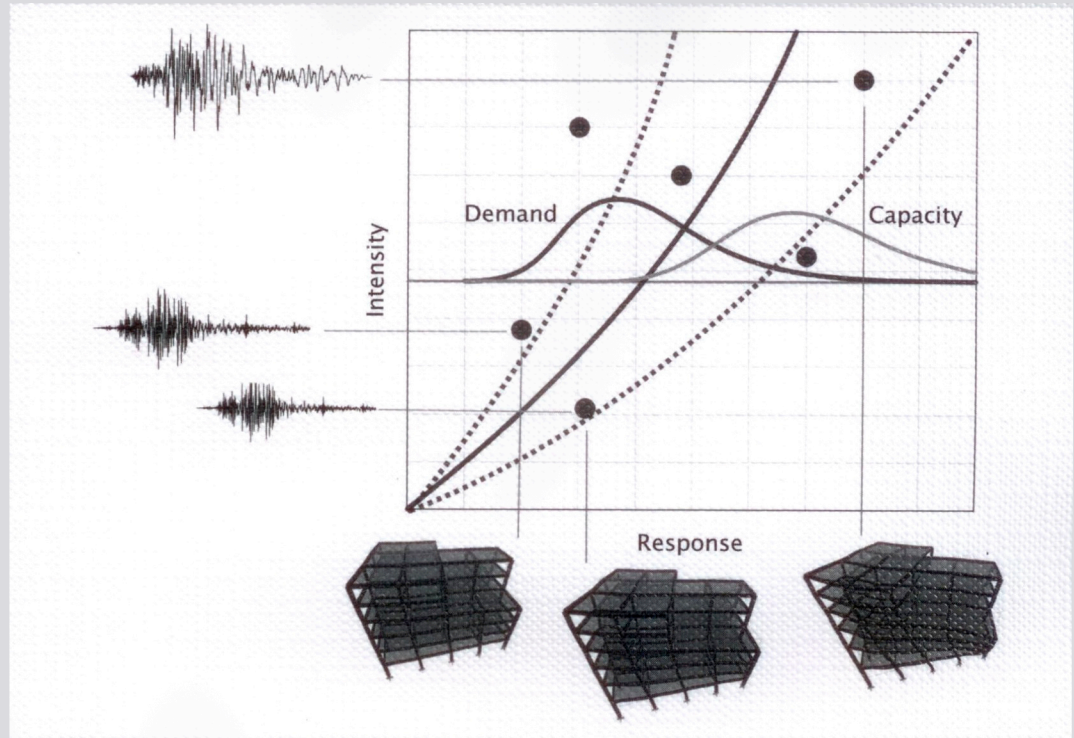




Probabilistic performance-based seismic design



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Technical Report prepared by
Task Group 7.7

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Preface

The now universally adopted design approach denominated as “Performance-based” is nothing but the formalization of the innate concept that buildings are made to satisfy a number of requisites related to their use (strength, durability, etc): scientific progress has only made the attainment of the objectives more efficient and more reliable, by translating the best available knowledge in the form of codified rules and procedures.

Restricting the attention to Performance-Based Design (PBD) against seismic actions, it is well known that current advanced formulations specify the requirements in terms of a number of so-called performance levels that must not be exceeded under seismic actions characterized in terms of mean return periods. The latter are the only quantities derived through probabilistic considerations.

Though existing procedures are intelligently conceived and well tested, they cannot be proved to ensure compliance with the stated performance objectives, as in fact they have not in a number of disastrous seismic events. The intrinsic inability of current procedures to provide a measure of compliance with the requirements of a given design becomes a particularly serious limitation when assessing existing buildings and when, as it is increasingly often the case, the requirements are formulated in terms that go beyond purely structural response, to include damage to non-structural components as well as repair costs.

In both cases, determination of performance involves consideration of several additional uncertain data, which makes recourse to a probabilistic approach unavoidable.

After a rather long history of partial progress, the last decade has seen reliability methods for seismic design become effective tools that can be used in practice with an acceptable amount of additional effort and competence. Mandatory adoption of Probabilistic-PBD¹ codes may still be quite far away: this time lag, however, should be regarded as an opportunity to familiarize with the approaches before actual application.

This is exactly the motivation that led to the decision to prepare this bulletin. Material on Probabilistic-PBD is dispersed in a myriad of journal papers, and comprehensive publications are yet to be written, a situation that disorients the potentially interested reader.

The authors of this bulletin have been active in the development of the approaches and hence have a clear picture of the present state of the art, they know the differences between the approaches and their respective advantages and limitations. They have tried to clearly distinguish and categorize the various classes of existing proposals, to explain them in terms aimed at non-specialized readers, and completed them with detailed realistic illustrative examples.

This bulletin is therefore neither a state of-the-art, nor a compendium of research results: its ambition is to provide an organic, educational text, readable with only a limited background knowledge on probability theory by structural engineers willing to raise their professional level, with the double aim of a greater understanding of the limitations in the current codes and of being prepared to apply more rigorous methods when they are needed for specific projects.

Paolo E. Pinto

Chair of *fib* Commission 7, *Seismic design*

Convener of *fib* Task Group 7.7, *Probabilistic performance-based seismic design*

¹ The attribute “probabilistic” is attached to PBD only here at the beginning of the document to highlight the difference with current PBD procedures, whereby consideration of uncertainty is not explicit and partial. In the remainder of the bulletin, PBD is to be understood as probabilistic.

Contents

1	Introduction	1
1.1	Historical development	1
1.1.1	Evolution of reliability concepts and theory in structural design	1
1.1.2	Introduction of reliability concepts in seismic design	4
1.2	On the definition of performance	5
1.3	Type and nature of models in performance-based engineering	7
1.3.1	Hazard models	8
1.3.2	Response models	9
1.3.3	Performance models	10
	References	11
2	Probabilistic seismic assessment	13
2.1	Introduction	13
2.2	Conditional probability approach (IM-based methods)	13
2.2.1	PEER formulation	15
2.2.1.1	Summary	15
2.2.1.2	Introduction	15
2.2.1.3	Formulation	17
2.2.1.4	Application of PEER formulation	24
2.2.1.5	Closure	34
2.2.2	SAC/FEMA formulation	35
2.2.2.1	Motivation for the SAC/FEMA method	36
2.2.2.2	The formal aspects of the SAC/FEMA method and its limitations	36
2.2.2.3	MAF format	39
2.2.2.4	DCFD methodology	41
2.2.2.5	Theoretical background on SAC/FEMA assumptions	43
2.2.2.6	Illustrative assessment example of the DCFD methodology	45
2.2.2.7	Illustrative assessment example of the MAF methodology	47
2.2.2.8	Future applications and concluding remarks	49
2.3	Unconditional probabilistic approach	49
2.3.1	Introduction	49
2.3.2	Simulation methods	50
2.3.2.1	Monte Carlo simulation methods	50
2.3.2.2	Application to the estimation of a structural MAF	51
2.3.2.3	Importance sampling with K-means clustering	52
2.3.3	Synthetic ground motion models	54
2.3.3.1	Seismologically-based models	54
2.3.3.2	Empirical models	57
2.3.4	Flow-chart of a seismic assessment by complete simulation	61
2.3.5	Example	63
2.3.5.1	Illustration of MCS, ISS and IS-K methods	63
2.3.5.2	Comparison with the IM-based approach	66
	References	70
3	Probabilistic seismic design	73
3.1	Introduction	73
3.2	Optimization-based methods	73
3.2.1	Terminology	74
3.2.2	Tools for solving optimization problems	75
3.2.3	A review of structural optimization studies	77
3.2.4	Illustrative example	79
3.3	Non-optimization-based methods	84

3.3.1	Introduction	84
3.3.2	Performance-based seismic design with analytical gradients	85
3.3.2.1	Gradients	85
3.3.2.2	Iterative search for a feasible solution	86
3.3.2.3	Design of reinforcement	87
3.3.3	Illustrative example	87
3.3.3.1	Design	87
3.3.3.2	Validation	89
	References	90
4	Appendix	93
4.1	Excerpts from MATLAB script for PEER PBEE calculations	93
4.2	Excerpts from MATLAB script for unconditional simulation calculations	100
4.3	Excerpts from MATLAB script for TS algorithm calculations	108

