,104	0,979		
25	3-5	6	Ramdane	98	0,600	100,3	0,233	1,097	0,805	0,226	1,754	1,221	1,056	0,877		
26	3-6	12	Ramdane	98	1,300	59,6	0,319	1,104	1,029	0,519	1,148	1,254	0,880	1,077		
27	3-7	13	Ramdane	98	1,300	43,1	0,297	1,178	1,095	0,670	1,077	1,440	0,854	1,192		
28	3-8	14	Ramdane	98	1,300	60,0	0,341	1,176	1,098	0,552	1,227	1,339	1,036	1,206		
29	3-9	16	Ramdane	98	1,300	97,2	0,362	1,017	0,967	0,362	1,360	1,161	0,923	1,142		
30	3-10	21	Ramdane	98	1,300	41,4	0,286	1,152	1,042	0,671	0,948	1,007	0,995	0,885		
31	3-11	22	Ramdane	98	1,300	83,2	0,405	1,211	1,104	0,473	1,314	0,988	1,409	1,012		
32	3-12	23	Ramdane	100	0,900	55,7	0,341	1,381	1,152	0,499	1,368	1,294	1,229	1,088		
33	3-13	25	Ramdane	100	1,200	32,5	0,244	1,069	0,975	0,613	0,848	1,150	0,976	0,857		
34	3-14	26	Ramdane	100	1,200	37,1	0,294	1,223	1,108	0,646	1,000	1,098	1,018	0,934		
35	3-15	27	Ramdane	102	1,000	33,3	0,227	1,023	0,901	0,608	0,837	0,986	0,835	0,768		
36	4-1	I. HS1	Marz./Hus.	95	0,400	63,7	0,178	1,415	0,865	0,262	1,861	1,231	1,414	0,749		
37	4-2	I. HS2	Marz./Hus.	95	0,700	66,5	0,249	1,318	0,900	0,351	1,583	1,213	1,146	0,864		
38	4-3	I. HS3	Marz./Hus.	95	1,200	65,6	0,356	1,353	1,054	0,509	1,455	1,233	1,126	1,034		
39	4-4	I. HS4	Marz./Hus.	90	2,100	62,7	0,418	1,394	1,175	0,678	1,272	1,133	1,297	1,099		

IV Table of comparison of test data with selected models

(2 of 6)