List of acronyms and symbols

C	= Generalized capacity in terms of EDP
CDF	= Cumulative Distribution Function
CCDF	= Complementary Cumulative Distribution Function
COV	= Coefficient of variation
D	= Generalized demand variable (demand in terms of EDP)
DCFD	= Demand and Capacity Factored Design
DM	= Damage Measure
DS	= Damage State; Directional Simulation
DV	= Decision Variable
EE	= Earthquake Engineering
EDP	= Engineering Demand Parameter
EDP_C	= Generalized EDP capacity
$\hat{EDP}_C$	= Median EDP capacity
$\hat{EDP}_{P_o}$	= Median EDP demand evaluated at probability level P_o
$f(x)$	= Probability of X (continuous) being in the neighborhood of x (PDF)
$f(x y)$	= Probability of X (continuous) being in the neighborhood of x given $Y = y$
$F(x)$	= Probability of non-exceedance of x (CDF)
$F(x y)$	= Conditional probability of non-exceedance of x given $Y = y$
FC	= Factored Capacity under total dispersion
FC_R	= Factored Capacity under aleatory variability
FD_{P_o}	= Factored Demand under total dispersion evaluated at probability level P_o
FD_{RP_o}	= Factored Demand under aleatory variability evaluated at probability level P_o
$G(x)$	= Probability of exceedance of x (CCDF)
$G(x y)$	= Conditional probability of exceedance of x given $Y = y$
$H(IM)$	= Hazard curve for IM, the MAF of exceedance of the IM
IM	= Intensity Measure
$\hat{IM}_C$	= Median capacity in terms of IM, or median IM capacity
IS	= Importance Sampling
K_x	= Standard normal variate
LS	= Limit state
LRFD	= Load and Resistance Factor Design
M	= Event magnitude
MAF	= Mean Annual Frequency
MC	= Monte Carlo
MIDR	= Maximum interstorey drift ratio
$P[A]$	= Probability of event A
$p(x)$	= Probability of $X = x$ (PMF)
$P(x)$	= Probability of exceedance of x (CCDF)
$P(x y)$	= Conditional probability of exceedance of value $X = x$ given $Y = y$
p_f	= Probability of failure
P_{GC}	= Probability of global collapse occurring
P_{PL}	= Probability of performance level violation
PBEE	= Performance Based Earthquake Engineering
PBSD	= Performance Based Seismic Design
PDF	= Probability Density Function
PGA	= Peak Ground Acceleration
PL	= Performance Level
PMF	= Probability Mass Function

POE	=	Probability of exceedance
PRA	=	Peak roof acceleration
R	=	Site distance from seismic fault
$S_a(T)$	=	Pseudo-spectral acceleration at period T and 5% damping
S_{aPo}	=	Value of S_a corresponding to a probability level P_o
$S_a(T_1)$	=	Spectral acceleration corresponding to period of first mode T_1
α	=	Confidence level
β	=	Standard dev. of the log of the data (often referred to simply as dispersion)
β_T	=	Total dispersion in EDP demand and capacity
β_{TU}	=	Dispersion in EDP demand and capacity due to epistemic uncertainty
β_{DT}	=	Total dispersion of EDP demand
β_{CT}	=	Total dispersion of EDP capacity
β_{DU}	=	Dispersion of EDP demand due to epistemic uncertainty
β_{CU}	=	Dispersion of EDP capacity due to epistemic uncertainty
β_{DR}	=	Dispersion of EDP demand due to aleatory variability
β_{CR}	=	Dispersion of EDP capacity due to aleatory variability
γ	=	“Safety” factor for demand under total dispersion (in the DCFD format)
γ_R	=	“Safety” factor for demand under aleatory variability (in the DCFD format)
δ	=	Coefficient of Variation
ε	=	A measure of dispersion of IM values generated by events of a given magnitude M and at a given distance R (model error in the attenuation law)
$\theta_{\max}$	=	Maximum interstorey drift ratio, over time and over all stories (MIDR)
$\theta_{\max,50}$	=	Median $\theta_{\max}$ evaluated at a given value of intensity IM.
$\theta_{\max,84}$	=	84 th percentile of $\theta_{\max}$ evaluated at a given value of intensity IM.
$\lambda(x)$	=	MAF of exceedance of the value x
λ_{PL}	=	Mean annual frequency of PL violation
λ_{LS}	=	Mean annual frequency of LS violation
μ	=	Mean
σ	=	Standard deviation
φ	=	“Safety” factor for capacity under total dispersion (in the DCFD format)
φ_R	=	“Safety” factor for capacity under aleatory variability (in the DCFD format)
$\cap$	=	Intersection
$\cup$	=	Union

1 Introduction

1.1 Historical development

1.1.1 Evolution of reliability concepts and theory in structural design

Starting from the second half of the last century Earthquake Engineering (EE) has progressively transformed itself from a sectorial field of engineering into a multidisciplinary area encompassing geophysics, geotechnics, structural engineering and, to an increasing extent, social sciences. This transformation has been the consequence of the scientific progress occurring naturally in all of the mentioned disciplines, with a pace, however, accelerated by the increasing relevance of the protection from the seismic threat in a world more densely populated and whose economy relies more and more on industrial production.

This brief section concentrates on one aspect of the progress made by the scientific community, namely the efforts towards an explicit probabilistic treatment of the whole process of seismic design, and on the relation between this scientific progress and the advance in seismic design codes.

It is well recognized that many areas in EE besides the reliability aspect are still today in need of substantial progress, one example for all being models for the behaviour of elements subjected to large inelastic deformation demands. Advances in these areas, however, are equally needed for deterministic as for probabilistic approaches to assessment or design, hence their consideration needs not and will not be included in the rest of this chapter.

A reliability approach to seismic design has appeared as the natural one since the very early age of EE. If we conventionally set the birth of modern EE at the First WCEE in 1956 in San Francisco, we discover already there a paper titled “*Some Applications of Probability Theory in Aseismic Design*”, by Emilio Rosenblueth (Rosenblueth, 1956), one of the future main fathers of EE. It may be superfluous to recall that the state of seismic codes at that time was rather primitive: codes had no explicit link with the physics of the phenomenon: essentially, they consisted on the prescription of static horizontal forces whose magnitude was based on “*tradition*”.

In the following years, a steady flow of probability-related studies emanated from research centers mainly in the US and in Japan, establishing results that have been later introduced in the codes and are now taken for granted as if they were original principles. To name but a few: the probabilistic definition of the hazard, the uniform hazard spectrum, the rules of modal combination, the spectrum-compatible accelerograms, etc. To provide an impression of the variety and of the increasing progress taking place in the period up to the late seventies references (Kanai 1957, Rosenblueth 1964, Amin and Ang 1966, Cornell 1968, Ruiz and Penzien 1969, Vanmarke *et al.* 1973, Pinto *et al.* 1979, Der Kiureghian 1981) are arbitrarily chosen among the myriad of research papers available in the literature.

In the face of such a progress, however, and in spite of the lessons that could have been learned from a number of destructive earthquakes such as the Alaskan one in 1964 and San Fernando in 1971, the international situation of seismic codes appears to have been rather static.

A notable exception is represented by New Zealand (Park and Paulay 1975, NZS 1976), where from the early 70's many of the definitive concepts now incorporated in all world codes were established: the force–reduction factor as function of the system overall ductility, the detailing for achieving component ductility, the ductility classes and, above all, the “capacity design” procedure.

For the US, the San Fernando earthquake of 1971 is generally considered to have provided a decisive stimulus for the improvement of existing seismic codes. Two activities are worth of special mention for the reach of their impact. The National Earthquake Hazard Reduction

Program (NEHRP), involving a large part of the building industry, that led to the so-called “*NEHRP Recommended Provisions for the Development of Seismic Regulations for New Buildings*”, a document first appeared in 1985 (BSSC, 1985), and periodically updated, and the ATC Document 3-06 (ATC, 1978, titled: “*Tentative Provisions for the Development of Seismic Regulation for Buildings*”. As will be seen in the following, the latter document has had an important influence internationally.

In Europe, the road to reliability-based design took a different path. It is widely known that the introduction in design codes of explicit performance requirements, formulated in terms of non-exceedance of a number of limit-states, and implemented through the use of characteristic (i.e. fractile) values for both loads and resistance variables, each one affected by appropriate factors, originates from the ideas of French and German engineers during the years of reconstruction after WWII.

In 1953, under the sponsorship of French contractors, the Comité Européen du Béton (CEB) was founded, with a board of six prominent designers and professors of continental Europe, whose mandate in the English translation reads as: “[...] *creating and orchestrating the international principles for the conception, calculation, construction and maintenance of concrete structures. Establishing codes, standards or other regulatory documents on an international unified basis progressively, through successive stages.*”

A first document titled: “CEB International Recommendations” was issued in 1964, and translated into fifteen languages, followed in 1970 by a second edition titled “CEB-FIP International Recommendations”, which also included provisions for prestressed.

The partial factors initially adopted in the previous documents were essentially of empirical origin, calibrated so as to produce designs comparable with those obtained from the old admissible stresses design. The need was clearly recognized of providing these values of firmer reliability bases. This task was assigned to a newly formed Committee (1971), called “Inter-Association Joint Committee on Structural Safety” (JCSS), sponsored by the following six international associations: CEB, Convention Européenne de la Construction Métallique (CECM), Conseil International du Bâtiment (CIB), Fédération Internationale de la Précontrainte (FIP), Association Internationale des Ponts et Chaussées (AIPC, also referred to as International Association of Bridge and Structural Engineering, IABSE), and Reunion Internationale des Laboratoires et Experts des Matériaux (RILEM), which included highly qualified experts and researchers in the field of structural reliability.

The result of the work of JCSS is reported in the document “CEB-FIP Model Code for Concrete Structures” (CEB-FIP, 1978), that can be seen as the third edition of the above mentioned documents dated 1964 and 1970, but having a much broader scope and official support from the countries of the European Community.

Volume I of (CEB-FIP, 1978) is the direct work of JCSS and is titled: “Common Unified Rules for Different Types of Construction and Material”. Excerpts from the initial part of this volume are reproduced below.

AIMS OF DESIGN

The aim of design is the achievement of acceptable probabilities that the structure being designed will not become unfit for the use for which it is required during some reference period and having regard to its intended life

DESIGN REQUIREMENTS

The criteria relating to the performance of the structure should be clearly defined; this is most conveniently treated in terms of limit states which, in turn, are related to the structure ceasing to fulfill the function, or to satisfy the condition for which it was designed. Etc.

LEVELS OF LIMIT STATE DESIGN

Level 1: a semi probabilistic process in which the probabilistic aspects are treated specifically in defining the characteristic values of loads or actions and strengths of materials, and these are

then associated with partial factors, the values of which, although stated explicitly, should be derived, whenever possible, from a consideration of probability aspects.

Level 2: a design process in which the loads and the strengths of materials are represented by their known, or postulated distributions, and some reliability level is accepted. It is thus a probabilistic design process. (in the Commentary : Level 2 should be used principally in assessing appropriate values for the partial safety factors in Level 1)

Level 3: a design process based upon “exact” probabilistic analysis for the entire structural system, using a full distributional approach, with safety levels based on some stated failure probability interpreted in the sense of relative frequency.

Though (CEB-FIP, 1978) was actually a Level 1 code, as is the Model Code 2010 (*fib*, 2012), it must be recognized that it was already founded on a rather modern reliability framework.

However, likely due to the fact that the large majority of the then small number of members of the European Community were almost immune to earthquake risk, the seismic action and the measures to counter it were not included in any of the CEB documents.

The situation changed abruptly in 1978, when the Economic Commission for Europe asked for a draft of a seismic model code that could be applied to different types of material and construction (Economic Commission for Europe, 1978). CEB responded to the request by setting up a panel of 19 members covering Europe, Argentina, Canada, New Zealand, US and Japan, with the remit of producing a document complying with the format of the “Common Unified Rules.” of 1978 (i.e. partial factors), while drawing the operative rules from the most recent available seismic codes, such as, *in primis*, the ATC 3-06, as well as the Australian, the Canadian and New Zealand ones.

A first draft of what would be finally called the *CEB Seismic Design Model Code* was presented in March of 1980 (CEB, 1980), while the printed volume accompanied by application examples was finalized in 1985.

The final act of the story of seismic codes in Europe consists of the advent of the Eurocodes, specifically of *Eurocode 8: Design of Structures for Earthquake Resistance*, whose Part 1 contains General rules, seismic actions and rules for buildings.

Officially started in 1990, the work was completed in 1994 (CEN, 2004). It was approved by the European Committee for Standardization (Comité Européen de Normalisation, CEN) which includes 28 countries (from the EU and EFTA), and it is meant to supplant, together with the other 60 parts comprising the whole Eurocode program, existing national codes.

The philosophy and the structure of EC8 is the same as that of the CEB Code, though the document is considerably more detailed and articulated. It can be said to be performance-based, since, as a principle, the fundamental requirements of no-collapse and damage limitation must be satisfied “*with an adequate degree of reliability*”, with target reliability values to be “*established by the National Authorities for different types of buildings or civil engineering works on the basis of the consequences of failure*”.

Actually, the reliability aspect is explicitly dealt with through the choice of the return period of the design seismic action only, hence at the end of the design process there is no way of evaluating the reliability actually achieved. One can only safely state that the adoption of all the prescribed design and detailing rules should lead to rates of failure substantially lower than the rates of exceedance of the design actions.

From the brief overview above on the state of seismic design codes internationally, it is possible to conclude that while a certain amount of concepts and procedures for performance-based design have by now found a stable place, the goal of fully probabilistic performance-based codes is still far from being attained. The motivations for pursuing such an objective, by also widening its scope to include economic considerations, together with a detailed account of the current main research streams will be illustrated in Chapter 3.

1.1.2 Introduction of reliability concepts in seismic design

Before considering the current state, however, the question could be reasonably asked about what has occurred in the area of seismic probabilistic research during the eighties and the nineties (we left above with a hint to the situation at the end of the seventies), if for no other reason but to understand why it took so long for specialists to finally arrive at something that could be proposed as a feasible approach outside the academic world, as is finally occurring.

High calibre scientists of the old guard, responsible for the advances of reliability theory for static problems, attempted to resolve the new and much more complex problems, and with them an ever increasing number of new adepts filled the journals with their attempts.

Two reviews of the large production of studies in these years are given in (Der Kiureghian, 1996) and (Pinto, 2001). Broadly speaking, the approaches can be grouped in three large categories.

The first one makes reference to the theory of random vibrations, and in particular to the Rice expression for the mean rate of outcrossing of a scalar random function from a given domain, and to its generalization to vector processes. The limitations of this approach are well known: existence of the exact solution only for the case of stationary Gaussian random processes (unrealistic for inelastic dynamic structural response), approximate value of the rate as a measure of the probability (upper bound) and, for the case of vector processes, availability of the solution for the rate only for time-invariant, deterministic safe domains bounded by planes. In the (usual) case of presence of random structural properties, it is necessary to solve the problem by separately conditioning on each of them, and then to have recourse to a convolution integral. This category of methods has not met with practical success, due to its relatively high requirements of specialist knowledge, and severe restrictions of use.

The second category includes the vast area of the simulation methods. Among the variance reduction techniques applicable to plain Monte Carlo (MC) procedures, only two appear to have received some attention in the field of earthquake engineering: Directional Simulation (DS) and Importance Sampling (IS), applied either separately or in combination. A good number of studies have been devoted to the crucial problem of finding an appropriate sampling density, which is difficult to guess in the case of dynamic problems. Adaptive techniques have been tried, one proposal being to transform the initial density, after a first run in which sample points in the failure domain have been obtained, into a multi modal density, with modes corresponding to the failure points, and a weight proportional to the contribution of each point to the probability integral. Application of these techniques to actual problems, however, are still computationally too expensive. Their efficiency is in fact always measured with respect to that of plain MC, which cannot represent a reference for EE problems, where each sample may imply a nonlinear analysis of a complex system.

The third category is represented by the well-known statistical technique called Response Surface. In a probabilistic context, a response surface represents an approximation of the limit state function, useful then when this latter is not obtainable in explicit form. This technique lends itself to obtain quite accurate results with much fewer computations than any of the enhanced MC methods. It automatically accounts for the correlation between the component responses and of all combinations of these responses that may lead to a pre-established state of failure. A limitation of the method lies in the number of structural random variables that can be explicitly introduced in the function (the order of magnitude being 5 or 6), though there is the possibility of accounting globally of the effect of a larger number of them, as well as of the effect of the variability of the ground motion, through the addition in the function of “random effect” terms. Calibration of such so-called “mixed” models, however, requires rather sophisticated techniques.

On the basis of the above sketchy panorama, it is not surprising that during that period (the 80's and 90's) code-making authorities preferred to stick to the traditional semi-probabilistic approach to performance-based design: there were actually no feasible alternatives ready for code implementation.

In the meantime, however, the search for affordable methods capable of calculating limit-state probabilities continued, conducted principally at Stanford University under the lead of C.A. Cornell, and by the middle of the 90's very promising results started to materialize (see, for example: Bazzurro and Cornell 1994, Cornell 1996). In these studies the problem was posed in terms of a direct (probabilistic) comparison between demand and capacity, as in the basic reliability formulation for the static case, with the demand being the maximum of the dynamic response of the system to a seismic action characterized in terms of a chosen return period. The streamlining of this approach has resulted in one of methods (the SAC-FEMA method) that will be described in full detail in Chapter 3. This method has the advantage of providing a closed-form expression for the failure probability (P_f), that can also be put in a partial factor format. The second approach illustrated in detail in Chapter 3, usually referred to as the PEER method, has several conceptual similarities with the first (in particular the need for a quite small number of simulations), is not in closed-form but it allows more flexibility and generality in the evaluation of the desired so-called “decision variable”, not necessarily coinciding with P_f .

In conclusion, it is possible today to rightly state that the goal of calculating structural and non-structural performance probabilities has been methodologically and operatively achieved in a way that is ready to be proposed for routine code-based design. It is not within the scope of this document to foresee time and modalities of this transition, which will require profound changes in the organization of the material in the present codes. It is however felt that engineers of modern education should already now be in possession of the probabilistic tools described in this document, for possible use in special cases, but also for the superior intellectual viewpoint they allow.

Efforts have been made to make this document directly readable with the most elementary probabilistic background, and all terms used are carefully defined from the start.

Teaching experience has shown that students absorb the “theory” of these methods with extreme facility: one should ensure that they do not forget, however, that recourse to probabilistic methods implies willingness of an approach closer to the “truth” which, by consequence, implies that the choice of all elements entering the procedure, from the model of the system to the type of analysis, to the capacity of the members, etc., be consistent with this direction. Replacement of the prescriptive, and quite often conservative, indications of deterministic codes with better, more physically-based expressions, generally represents the truly challenging part of a probabilistic analysis.