running number	serial no.	slab no.	author	effective depth	reinforcement ratio (inner section)	compressive uniaxial strength (prism strength)	failure load	Model by Kinnunen/Nylander	Model by Moe	Model by Braestrup/Nielsen et al.	Model by Georgopoulos	Model by Men��trety	Model by Hallgren	Model by St��tler
no.	notat.	n	A	d	ρ_{li}	f_{lc}	V_{exp}	$V_{exp}/V_{cal,u}$						
unit	-	-	-	mm	%	MPa	MN	-						
40	4-5	I. HS7	Marz./Hus.	95	0,900	70,3	0,356	1,573	1,139	0,474	1,846	1,390	1,289	1,128
41	4-6	II. HS5	Marz./Hus.	125	0,500	64,6	0,365	1,320	1,003	0,345	2,131	1,450	1,342	0,932
42	4-7	II. HS6	Marz./Hus.	120	0,500	66,5	0,489	1,922	1,422	0,479	3,049	1,988	1,950	1,326
43	4-8	II. HS8	Marz./Hus.	120	1,000	65,6	0,436	1,047	0,975	0,433	1,463	1,201	0,968	0,940
44	4-9	II. HS9	Marz./Hus.	120	1,500	70,3	0,543	1,006	1,061	0,503	1,339	1,139	1,027	1,014
45	4-10	II. HS10	Marz./Hus.	120	2,100	76,0	0,645	0,986	1,139	0,553	1,260	1,025	1,089	1,066
46	4-11	III. HS11	Marz./Hus.	70	0,700	66,5	0,196	1,909	1,174	0,440	1,879	1,576	1,666	1,061
47	4-12	III. HS12	Marz./Hus.	70	1,200	71,3	0,258	1,760	1,187	0,541	1,627	1,329	1,342	1,156
48	4-13	III. HS13	Marz./Hus.	70	1,600	64,6	0,267	1,608	1,147	0,617	1,375	1,214	1,349	1,102
49	4-14	IV. HS14	Marz./Hus.	95	1,200	68,4	0,498	1,426	1,223	0,569	1,617	1,500	1,271	1,200
50	4-15	IV. HS15	Marz./Hus.	95	1,200	67,5	0,560	1,300	1,267	0,553	1,467	1,482	1,300	1,130
51	4-16	I. NS1	Marz./Hus.	95	1,200	39,9	0,320	1,503	1,137	0,751	1,295	1,365	1,133	1,024
52	4-17	II. NS2	Marz./Hus.	120	0,500	28,5	0,396	1,861	1,483	0,905	2,069	2,341	1,596	1,320
53	5-1	F1	Lov./McL.	83	1,740	37,3	0,480	1,929	3,020	-	2,406	2,212	1,030	1,688
54	5-2	F2	Lov./McL.	83	1,740	37,3	0,204	0,882	1,284	-	1,023	0,940	0,644	0,778
55	5-3	F3	Lov./McL.	83	1,740	37,3	0,149	0,706	0,937	-	0,747	0,687	0,605	0,602
56	5-4	F4	Lov./McL.	83	1,740	37,3	0,129	0,684	0,812	-	0,647	0,595	0,615	0,545
57	5-5	F5	Lov./McL.	83	1,740	37,3	0,139	0,901	0,875	-	0,697	0,641	0,886	0,628
58	6-1	S2.1	Tolf	200	0,800	19,4	0,603	0,922	0,916	-	0,752	1,160	0,754	0,815
59	6-2	S2.2	Tolf	199	0,800	18,9	0,600	0,935	0,926	-	0,752	1,168	0,768	0,821
60	6-3	S2.3	Tolf	200	0,450	19,3	0,489	0,988	0,825	-	0,775	1,328	0,774	0,798
61	6-4	S2.4	Tolf	197	0,450	21,6	0,444	0,896	0,733	-	0,719	1,138	0,713	0,713
62	6-5	S1.1	Tolf	100	0,800	30,1	0,216	1,142	1,072	-	0,972	1,264	0,895	0,850
63	6-6	S1.2	Tolf	99	0,800	23,8	0,194	1,137	1,081	-	0,918	1,305	0,851	0,839
64	6-7	S1.3	Tolf	98	0,400	26,0	0,145	1,180	0,907	-	1,047	1,417	0,850	0,773
65	6-8	S1.4	Tolf	99	0,400	25,5	0,148	1,186	0,922	-	1,059	1,453	0,843	0,784
66	7-1	I/1	Regan	77	1,002	25,4	0,194	1,620	1,024	0,856	0,870	0,951	1,299	0,981
67	7-2	I/2	Regan	77	1,200	23,1	0,176	1,411	0,910	0,854	0,885	1,450	1,115	0,865
68	7-3	I/3	Regan	77	0,701	27,1	0,194	1,899	1,153	0,804	0,879	1,376	1,435	1,083
69	7-4	I/4	Regan	77	1,200	31,9	0,194	1,373	0,889	0,682	0,913	1,340	1,193	0,861
70	7-5	I/5	Regan	79	0,630	27,8	0,165	1,642	0,996	0,641	0,708	1,066	1,258	0,927
71	7-6	I/6	Regan	79	0,800	21,6	0,165	1,530	0,971	0,823	1,001	1,769	1,163	0,929
72	7-7	I/7	Regan	79	0,800	30,0	0,186	1,572	0,986	0,669	1,093	1,659	1,222	0,942
73	7-8	II/1	Regan	200	1,000	34,4	0,825	0,948	0,829	0,544	0,906	1,223	1,064	0,857
74	7-9	II/2	Regan	128	1,000	32,9	0,390	1,131	0,995	0,679	1,108	1,309	0,988	0,911
75	7-10	II/3	Regan	128	1,000	33,9	0,365	1,046	0,920	0,616	1,036	1,451	1,055	0,893
76	7-11	II/4	Regan	64	1,000	32,9	0,117	1,360	1,197	0,816	1,338	1,307	0,934	0,873
77	7-12	II/5	Regan	64	1,000	33,9	0,105	1,206	1,061	0,710	1,200	1,391	0,890	0,828
78	7-13	II/6	Regan	64	1,000	35,7	0,105	1,182	1,040	0,674	1,199	1,629	0,832	0,845