1.2 On the definition of performance

The primary concern for most structural engineers is the structural integrity of the structures that they design. In modern engineering practice, particular consideration is given to the extreme loading events that are imposed by natural hazards, such as earthquakes. The design problem is usually addressed by designing structural components to meet code-prescribed limit-states that are related to strength and deformation. Meeting the required limit-states, with safety coefficients provided by codes, implies a low probability that demands emerge greater than the corresponding capacities. Notably, traditional limit-states, regardless of whether they address safety or functionality, measure capacity and demand at the component level. Examples include bending moment, shear force, and deflections. These are

deemed to be reasonable proxy measures of performance when acceptable performance means preservation of structural integrity.

However, the component-oriented code-based approach has come under criticism, for two reasons. One concern is that exclusive use of component-level limit-states may fail to capture significant system-level effects, i.e. global structural behaviour. Another concern is that component-based limit-states may represent an unsatisfactory proxy of “performance.” In particular, traditional limit-states do not convey information that is understandable for a broader audience, such as owners and developers, except that the structure “meets the code.” Although these concerns are particularly pressing in Earthquake Engineering (EE), they have gained the attention of the structural engineering profession at large. A revealing but not unique example is the 2005 version of the National Building Code of Canada, in which each limit-state is linked with statements that identify performance objectives. This has the added advantage of making it feasible to consider innovative, non-traditional design solutions.

In EE the negative ramifications of absent or obscure performance targets are especially evident. In North America, the Northridge earthquake that occurred in a Los Angeles neighbourhood in 1994 is a case in point. Although the structural integrity was preserved for the vast majority of structures, the direct and indirect economic losses associated with structural under-performance were dramatic. In other words, even buildings that apparently met the code were associated with performance that was considered unacceptable, or at least unexpected, by the general population. This motivates the following discussion of what constitutes adequate performance measures, and how structural engineers can address them.

The label “performance-based” is attached to many recent developments in structural engineering. Although sometimes misconstrued, the phrase has a particular implication. It implies that the traditional considerations of structural engineering are amended with the consideration of “impacts” or “consequences.” Direct and indirect losses due to damage represent an illustrative example. The fundamental definition adopted in this bulletin is that structural performance is to be understood by a broad audience. This audience includes owners, developers, architects, municipal governments, and regular citizens who take an interest in seismic safety. Therefore, the value of structural response quantities, such as interstorey displacement, is not by itself a measure of performance. It must be linked with other measures or interpretations that are understood by a non-engineering audience. One illustration of this concept is presented in Section 2.2.1, where performance is presented in terms of economic loss. It follows that a structural design that is based on performance has undergone considerations of global structural behaviour. In short, a performance-based design comes with more information than code compliance; it comes with information that invites a broad audience to understand the implications of the design decisions.

Clearly, the definition of performance that is adopted above will seem unusual and onerous for the contemporary structural engineer who wants to adopt a performance-based design philosophy. Three remarks are provided to address this potential frustration. First, the purpose of this bulletin is to provide hands-on methodologies and examples to assist in the development and communication of performance-based designs. Second, the adopted definition of performance should serve to highlight the broad scope and multi-disciplinary nature of modern EE. This is an important understanding for the 21st century earthquake engineer. Third, the challenges that engineers face motivate a number of research projects that are currently under way. In fact, a valuable consequence of performance-based design that cannot be overemphasized is that it brings engineering practice and academic research closer together.

1.3 Type and nature of models in performance-based engineering

Structural engineering requires iteration between design and analysis. Trial designs, often based on a mix of experience, judgment, and input from architects and developers, are formulated, followed by analysis (referred to also as assessment) to verify their suitability.

The fundamental problem in the assessment phase is to predict structural performance according to the broad definition of performance outlined in the previous section. Typical results are event-probabilities, such as the probability that the structure is operational after an earthquake that has uncertain characteristics. Another example is represented by entire loss-probability curves. The computation of such results requires two ingredients: analysis *models* and analysis *methods*.

In this section, concepts related to the models are described. This is appropriate because it is the quality of the models that determines the quality of the performance assessments. In fact, in the performance-based paradigm the efforts spent on modelling are usually more productive than refinement of the analysis techniques. The analysis methods are not specific to performance analysis and serve the utilitarian purpose of coordinating the models and computing performance-probabilities. Furthermore, the analysis methods must be selected to match the type of models that are available (as shown in the following chapter).

To illustrate the modelling problem that is at the heart of this bulletin, consider a developer of a new building who asks an engineer to estimate the probability that the monetary loss due to earthquake-damage in the lifespan of the building exceed a given amount, in present-value currency. Obviously, this is a departure from traditional limit-state design and the following list summarizes the novel aspects that pertain to modelling:

1. Instead of models with *conservative bias* and *safety coefficients*, which are found in the codes, *predictive* models are required, i.e. models that aim to provide an unbiased estimate of the quantity of interest, together with a measure of the associated uncertainty. There are two types of predictive models. One type of models simulates possible events; a nonlinear dynamic structural model is an example (where the event is defined in terms of structural response). Another type of predictive models produces the probability that some event will occur; so-called *fragility functions* are of this type because they yield the probability of a specific failure at a given demand.
2. Uncertainty in the prediction of structural performance is unavoidable and must be exposed and accounted for. Several non-deterministic approaches are available, but the use of probabilistic techniques is preferred. It is particularly appealing to characterize the uncertainty by means of random variables because this facilitates reliability analysis to compute the probability of response events of interest. Furthermore, it is desirable to characterize the uncertainty in the hazard, structure, and performance individually, and in a consistent manner. A perception exists that the uncertainty in the ground motion is greater than that associated with the structural performance. Except for the uncertainty in the occurrence time of seismic events, this assumption cannot be made *a priori* and all uncertainty must be included.
3. Some uncertainty is reducible, and this uncertainty should be identified and characterized by probabilistic means. While there are numerous sources of uncertainty in performance predictions, the nature of the uncertainty is either reducible (“epistemic”) or irreducible (“aleatory”). This distinction is important for the practical reason that resources can be allocated to reduce the epistemic uncertainty. One example of epistemic uncertainty is model uncertainty. A candid representation of this uncertainty allows resources to be allocated to reduce it, i.e. to develop better models. Another example is statistical uncertainty due to limited number of observations of a phenomenon. It is desirable that all

such uncertainty is reduced over time, as new knowledge and observations become available.

4. Finally, the uncertainty affecting all elements in the analysis implies that its outcome is probabilistic in nature. This bulletin is based on the understanding that probabilities are integral to performance-based design. Therefore, instead of having misgivings about presenting probabilities, performance predictions should be provided, as probabilities or as probability distributions, or at the very lowest level as means and dispersions of performance measures.

It is understood from the previous text that the assessment phase of performance-based design requires predictive models. The three categories of models that are needed are discussed in the following subsections.

1.3.1 Hazard models

The fundamental cause of earthquakes is the movement of tectonic plates in the Earth's crust. Although there are exceptions, most strong earthquakes occur in the boundary regions between the tectonic plates. In these locations, strains accumulate over time and are suddenly released in brittle rupture events. This causes the ground to shake, with varying intensity in the surrounding area. The amount of energy released and the distance from the rupture influences the ground motion at a specific site. Moreover, the shaking depends strongly on the material and geometrical properties of the ground through which the energy propagates. As a result, the ground shaking at a site is highly complex and, obviously, the prediction of future ground motions are associated with significant uncertainty.

As mentioned in Section 1.1.2, structural engineers have historically incorporated the seismic hazard by the application of static horizontal forces, similar to wind loading. In many codes this remains a primary approach for verifying the seismic resistance of several buildings types. In this approach, the force level is related to some scalar measure of the intensity of the ground shaking. Although most engineers are familiar with at least a few such measures, the venture into performance-based design makes it worthwhile to revisit this practice. From the viewpoint of accurate probabilistic prediction of performance it seems desirable to develop a probabilistic seismic hazard model that produces any possible ground motion, with appropriate variation in time and space, and with associated occurrence probabilities. However, at present this is a utopian notion and simpler models are both necessary and appropriate. In particular, simplifications are appropriate if they capture vital ground motion characteristics that are important for the subsequent prediction of performance. The simplified models come in several categories, which include the following:

1. Single scalar intensity measures: Historically the peak horizontal ground acceleration was the most popular such measure, and it still is in geotechnical engineering. However, in structural engineering it is found that the spectral acceleration, i.e. the peak acceleration response of a single-degree-of-freedom system at the first natural period of the structure, is more suitable. Attenuation models are available to express these intensity measures in terms of the magnitude and distance to potential sources of ruptures. By means of probabilistic seismic hazard analysis (PSHA) this information is combined with the occurrence rate of earthquakes and intensity attenuation models to create a probability distribution of the intensity measure. These are called hazard curves, and examples are provided in Chapter 2.
2. Multiple scalar intensity measures: Although seismologists have grown accustomed to providing only one scalar intensity measure to structural engineers, research shows that the use of several values may significantly improve the correlation with observed damage.

A variety of options are put forward. One approach is to utilize the spectral acceleration at several periods, which is straightforward because seismologists already provide this information.