IV Table of comparison of test data with selected models

(3 of 6)

running number	serial no.	slab no.	author	effective depth	reinforcement ratio (inner section)	compressive uniaxial strength (prism strength)	failure load	Model by <i>Kinnunen/Nylander</i>	Model by <i>Moe</i>	Model by <i>Braestrup/Nielsen et al.</i>	Model by <i>Georgopoulos</i>	Model by <i>Men��trety</i>	Model by <i>Hallgren</i>	Model by <i>Staller</i>
no.	notat.	n	A	d	$\rho_{l,i}$	f_{lc}	V_{exp}	$V_{exp}/V_{cal,u}$						
unit	-	-	-	mm	%	MPa	MN	-						
79	7-14	III/1	Regan	95	0,800	22,9	0,197	1,288	0,963	0,805	1,051	1,456	0,863	0,838
80	7-15	III/2	Regan	95	0,800	9,4	0,123	1,040	0,868	1,225	0,914	1,935	0,614	0,764
81	7-16	III/3	Regan	95	0,800	37,4	0,214	1,169	0,877	0,536	1,128	1,213	0,867	0,780
82	7-17	III/4	Regan	93	1,500	11,8	0,154	0,987	0,930	1,265	1,016	1,299	0,663	0,724
83	7-18	III/5	Regan	93	1,500	26,5	0,214	1,049	0,890	0,782	0,865	1,044	0,784	0,742
84	7-19	III/6	Regan	93	1,500	42,1	0,248	1,009	0,853	0,570	0,933	0,950	0,775	0,746
85	7-20	V/1	Regan	118	0,800	33,9	0,170	0,877	1,073	-	0,780	0,849	0,756	0,564
86	7-21	V/2	Regan	118	0,800	31,8	0,280	0,982	0,774	-	0,858	1,179	0,748	0,694
87	7-22	V/3	Regan	118	0,800	32,0	0,265	1,041	0,975	-	0,979	1,188	0,875	0,762
88	7-23	V/4	Regan	118	0,800	35,8	0,285	1,106	1,057	-	1,083	1,217	0,953	0,809
89	8-1	S1	Swamy/Ali	100	0,600	40,1	0,198	1,293	0,838	0,427	1,281	1,451	1,044	0,819
90	8-2	S7	Swamy/Ali	100	0,600	37,4	0,222	1,476	0,960	0,513	1,260	1,592	1,171	0,938
91	9-1	P2	ETH	143	1,500	35,4	0,628	1,307	0,930	0,719	0,866	1,121	1,284	1,024
92	9-2	P5	ETH	171	1,200	27,1	0,626	1,033	0,863	0,709	0,802	1,189	1,009	0,905
93	10-1	0	Schaeffers	113	0,800	21,9	0,280	1,351	1,075	0,875	1,142	1,372	0,954	0,996
94	10-2	3	Schaeffers	170	0,600	22,1	0,460	0,993	1,054	-	1,163	1,251	0,890	0,857
95	11-1	DA 6	Ladner	80	1,800	29,6	0,183	1,045	1,104	0,905	1,037	1,084	0,854	0,772
96	11-2	DA 7	Ladner	80	1,800	33,1	0,288	1,035	1,070	0,919	1,099	1,313	0,846	0,854
97	11-3	DA 10	Ladner	80	1,800	31,6	0,281	0,867	0,982	0,842	0,997	1,276	0,729	0,757
98	11-4	DA 11	Ladner	80	1,800	30,0	0,324	0,759	1,060	0,821	0,993	1,239	0,661	0,728
99	12-1	P1	Ladner	240	1,300	27,6	1,662	1,053	1,008	0,867	0,929	1,258	0,981	1,094
100	12-2	M1	Ladner	109	1,200	31,4	0,362	1,100	1,030	0,806	0,999	1,096	0,729	0,892
101	13-1	AN-1	Corl./Hawk.	111	1,500	42,2	0,334	0,782	0,700	0,455	0,787	0,840	0,703	0,678
102	13-2	AN-2	Corl./Hawk.	111	1,000	42,2	0,266	0,903	0,686	0,404	0,875	0,927	0,734	0,669
103	14-1	A1/M1	Base	114	1,100	15,5	0,322	1,276	1,318	1,278	1,509	1,940	1,174	1,190
104	14-2	A1/M2	Base	117	1,500	14,7	0,346	1,066	1,238	1,387	1,287	1,704	0,835	1,097
105	14-3	A1/M3	Base	121	1,900	13,5	0,307	0,834	1,037	1,274	1,136	1,344	0,623	0,891
106	14-4	A1/M4	Base	124	1,000	13,3	0,259	0,877	0,886	1,048	0,932	1,513	0,608	0,785
107	14-5	A1/M5	Base	117	1,200	20,0	0,346	1,059	1,042	1,024	1,097	1,456	0,795	0,911
108	14-6	A2/M1	Base	124	1,000	33,6	0,409	1,343	1,186	0,655	1,831	1,518	1,364	1,052
109	14-7	A2/M2	Base	117	1,500	31,2	0,419	1,062	1,125	0,794	1,409	1,321	0,966	1,016
110	14-8	A2/M3	Base	121	1,900	30,9	0,430	0,880	1,029	0,780	1,191	1,142	0,745	0,920
111	14-9	A2/T1	Base	124	1,000	37,3	0,419	0,994	0,961	0,604	1,275	1,309	0,837	0,875
112	14-10	A2/T2	Base	124	1,700	39,3	0,439	0,806	0,854	0,601	0,957	0,949	0,693	0,756
113	14-11	A3/M1	Base	124	1,000	17,9	0,247	0,869	0,874	0,745	1,068	1,304	0,829	0,789
114	14-12	A3/M2	Base	102	1,700	18,3	0,336	1,177	1,304	1,336	1,386	1,578	0,949	1,139
115	14-13	A3/M3	Base	117	1,900	25,9	0,298	0,685	0,801	0,678	0,887	0,911	0,573	0,709
116	14-14	A3/T1	Base	121	1,000	19,6	0,328	1,030	0,991	0,939	1,082	1,488	0,742	0,880

IV Table of comparison of test data with selected models

(4 of 6)

running number	serial no.	slab no.	author	effective depth	reinforcement ratio (inner section)	compressive uniaxial strength (prism strength)	failure load	Model by <i>Kinnunen/Nylander</i>	Model by <i>Moe</i>	Model by <i>Braestrup/Nielsen et al.</i>	Model by <i>Georgopoulos</i>	Model by <i>Menétrey</i>	Model by <i>Hallgren</i>	Model by <i>Staller</i>
no.	notat.	n	A	d	$\rho_{l,i}$	f_{lc}	V_{exp}	$V_{exp}/V_{col,u}$						
unit	-	-	-	mm	%	MPa	MN	-						
117	14-15	A3/T2	Base	119	1,200	15,2	0,298	0,967	0,983	1,127	1,046	1,484	0,699	0,851
118	14-16	A4/M1	Base	114	1,100	36,4	0,259	0,915	0,806	0,438	1,225	0,959	0,932	0,713
119	14-17	A4/M2	Base	119	1,500	27,7	0,341	0,859	0,931	0,706	1,123	1,130	0,761	0,837
120	14-18	A4/M3	Base	117	1,900	30,6	0,541	1,164	1,319	1,044	1,467	1,464	0,967	1,167
121	14-19	A4/T1	Base	114	1,100	31,2	0,384	1,087	1,034	0,758	1,248	1,361	0,866	0,930
122	14-20	A4/T2	Base	117	1,200	27,8	0,402	1,091	1,060	0,852	1,207	1,397	0,839	0,944
123	15-1	P1-S1	Manterola	107	1,100	24,3	0,216	1,731	1,172	-	1,271	1,342	1,513	1,084
124	15-2	P2-S1	Manterola	107	1,100	32,1	0,257	1,332	0,792	-	0,946	1,028	1,041	0,824
125	15-3	P3-S1	Manterola	107	1,100	28,2	0,301	1,089	0,886	-	0,738	1,040	1,068	0,709
126	15-4	P1-S2	Manterola	107	1,100	23,0	0,196	1,574	1,073	-	1,106	1,242	1,405	0,981
127	15-5	P2-S2	Manterola	107	1,100	31,4	0,283	1,449	0,853	-	0,999	1,128	1,144	0,894
128	15-6	P3-S2	Manterola	107	1,100	30,3	0,397	1,371	1,108	-	0,932	1,302	1,326	0,895
129	15-7	P1-S3	Manterola	107	1,100	37,7	0,184	1,280	0,829	-	1,066	0,899	1,249	0,787
130	15-8	P2-S3	Manterola	107	1,400	34,0	0,211	1,318	0,937	-	1,014	0,907	1,368	0,859
131	15-9	P3-S3	Manterola	107	0,500	37,2	0,165	1,870	0,975	-	1,911	1,284	1,526	0,921
132	15-10	P1-S4	Manterola	107	0,500	25,1	0,175	2,026	1,041	-	1,385	1,480	1,591	0,991
133	15-11	P2-S4	Manterola	107	0,500	29,7	0,246	1,855	0,940	-	1,197	1,490	1,414	0,921
134	15-12	P3-S4	Manterola	107	0,500	32,5	0,294	1,479	1,073	-	0,935	1,389	1,509	0,756
135	16-1	II-5	Yitzhaki	82	0,500	16,0	0,152	1,407	1,185	1,042	1,204	2,041	1,241	1,049
136	16-2	II-8	Yitzhaki	82	0,600	17,4	0,218	1,338	1,395	1,081	1,201	2,184	1,326	1,120
137	16-3	IIS20-1	Yitzhaki	78	0,700	10,2	0,128	1,464	1,121	-	1,136	2,450	1,077	0,994
138	16-4	II-1	Yitzhaki	82	1,200	9,7	0,181	1,265	1,355	2,037	1,429	2,194	0,890	1,176
139	16-5	II-4a	Yitzhaki	82	0,900	16,6	0,245	1,657	1,465	1,613	1,352	2,261	1,172	1,293
140	16-6	II-4b	Yitzhaki	82	0,900	9,1	0,162	1,135	1,222	1,799	1,407	2,836	0,779	0,995
141	16-7	II-4c	Yitzhaki	82	0,900	12,9	0,215	1,376	1,365	1,687	1,479	2,487	1,031	1,076
142	16-8	IIR20-2	Yitzhaki	83	0,900	13,9	0,307	2,327	2,097	2,487	1,975	3,334	1,576	1,847
143	16-9	IIR30-1	Yitzhaki	80	2,000	16,3	0,239	1,284	1,175	1,426	1,148	1,298	1,160	1,068
144	16-10	II-2	Yitzhaki	82	1,300	9,1	0,152	1,408	1,201	1,839	1,252	2,118	0,904	1,184
145	16-11	II-3	Yitzhaki	82	1,300	12,5	0,244	1,352	1,282	-	1,400	1,983	1,236	1,000
146	16-12	II-6	Yitzhaki	82	1,300	20,1	0,240	1,755	1,278	1,312	1,166	1,590	1,326	1,237
147	16-13	II-9	Yitzhaki	79	0,850	8,6	0,157	1,220	1,264	1,948	1,494	0,502	0,634	1,013
148	16-14	III-3	Yitzhaki	82	1,200	16,8	0,201	1,201	1,126	1,311	1,100	1,450	0,891	0,960
149	16-15	7	Yitzhaki	82	0,700	9,3	0,117	1,393	1,427	1,786	1,287	2,865	0,824	1,175
150	16-16	II-10	Yitzhaki	82	1,000	10,8	0,098	1,260	1,081	1,059	0,976	1,503	0,787	0,929
151	17-1	S1-60	Moe	114	1,100	22,1	0,389	1,295	1,094	0,971	1,137	1,346	0,997	0,945
152	17-2	S2-60	Moe	114	0,531	21,0	0,356	1,854	1,369	0,937	0,914	1,214	1,829	1,122
153	17-3	S3-60	Moe	114	0,627	21,5	0,364	1,692	1,286	0,937	0,909	0,993	1,567	1,077
154	17-4	S4-60	Moe	114	0,742	22,6	0,334	1,376	1,079	0,816	0,806	0,769	1,238	0,918
155	17-5	S1-70	Moe	114	1,100	23,3	0,393	1,310	1,023	0,933	1,038	1,272	0,993	0,881