3. Recorded ground motions: The utilization of accelerograms from past earthquakes has the significant advantage that all characteristics of the ground motion are included in the record. Often, the recording is synthesized by considering only the ground motion component in one horizontal direction, which is of course associated with some loss of information. Nevertheless, given recent advances in structural analysis the record can be applied to advanced structural models, and a host of results can be obtained. However, from a probabilistic viewpoint, this approach has a serious shortcoming: A recorded ground motion represents only one point in a continuous outcome space and the specific realization will not reoccur in the exact same form in the future.
4. Scaled recorded ground motions: The scaling of recorded ground motions is intended to remedy the problem that was identified in the previous item. This technique is usually combined with the aforementioned hazard curves to scale records to an intensity level that is associated with a selected probability of exceedance. This approach is popular and is demonstrated in Chapter 3. It is important to note, however, that the scaling of ground motions must be carried out with caution, particularly in probabilistic analysis. Too severe scaling yields unphysical realizations, and the outcome space of possible ground motions is by no means comprehensively covered. Grigoriu (2011) outlines other issues related to the use of scaled records in probabilistic analysis.
5. Artificially generated ground motions: Several models exist for the generation of ground motions that have the same characteristics as actual accelerograms. The approaches include wavelet-based methods and filtered white noise techniques. One example of the latter is presented in Section 3.3. The approach presented in that section is an example of a particular class of techniques that produces ground motions that correspond to the realization of a set of random variables. In other words, the uncertainty in the ground motion is discretized in terms of random variables. This is especially appealing because this type of model is amenable to reliability analysis, as described in Section 3.3. It is noted, however, that the development of this type of models is still an active research field and caution must be exercised to avoid the inclusion of unphysical or unreasonable ground motions.

Although other approaches exist, such as the use of comprehensive numerical simulations to model the propagation of waves from the rupture to a site, it is easy to find fault with any of the seismic hazard models that are currently available. In short, the merit of a simplified model should be evaluated based on how well it captures the ground motion characteristics that ultimately influence the structural performance. In this bulletin it is argued that performance-based seismic design requires ground motions, recorded or artificial, to capture the complexity of the structural response and ultimately the performance.

1.3.2 Response models

Structural responses quantify deformations and internal forces. In performance-based design they ultimately serve to expose the performance of the structure, such as damage, which enter into downstream models for performance. In classical structural engineering, the consideration of ultimate limit-states requires the computation of internal forces. In contrast, serviceability limit-states entail the computation of deformations. Deformations receive still greater emphasis in modern seismic design with the computation and restriction of inelastic deformations.

The responses that are utilized in performance-based seismic design depend upon three factors: 1) the level of refinement of the structural model, 2) the output from the upstream hazard model, and 3) the input to the downstream performance models. It is possible to model the structure with a single-degree-of-freedom model or a parameterized pushover curve. However, in this bulletin it is contended that this is insufficient to capture the relevant global structural response characteristics and local damage in the structural and non-structural components. It follows that performance-based seismic design requires a detailed structural (finite-element) model. Furthermore, for the downstream prediction of damage, this model must capture the inelastic behavior of the structural components. A host of material models are available for this purpose.

To simulate the response in an earthquake event, the finite element structural model can be subjected to static or dynamic loading. The former is referred to as pushover analysis; the latter is called time history analysis. As implied in the previous section, in this bulletin it is asserted that dynamic analysis is necessary to capture the details of the structural response and ensuing damage. It follows that the hazard model must yield a ground motion as input to the structural model. In turn, the raw output from the structural model is time histories of deformations and internal forces.

1.3.3 Performance models

According to the definition established in Section 1.2, performance measures serve the purpose of informing stakeholders about the performance of the structure. The raw measure of performance is “damage.” In turn, information about damage enables the prediction of functionality of the building and safety of the occupants. In the past, structural damage was typically addressed by damage indices, which combine structural responses to produce a scalar value that exposes the severity of the damage. Prompted by the presence of uncertainties and developments in probabilistic analysis the concept of “fragility curves” has recently emerged as a popular alternative. Formally, the ordinate of a fragility curve is the probability of failure according to some limit state given the hazard intensity at the abscissa axis, accounting for uncertainty in both demand and capacity. A less strict definition is employed in damage modeling. The damage fragility curve displays the probability that a component is in a particular damage state—or higher—for a given structural response. As an example, consider a structural column with possible damage scenarios ranging from 0 (no damage) to 4 (severe damage). Suppose the damage is determined by the maximum interstorey displacement that the column undergoes. Four fragility curves are employed as a damage model in this case. The first curve displays the probability that the column is in damage state 1 or higher for given interstorey displacement. Similarly, the last curve displays the probability that the column is in damage state 4 for given interstorey displacement.

The prediction of performance requires models that are new to most structural engineers. In fact, it is in this category that the interdisciplinary nature of EE manifests most strongly. Although examples are provided in this bulletin, it is important to note that the selection of performance measure depends on the audience. Furthermore, the predictions are associated with significant uncertainty. At present, the quality of several models is explored and improved in academia. In particular, models that formulate the uncertainty in terms of random variables rather than conditional probabilities are a versatile and powerful option. Therefore, as mentioned in Section 1.1, comprehensive communication between the academic community and engineering practice is imperative in order to steadily improve the range and quality of performance predictions. The remainder of this bulletin provides examples that are intended to establish a common starting point and to motivate subsequent improvements.

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2 Probabilistic seismic assessment

2.1 Introduction

This chapter presents the two classes of methods available today for performing a probabilistic seismic assessment. The methods in Section 2.2 have a distinct practice-oriented character, they are currently employed as the standard tool in the research community and are expected to gain ever increasing acceptance in professional practice. These methods are described in a quite detailed manner and are provided with examples so as to make the reader capable of using them. Methods in Section 2.3, on the other hand, have a more advanced character, and their presentation reflects it. Though an effort to present all concepts in a sufficiently plain way is made, their full understanding requires theoretical notions in the field of probability and random processes that are not expected to be part of most of the readers' background. The reason for including the latter methods is to show how they compare with the practice-oriented ones, so as to better appreciate their effectiveness in light of the considerably minor effort required.

2.2 Conditional probability approach (IM-based methods)

The philosophy behind this class of methods is based on the use of a scalar (or vector) intensity measure (IM) as an interface between seismology and structural engineering. In this view of the performance-based assessment problem, the seismologist would model the faults causing earthquakes that may affect the site under investigation and would summarize all information into a single hazard curve (or multiple curves in the case of a vector) representing the mean annual rate of exceeding different levels of the intensity measure. For example, two points along one such a curve developed for horizontal Peak Ground Acceleration (PGA) or spectral acceleration at a given period, $S_a(T)$, may say that values of 20% and 40% of the acceleration of gravity, g , have average rates of 0.01 and 0.001, respectively, to be exceeded at least once every year at the site. An alternative, equivalent, and often useful way of interpreting these values can be obtained by considering the reciprocal of the rates. Hence, in the example above the values of 20% and 40% of g are exceeded at the site, on average, once every 100 and 1,000 years, respectively. This seismic hazard curve is formally computed via an approach known as probabilistic seismic hazard assessment (PSHA) (Cornell, 1968), whereby the seismologist aggregates the distribution of ground motion levels (measured by IMs such as PGA or $S_a(T)$) that may result at the site from all possible future earthquake scenarios appropriately weighted by their annual rate of occurrence to arrive at a probabilistic distribution for the IM at the site.

In the past structural engineers have operated downstream from the probabilistically derived representation of the IM, by estimating ranges of structural response, damage or loss that the structure of interest may experience should it be subjected to given values of the IM. However, most of the time engineers were completely oblivious of all the aspects of the seismology work that led to the definition of the site seismic hazard they were using as input to their analyses, including those that would have been important to know to inform the selection of the methods to be utilized for structural response computations. As a result, often times the engineers armed only with the notion of a seismic hazard curve for one or more IMs at the site utilized methods not entirely consistent with the seismological work that led to the site hazard representation. The potential consequences of such a mismatch are estimates of structural performance that are inaccurate at best and erroneous at worst.

In the past 20 years, however, significant efforts have been made in bridging the dichotomy between seismology and engineering to ensure that engineers use all the information about the seismic hazard that is relevant to their structural assessment task. Hence, while this document is mainly geared towards a structural engineering audience, it contains enough notions of seismology beyond the mere definition of an IM or of a hazard curve to guide engineers in the correct applications of their structural analysis approaches. These additional notions include, in one form or another, information about the most likely earthquake scenarios that can cause the exceedance of the IM of choice at the site. These scenarios are conveniently described in terms of the event magnitude, M , the source-to-site distance, R , and a measure of the dispersion of the IM around its expected value caused at sites at that distance from the earthquake rupture by past earthquakes of that magnitude. A measure of such dispersion is conventionally called ε .

As alluded to earlier, the methods that the engineers can use to estimate the structural performance hinge on checking whether the response of the structure is acceptable when it is subject to specific levels of IMs. These methods often use nonlinear dynamic analysis of a structural model subject to a carefully selected suite of ground motion records that have the “right” level of IM for the site of interest. Hence, since the analyses are performed for a given level of IM (or, in probabilistic lingo, are “conditioned on a level of IM”), it is implicitly assumed that structural response is independent of the M , R and ε that describe the earthquake scenarios more likely to cause that level of IM at the site. This assumption is tenable when the selected IM has characteristics that make it “sufficient” (as defined by Luco & Cornell, 2007) for the structural response analysis at hand. In simple words, an IM is sufficient if different sets of ground motion records having the same IM value but systematically differing in other characteristics (e.g. M of the causative event, duration, frequency content, etc.) cause, on average, the same level of structural response. There are two ways to proceed.