IV Table of comparison of test data with selected models

(5 of 6)

running number	serial no.	slab no.	author	effective depth	reinforcement ratio (inner section)	compressive uniaxial strength (prism strength)	failure load	Model by Kinnunen/Nylander	Model by Moe	Model by Braestrup/Nielsen et al.	Model by Georgopoulos	Model by Menétrey	Model by Hallgren	Model by Staller
no.	notat.	n	A	d	$\rho_{l,i}$	f_{lc}	V_{exp}	$V_{exp}/V_{cal,u}$						
unit	-	-	-	mm	%	MPa	MN	-						
156	17-6	S3-70	Moe	114	0,627	24,1	0,378	1,617	1,186	0,866	0,874	0,919	1,296	1,010
157	17-7	S4-70	Moe	114	0,742	33,4	0,374	1,322	0,989	0,618	0,696	0,662	1,082	0,853
158	17-8	S4-70A	Moe	114	0,742	19,5	0,312	1,314	0,983	0,885	0,831	0,745	0,945	0,847
159	17-9	S5-60	Moe	114	1,100	21,1	0,343	1,400	1,094	1,000	1,142	1,337	0,990	0,944
160	17-10	S5-70	Moe	114	1,100	21,9	0,378	1,555	1,132	1,064	1,140	1,393	1,069	0,965
161	17-11	R1	Moe	114	1,400	25,3	0,312	1,159	1,083	0,866	1,140	1,527	1,092	1,043
162	17-12	R2	Moe	114	1,400	26,2	0,394	1,107	1,027	0,830	1,098	1,565	1,030	1,035
163	17-13	H1	Moe	114	1,100	24,8	0,372	1,198	1,063	0,814	1,167	1,243	1,069	0,911
164	17-14	M1A	Moe	114	1,500	19,8	0,433	1,083	1,026	1,105	1,028	1,227	0,915	0,838
165	18-1	IA15a-5	Kinn./Nyl.	117	0,800	27,6	0,255	1,123	1,035	-	1,119	1,115	0,885	0,888
166	18-2	IA15a-6	Kinn./Nyl.	118	0,800	25,4	0,275	1,211	1,132	-	1,167	1,240	0,936	0,959
169	18-3	IA15c-11	Kinn./Nyl.	121	0,697	31,0	0,334	1,446	1,286	-	0,862	0,879	1,171	1,114
170	18-4	IA15c-12	Kinn./Nyl.	122	0,678	28,4	0,332	1,463	1,314	-	0,881	0,917	1,171	1,131
171	18-5	IA30a-24	Kinn./Nyl.	128	1,000	25,6	0,430	1,104	0,983	0,807	0,997	1,195	0,821	0,971
172	18-6	IA30a-25	Kinn./Nyl.	124	1,100	24,3	0,408	1,086	0,973	0,846	0,955	1,164	0,803	0,955
175	18-7	IA30c-30	Kinn./Nyl.	120	0,916	29,2	0,491	1,432	1,234	0,893	0,905	0,960	1,167	1,224
176	18-8	IA30c-31	Kinn./Nyl.	119	0,913	29,2	0,540	1,590	1,363	0,995	1,008	1,052	1,277	1,352
177	18-9	IA30d-32	Kinn./Nyl.	123	0,500	25,5	0,258	1,085	0,823	0,516	0,994	1,159	1,019	0,784
178	18-10	IA30d-33	Kinn./Nyl.	125	0,500	25,8	0,258	1,034	0,787	0,497	0,952	1,130	0,957	0,755
179	18-11	IA30e-34	Kinn./Nyl.	120	0,684	26,5	0,332	1,167	0,940	0,663	0,835	1,046	0,990	0,922
180	18-12	IA30e-35	Kinn./Nyl.	122	0,686	24,3	0,332	1,154	0,943	0,707	0,827	1,086	0,960	0,928
181	19-1	I A-1a	Elst./Hog.	118	1,200	11,1	0,303	1,079	1,086	1,427	1,118	1,913	0,815	1,023
182	19-2	I A-1b	Elst./Hog.	118	1,200	19,9	0,365	1,114	1,043	0,962	1,085	1,474	0,918	0,965
183	19-3	I A-1c	Elst./Hog.	118	1,200	22,9	0,356	1,050	0,967	0,815	1,037	1,322	0,882	0,896
184	19-4	I A-1d	Elst./Hog.	118	1,200	29,1	0,351	0,976	0,879	0,633	1,001	1,143	0,863	0,817
185	19-5	I A-1e	Elst./Hog.	118	1,200	16,0	0,356	1,143	1,102	1,164	1,104	1,658	0,920	1,021
186	19-6	I A-2a	Elst./Hog.	114	2,500	10,7	0,334	0,975	1,149	1,718	1,327	1,610	0,893	0,953
187	19-7	I A-2b	Elst./Hog.	114	2,500	15,4	0,400	1,042	1,139	1,435	1,247	1,441	0,927	0,968
188	19-8	I A-2c	Elst./Hog.	114	2,500	29,5	0,467	0,950	0,995	0,873	0,975	1,144	0,873	0,899
189	19-9	I A-7b	Elst./Hog.	114	2,500	22,0	0,512	1,172	1,236	1,284	1,258	1,474	1,067	1,085
190	19-10	I A-3a	Elst./Hog.	114	3,700	10,1	0,356	0,930	1,326	1,945	1,470	1,286	0,901	0,921
191	19-11	I A-3b	Elst./Hog.	114	3,700	17,9	0,445	0,960	1,126	1,377	1,255	1,072	0,917	0,892
192	19-12	I A-3c	Elst./Hog.	114	3,700	20,9	0,534	1,084	1,244	1,409	1,358	1,172	1,050	1,011
193	19-13	I A-3d	Elst./Hog.	114	3,700	27,3	0,547	0,995	1,123	1,109	1,164	1,040	0,990	0,948
194	19-14	II A-4	Elst./Hog.	118	1,200	20,6	0,400	0,951	1,004	0,858	0,962	1,401	0,898	0,864
195	19-15	II A-5	Elst./Hog.	114	2,500	22,0	0,534	0,907	1,109	1,132	1,075	1,253	0,862	0,934
197	19-16	V A-9	Elst./Hog.	114	6,900	23,6	0,445	0,704	0,944	1,041	1,044	0,604	0,983	0,657
198	19-17	V A-10	Elst./Hog.	114	6,900	23,5	0,489	0,579	0,871	0,970	0,942	0,557	0,897	0,596
199	19-18	VII A-13	Elst./Hog.	121	0,500	20,7	0,236	1,260	0,953	0,486	1,022	1,326	1,318	0,687