The first approach is to make sure that structural analysis is performed by employing an IM which is “sufficient” *across all levels* of ground motions. This means that ground motion records that have (either naturally or when scaled) any given desired value of the IM, cause structural responses that are independent of values of M , R and ε that define them. For example, the inelastic spectral displacement of an equivalent SDOF system, especially when modified to account for higher mode contributions, has been shown by Tothong & Cornell (2008) to possess such qualities both for far-field and near-field excitations acting on regular moment-resisting frames of up to 9 stories. Similarly, $S_a(T_1)$ is usually an adequate choice for assessing the response of first-mode dominated structures to ground motions from far-field scenarios. The adequacy of $S_a(T_1)$ may break apart when not enough real records are available for the desired IM level (which is usually the case for very high values, such as PGA of 1.5 g) and, therefore, the input motions need to be scaled by a large amount. Records that are scaled by “excessively” large factors are, on average, more aggressive than those that naturally have large IM values and, therefore, their use will generally introduce bias towards higher estimates of displacement response (Bazzurro & Luco 2007). On the other hand, PGA is not useful for assessing the response of any but the short period structures at low levels of intensity.

A second approach seeks to employ multiple sets of ground motions, each one appropriate *for a given level* of IM and not across all levels of IMs as the former approach does. Since PSHA employs myriads of scenarios of M , R and ε , and the characteristics of the motions (e.g. its frequency content) are very much dependent on the basic parameters of the causative earthquake scenario (e.g. M , and R) and the soil conditions at the site, it is intuitive to understand that generally different scenario events control the hazard at different levels of IM. Utilizing existing ground motion catalogues (e.g. the PEER NGA 2005 database available at <http://peer.berkeley.edu/nga/>) it is possible to cherry-pick appropriate sets of records that satisfy the parameters of the hazard-controlling scenario events to achieve sufficiency for

different levels of IM. Baker and Cornell (2005) have shown that, in addition to M and R , also the distribution of ε is particularly important to achieve IM sufficiency in the record selection. Since the details of such work are less intuitive and this concept is not crucial for the purpose at hand, it will not be discussed further here.

For completeness, it is important to mention that when only few or no real records from past events exist in the catalogue, as may happen for certain soil conditions or high intensity levels, it is often necessary to simulate them to reach a number of records large enough to allow a robust estimate of the structural response. Simulation of synthetic ground motion records is a rapidly emerging area harboring different methods to obtain artificial ground motions compatible with the desired seismic scenarios (details about two such models are given later in §2.3.3).

Obviously, any of the above methods can be viable in a performance-based assessment framework, depending on the tools and resources available. In the sections to follow, we choose to take the first path, which is simpler and best suited to general design applications. Essentially, we will use $S_a(T_1)$ as the IM of choice and employ a limited scaling on a suite of “ordinary” ground motion records (i.e. records bearing no marks of near source-effects, such as velocity pulses, or narrow-band frequency amplification due to soft soil conditions) to estimate structural response. As mentioned earlier, however, this method may provide some usually minor bias in the response at high $S_a(T_1)$ levels due to the less-than-perfect sufficiency of $S_a(T_1)$. However, the introduced bias, if it exists at the scaling levels adopted here, is on the conservative side and, if not excessive, may be considered by some even welcome in design applications such as those targeted in this document.

2.2.1 PEER formulation

2.2.1.1 Summary

Considering the shortcomings of the first-generation performance-based earthquake engineering (PBEE) procedures in USA, a more robust PBEE methodology has been developed in the Pacific Earthquake Engineering Research (PEER) Center which is based on explicit determination of system performance measures meaningful to various stakeholder groups, such as monetary losses, downtime and casualties, in a rigorous probabilistic manner. PEER PBEE methodology and the analysis stages that comprise the methodology are summarized in this section. An example of a real building demonstrates the application of this methodology at the end of the section.

2.2.1.2 Introduction

Traditional earthquake design philosophy is based on preventing structural and non-structural elements of buildings from any damage in low-intensity earthquakes, limiting the damage in structural and non-structural elements to repairable levels in medium-intensity earthquakes, and preventing the overall or partial collapse of buildings in high-intensity earthquakes. After 1994 Northridge and 1995 Kobe earthquakes, the structural engineering community started to realize that the amount of damage, the economic loss due to downtime, and repair cost of structures were unacceptably high, even though those structures complied with available seismic codes based on traditional design philosophy (Lee and Mosalam 2006). Recent earthquakes, once again have shown that traditional earthquake design philosophy have fallen short of meeting the requirements of sustainability and resiliency. As an example, a traditionally designed hospital building was evacuated immediately after the 2009 L’Aquila, Italy earthquake, while ambulances were arriving with injured people (Günay and Mosalam

2010). Similarly, some hospitals were evacuated due to non-structural damage and damage to infill walls after the 8.8 moment magnitude Chile, 2010 earthquake (Holmes 2010). In addition, some of the residents did not want to live in their homes anymore although these buildings had satisfactory performance according to the available codes (Moehle 2010).

Vision 2000 report (SEAOC 1995) is one of the early documents of the first generation performance-based earthquake engineering in USA. In this report, Performance-based earthquake design (PBED) is defined as a design framework which results in the desired system performances at various intensity levels of seismic hazard (Fig. 2-1). The system performance levels are classified as fully operational, operational, life safety, and near collapse. Hazard levels are classified as frequent (43 years return period (RP), 50% probability of exceedance (POE) in 30 years), occasional (72 years RP, 50% POE in 50 years), rare (475 years RP, 10% POE in 50 years), and very rare (949 years RP, 10% POE in 100 years) events. The designer and owner consult to select the desired combination of performance and hazard levels (performance or design objectives) to use as design criteria. The intended performance levels corresponding to different hazard levels are either determined based on the public resiliency requirements in the case of, for example, hospital buildings, or by the private property owners in the case of residential or commercial buildings.

Subsequent documents of the first generation PBEE; namely ATC-40 (1996), FEMA-273 (1997) and FEMA-356 (2000) documents express the design objectives using a similar framework, with slightly different performance descriptions (e.g. operational, immediate occupancy, life safety, and collapse prevention in FEMA-356) and hazard levels (e.g. 50%, 20%, 10%, and 2% POE in 50 years in FEMA-356). The member deformation and force acceptability criteria corresponding to the performance descriptions are specified for different structural and non-structural components from linear, nonlinear, static, and/or dynamic analyses. These criteria do not possess any probability distribution, i.e. member performance evaluation is deterministic. The defined relationships between engineering demands and component performance criteria are based somewhat inconsistently on relationships measured in laboratory tests, calculated by analytical models, or assumed on the basis of engineering judgment (Moehle 2003). In addition, member performance evaluation is not tied to a global system performance (as already pointed out in §1.2). It is worth mentioning that FEMA-273 guidelines and FEMA-356 prestandard are converted to a standard in the more recent ASCE-41 (2007) document.

		System Performance			
		Fully Operational	Operational	Life Safety	Near Collapse
Hazard Levels (Return Period)	Frequent (43 years)	●	○	○	○
	Occasional (72 years)	△	●	○	○
	Rare (475 years)	◆	△	●	○
	Very rare (949 years)		◆	△	●

○: unacceptable performance
●: basic safety objective
△: essential hazardous objective
◆: safety critical objective

Fig. 2-1: Vision 2000 recommended seismic performance objectives for buildings

2.2.1.3 Formulation

One of the key features of PEER PBEE methodology is the explicit calculation of *system* performance measures which are expressed in terms of the direct interest of various stakeholder groups such as monetary losses, downtime (duration corresponding to loss of function), and casualties. Unlike earlier PBEE methodologies, forces and deformations of components are not directly used for performance evaluation. Another key feature of the methodology is the calculation of performance in a rigorous probabilistic manner without relying on expert opinion. Moreover, uncertainties in earthquake intensity, ground motion characteristics, structural response, physical damage, and economic and human losses are explicitly considered in the methodology (Lee and Mosalam 2006). PEER performance assessment methodology has been summarized in various publications (Cornell and Krawinkler 2000; Krawinkler 2002; Moehle 2003; Porter 2003; Krawinkler and Miranda 2004; Moehle and Deierlein 2004) and various benchmark studies have been conducted as discussed in (Comerio 2005; Krawinkler 2005; Goulet et al. 2006; Mitrani-Reiser et al. 2006; Bohl 2009).

As presented schematically in Fig. 2-2, PEER PBEE methodology consists of four successive analyses, namely hazard analysis, structural analysis, damage analysis, and loss analysis. The methodology focuses on the probabilistic calculation of meaningful system performance measures to facilitate stakeholders by considering all the four analysis stages in an integrated manner. It should be noted that Fig. 2-2 represents one idealization of the outline of the methodology, where variations are also possible. For example, the probability distribution functions shown in Fig. 2-2 can be replaced by probability mass functions and the integrals can be replaced by summations when the probabilities are defined with discrete values instead of continuous functions. In addition, after obtaining POE values for decision variables (loss curves), DV's, simple point metrics such as the expected economic loss during the owner's planning period, can be extracted from these DV's (Porter 2003) to arrive to a final decision. The different analysis stages that comprise PEER PBEE methodology are explained in the following sub-sections.