IV Table of comparison of test data with selected models

(6 of 6)

running number	serial no.	slab no.	author	effective depth	reinforcement ratio (inner section)	compressive uniaxial strength (prism strength)	failure load	Model by <i>Kinnunen/Nylander</i>	Model by <i>Moe</i>	Model by <i>Braestrup/Nielsen et al.</i>	Model by <i>Georgopoulos</i>	Model by <i>Menétrey</i>	Model by <i>Hallgren</i>	Model by <i>Staller</i>
no.	notat.	n	A	<i>d</i>	ρ_{ti}	f_{ic}	V_{exp}	$V_{exp}/V_{cal,u}$						
unit	-	-	-	mm	%	MPa	MN	-						
200	19-19	VIII B-1	Elst./Hog.	114	0,500	11,2	0,178	1,158	0,928	0,877	0,974	1,744	1,134	0,786
201	19-20	VIII B-2	Elst./Hog.	114	0,500	37,6	0,200	1,199	0,788	0,294	1,209	0,876	1,262	0,576
202	19-21	VIII B-4	Elst./Hog.	114	0,900	37,7	0,334	1,221	0,970	0,490	1,308	1,079	1,254	0,806
203	19-22	VIII B-9	Elst./Hog.	114	2,000	34,7	0,505	1,049	1,052	0,805	1,099	1,056	0,905	0,920
204	19-23	VIII B-11	Elst./Hog.	114	3,000	10,7	0,329	0,905	1,176	1,705	1,308	1,163	0,829	0,759
205	19-24	VIII B-14	Elst./Hog.	114	3,000	39,9	0,578	0,960	1,042	0,801	1,033	0,951	0,881	0,897

V List of notations for "Collected Data"

	notation	unit	description
geometry of slab and column	c	mm	total diameter of circular slab
	c_1	mm	diameter of circle along the load was distributed
	h	mm	total height of slab
	c_2	mm	total diameter of circular column stub
	a	mm	side length of quadratic slab
	a_1	mm	span of quadratic slab
	a_2	mm	side length of quadratic column stub
layout of reinforcement	d	mm	effective depth
	d_s	mm	diameter of reinforcing steel
	s_x / s_y	mm	layout of reinforcement (x- / y-direction)
	s_{x2} / s_{y2}	mm	layout of compressive reinforcement
	ρ_r / ρ_t	%	reinforcement ratio (radial- / tangential-direction)
material properties of concrete	$f_{c,cyl}$	MPa	compressive cylinder strength
	$f_{c,cube}$	MPa	compressive cube strength
	$f_{c,prism}$	MPa	compressive prism strength
	$f_{ct,sp}$	MPa	tensile splitting strength
	$f_{ct,ax}$	MPa	tensile axial strength
	$f_{ct,fl}$	MPa	tensile flexural strength
	E_c	GPa	modulus of elasticity
	G_f	N/m	fracture energy
	d_a	mm	maximum aggregate size
material properties of reinforcing steel	f_y	MPa	yield stress of reinforcing steel
	f_t	MPa	tensile strength of reinforcing steel
	E_s	GPa	modulus of elasticity of reinforcing steel
failure	P_u	MN	failure load
	$P / FP / DP$	-	type of failure (P = Brittle Punching; FP = Flexural Punching; DP = Ductile Punching)

	notation	unit	description
shape of the shear reinforcement	<i>SE or SB</i>	-	bent up bars
	<i>HA or HO</i>	-	hooks
	<i>SZ or SL</i>	-	shear assemblies like shear ladders
	<i>BÜ or ST</i>	-	stirrups
distribution of the shear reinforcement	<i>HA</i>	-	placed only in the orthogonal direction
	<i>ST</i>	-	placed in a star shape
	<i>U</i>	-	placed around the column face in one ore more layers
place of the major shear crack	<i>PC or Pc</i>	-	failure at the column face
	<i>PO or Po</i>	-	failure outside of the shear reinforced area
	<i>PS or PS</i>	-	failure within the shear reinforcement
	<i>PBS or Pbs</i>	-	failure between the shear reinforcement rows

V List of notations for "Input data for calculation"

	notation	unit	description
geometry of slab	BC	-	boundary condition k = supported with circular load or circular restraint boundary geometry; q = quadratic flat slabs with up lifting corners
	c_1	mm	diameter of circle along the load was distributed
	a_1	mm	span of quadratic slab
	h	mm	total height of slab
geometry of column	SC	-	shape of column k = circular; q = quadratic
	c_2	mm	total diameter of circular column stub
	a_2	mm	side length of quadratic column stub
reinforcement - dimensions	d	mm	effective depth
	d_s	mm	diameter of reinforcing steel
	d_{s2}	mm	diameter of compression reinforcing steel
	$\rho_{l,i} / \rho_{l,a}$	-	longitudinal reinforcement ratio (inner / outer section)
	$\rho_{l2,i} / \rho_{l2,a}$	-	compression reinforcement ratio (inner / outer section)
material properties of concrete	f_{lc}	MPa	compressive uniaxial strength (prism strength)
	f_{lct}	MPa	tensile uniaxial strength
	E_c	GPa	modulus of elasticity
	d_a	mm	maximum aggregate size
material properties of reinforcing steel	f_y	MPa	yield stress of reinforcing steel
	f_t	MPa	tensile strength of reinforcing steel
	E_s	GPa	modulus of elasticity of reinforcing steel
failure	P_u	MN	failure load
	P_u / P_{flex}	MN	failure load over flexure load (plasticity)
	$f_{lc} - 12$	MPa	lower bound of concrete strength