2.2.1.3.1 Hazard analysis

Probabilistic seismic hazard analysis takes as input the nearby faults, their magnitude-recurrence rates, fault mechanism, source-site distance, site conditions, etc., and employs attenuation relationships², such as next generation attenuation (NGA) ground motion prediction equations (Power et al. 2008), to produce an hazard curve which shows the variation of the selected intensity measure (ground motion parameter) against the mean annual frequency (MAF) of exceedance (Bommer and Abrahamson 2006). Considering the common assumption that the temporal occurrence of an earthquake is described by a Poisson model (Kramer 1996), POE of an intensity parameter in “t” years corresponding to a given mean annual frequency of exceedance is calculated with Equation 2-1 where “t” can be selected as the duration of life cycle of the facility.

$$P(IM) = 1 - e^{-\lambda(IM) t} \quad (2-1)$$

² Attenuation relationships, also denominated ground motion prediction equations (GMPE), are empirical relationships relating source and site parameters, such as the event magnitude, the faulting style, the source-to-site distance, and the site soil conditions, to the ground motion intensity at the site. Peak ground acceleration, PGA, peak ground velocity, PGV, and spectral acceleration at the period of the first mode, Sa(T₁), are examples of parameters that have been used as intensity measures. The reason of these parameters being generally used as IM is that most of the available attenuation relationships are developed for these parameters.

where IM is the intensity measure, $\lambda(\text{IM})$ is the annual frequency of exceedance of IM and $P(\text{IM})$ is the POE of IM in “t” years (Fig. 2-3). Probability density or probability mass function of IM is calculated with Equation 2-2 or algorithmically using Equation 2-3 for the continuously and discretely expressed POE of IM, respectively.

$$p(\text{IM}) = -\frac{dP(\text{IM})}{d\text{IM}} \quad (2-2)$$

$$p(\text{IM}_m) = \begin{cases} P(\text{IM}_m) & \text{if } m = \# \text{ of IM data points} \\ P(\text{IM}_m) - P(\text{IM}_{m+1}) & \text{otherwise} \end{cases} \quad (2-3)$$

where “p” represents the probability density function (PDF) or probability mass function (PMF) for the continuous and discrete cases, respectively, and “P” represents the probability of exceedance (POE).

Other than the generation of the hazard curve, hazard analysis also includes the selection of a number of ground-motion time histories “compatible” with the hazard curve (see §2.1). For example, if $S_a(T_1)$ is utilized as IM, for each $S_a(T_1)$ value in the hazard curve, an adequate number of ground motions should be selected which possess that value of $S_a(T_1)$. Here, adequate number refers to the number of ground motions which would be adequate to provide meaningful statistical data in the structural analysis phase. In order to be consistent with the probabilistic seismic hazard analysis, selected ground motions should be compatible with the magnitude and distance combination which dominates the hazard for a particular value of IM (Sommerville and Porter 2005).

For practicality purposes, instead of selecting ground motions for each IM value, an alternative way is to select a representative set of ground motions and scale them for different IM values. However, as anticipated, this alternative might lead to unrealistic ground motions when large scales are needed. The following taken from (Sommerville and Porter 2005) is an example for this situation: Higher magnitudes correspond to higher $S_a(T_1)$ values. However, higher magnitudes also correspond to longer durations. When the amplitude of a ground motion is scaled considering $S_a(T_1)$, the duration of the ground motion does not change which may make the ground motion unrealistic.

Tothong and Cornell (2007) investigated use of IM’s other than the common ones mentioned in the previous paragraphs. They investigated the use of inelastic spectral displacement and inelastic spectral displacement corrected with a higher-mode factor as IM’s. They stated that ground motion record selection criteria for accurate estimation of the seismic performance of a structure and the problems related to scaling become less important with these advanced IM’s. They developed attenuation relationships for these advanced IM’s such that they can be used in probabilistic seismic hazard analysis.

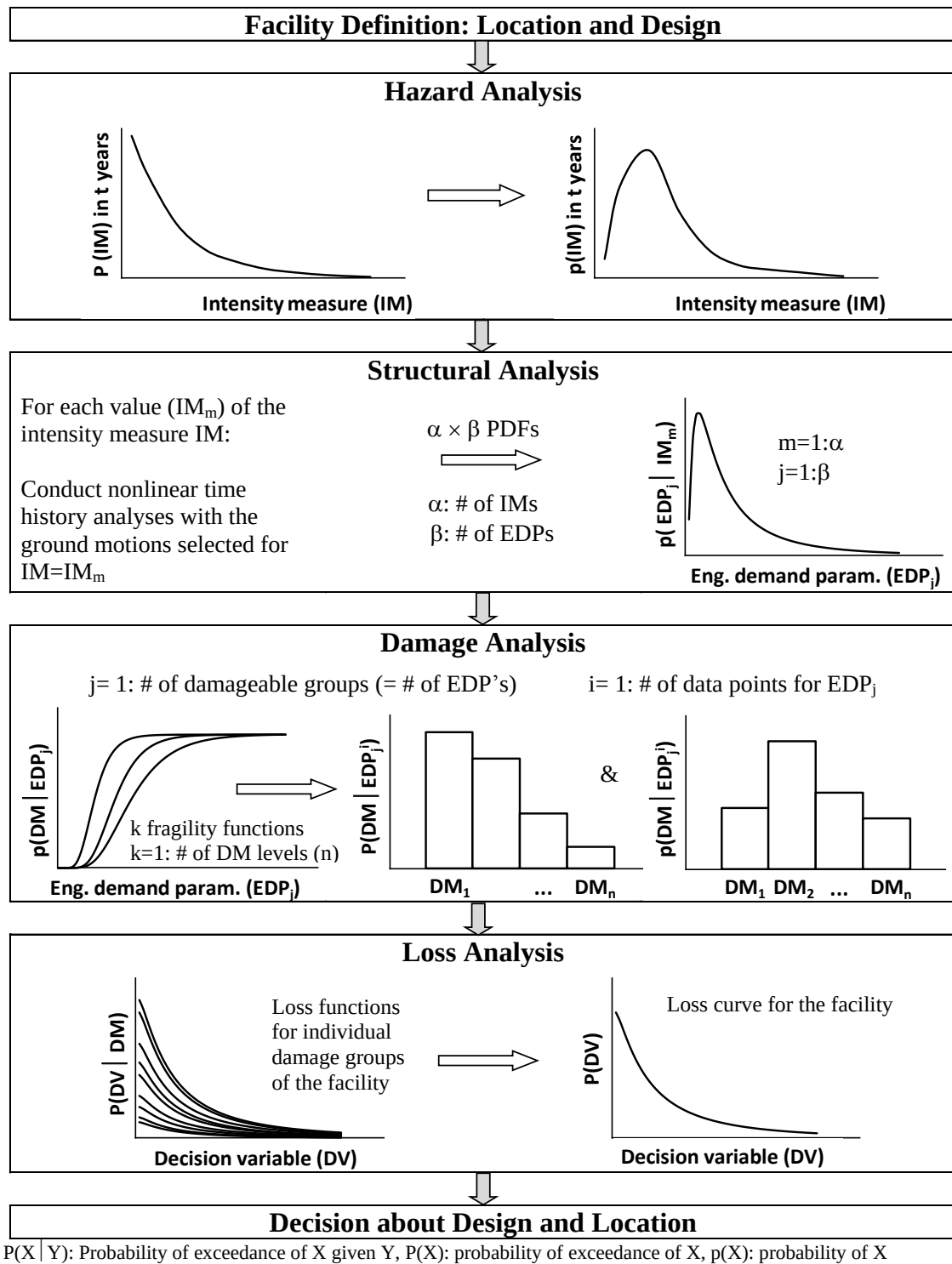


Fig. 2-2: PEER PBEE methodology

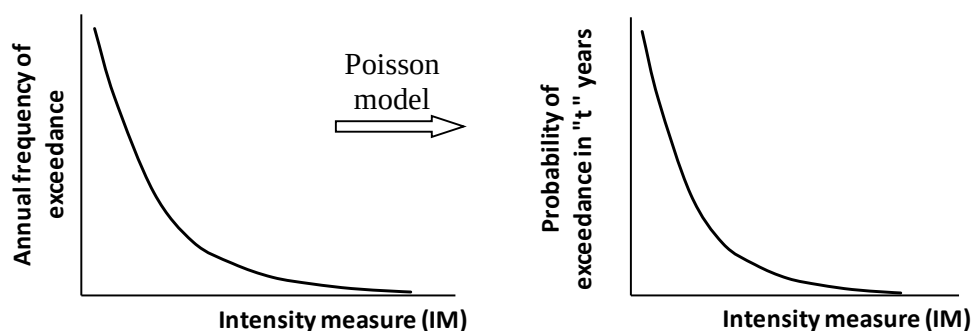


Fig. 2-3: Correspondence between annual frequency and POE of IM

2.2.1.3.2 Structural analysis

In the structural analysis phase, a computational model of the structure is developed. For each intensity level, nonlinear time history analyses are conducted to estimate the structural responses in terms of selected engineering demand parameters (EDP), using the ground motions selected for that intensity level. EDP's may include local parameters such as member forces or deformations, or global parameters such as floor acceleration and displacement, and interstorey drift. For the structural components, member forces (such as the axial or shear forces in a non-ductile RC column) or deformations (such as plastic rotations for ductile flexural behavior) are more suitable, whereas global parameters such as floor acceleration are better suited for non-structural components, e.g. equipment. On the other hand, interstorey drift is a suitable parameter that can be used for the analyses focusing both on structural and non-structural components.

It is possible to use different EDPs for different damageable components of a structure (denoted by EDP_j in Fig. 2-4). For example, interstorey drift can be used for the structural system of a building (Krawinkler 2005), while using floor acceleration for office or laboratory equipment (Comerio 2005) of the same building. As a result of nonlinear time history simulations, the number of data points for each of the selected EDP's (i.e. EDP_j) at an intensity level is equal to the number of simulations conducted for that intensity level.

It may happen that for higher intensity levels *global collapse* occurs. Global collapse is a performance level that is at the very limit of the current prediction capabilities of standard structural analysis tools. There is some consensus on the notion that collapse can be identified in terms of dynamic response as the flattening of the relationship between intensity and displacement response. This event corresponds to infinite increases of response for infinitesimal increases in input intensity and is called *global dynamic instability*. In order to have a more realistic representation of global collapse, Talaat and Mosalam (2009) developed a progressive collapse algorithm based on element removal and implemented it into the structural and geotechnical simulation framework, OpenSees (2010), which is one of the main tools utilized for the application of PEER PBEE methodology.

The probability of the global collapse event, $p(C|IM)$, can be approximately determined as the number of simulations leading to it divided by the number of simulations conducted for the considered intensity level. Probability of having no global collapse is defined as, $p(NC|IM) = 1.0 - p(C|IM)$. These probabilities are employed in the loss analysis stage as explained later.

A suitable probability distribution, e.g. lognormal, is used for each EDP (e.g. EDP_j) by calculating the parameters of this distribution from the data obtained from simulations with no global collapse (Fig. 2-4). The number of PDF's available as a result of structural analysis is $\alpha \times \beta$, where α indicates the number of IM data points and β indicates the number of considered EDP's.

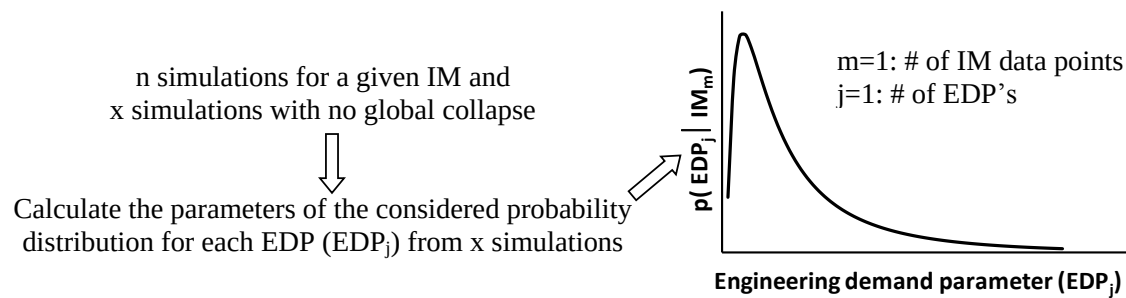


Fig. 2-4: Determination of probability distribution functions from structural analysis

Finally, for the nonlinear time history simulations, uncertainties in parameters defining the structural model (e.g. mass, damping, stiffness, and strength) can also be considered in addition to the ground motion variability (see §2.3). Lee and Mosalam (2006), however, showed that the ground motion variability is more significant than the uncertainty in structural parameters in affecting the local EDPs, based on analyses conducted for one of the test beds of PEER PBEE methodology application.

2.2.1.3.3 Damage analysis

As stated before, the improvement of PEER PBEE methodology with respect to the first generation PBEE methods is the determination of DV's meaningful to stakeholders, e.g. monetary losses, downtime, casualties, etc., rather than the determination of only engineering parameters, e.g. forces or displacements. Therefore, after the determination of PDF's for EDP's in the structural analysis phase, these probabilities should be used to determine the POE for DV's or expected values for DV's. This is achieved from the damage analysis and loss analysis stages as explained in the following.

The purpose of the damage analysis is to estimate physical damage at the component or system levels as function of the structural response. While it is possible to use other definitions, damage measures (DM) are typically defined in terms of damage levels corresponding to the repair measures that must be considered to restore the components of a facility to the original conditions (Porter 2003). For example, Mitrani-Reiser et al. (2006) defined damage levels of structural elements as light, moderate, severe, and collapse corresponding to repair with epoxy injections, repair with jacketing, and replacement of the member (for the latter two), respectively. They defined the damage levels of non-structural drywall partitions as visible and significant corresponding to patching and replacement of the partition, respectively.

In the damage analysis phase, the POE and the probability values for the DM are calculated by using fragility functions. A fragility function represents the POE of a damage measure for different values of an EDP. Fragility functions of structural and non-structural components can be developed for the considered facility using experimental or analytical models. Alternatively, generic fragility functions corresponding to a general structure or component type can be used.

The damageable parts of a facility are divided into groups which consist of components that are affected by the same EDP in a similar way, meaning that the components in a group should have the same fragility functions. For example, Bohl (2009) used 16 different groups for a steel moment frame building including the structural system, exterior enclosure, drift sensitive non-structural elements, acceleration sensitive non-structural elements, and office content in each floor. For each (index j) *damageable group* and each (index i) EDP data point (EDP_j^i), the POE of a damage level is available as a point on the related fragility curve (Fig. 2-5). Probability of a damage level is calculated from the POE using the algorithm in Equation 2-4.

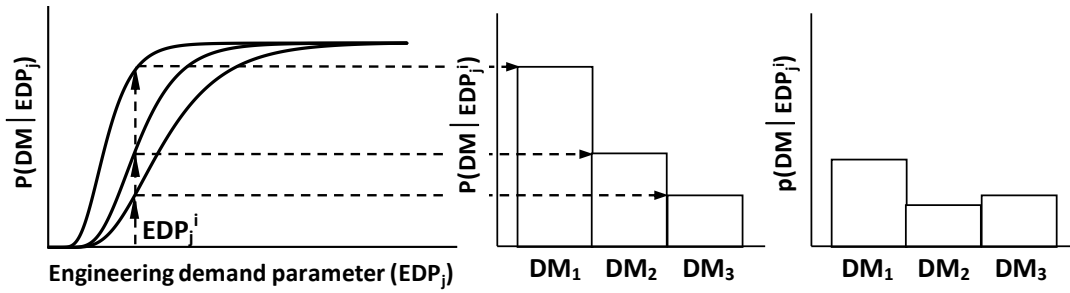


Fig. 2-5: Probability of exceedance P and probability p of a damage level from fragility curves

for $k = 1 : \# \text{ of DM levels}$

$$p(\text{DM}_k | \text{EDP}_j^i) = P(\text{DM}_k | \text{EDP}_j^i) \quad \text{if } k = \# \text{ of DM levels} \quad (2-4)$$

$$p(\text{DM}_k | \text{EDP}_j^i) = P(\text{DM}_k | \text{EDP}_j^i) - P(\text{DM}_{k+1} | \text{EDP}_j^i) \quad \text{otherwise}$$

where “ p ” represents the PMF. Hence, the number of PMF’s obtained for a damageable group from the damage analysis is equal to the number of EDP values that define the fragility function for that group. The total number of PMF’s is equal to the sum of the PMF’s for all damageable groups.

2.2.1.3.4 Loss analysis

Loss analysis is the last stage of PEER PBEE methodology, where damage information obtained from the damage analysis (Fig. 2-5) is converted to the final decision variables. These variables can be used directly by the stakeholders for decision about the design or location of a facility or for other purposes such as comparison with the premium rates. Most commonly utilized decision variables are stated as follows:

1. *Fatalities*: Number of deaths as a direct result of facility damage.
2. *Economic loss*: Monetary loss which is a result of the repair cost of the damaged components of a facility or the replacement of the facility.
3. *Repair Duration*: Duration of repairs during which the facility is not functioning.
4. *Injuries*: Number of injuries, as a direct result of facility damage.

First three of these decision variables are commonly known as deaths, dollars and downtime.

In the loss analysis, the probabilities of exceedance of the losses for different damageable groups at different damage levels (loss functions) are combined to obtain the facility loss curve. This requires use of PDF’s and PMF’s from the hazard, structural and damage analyses (Fig. 2-6, Equation 2-5). Calculation of the loss curve can be summarized in the following steps:

- Determine the loss functions (Fig. 2-6) for each damageable group of the facility for each considered damage level, $P(\text{DV}_j^n | \text{DM}_k)$.
- Determine the probability of exceedance of the n th value of DV for each damageable group of the facility for each value of the EDP utilized in the fragility function of the group, $P(\text{DV}_j^n | \text{EDP}_j^i)$, with Equation 2-5a by considering the loss functions of step 1, $P(\text{DV}_j^n | \text{DM}_k)$, and probabilities for each damage level when subjected to EDP_j^i , i.e. $p(\text{DM}_k | \text{EDP}_j^i)$.
- Determine the probability of exceedance of the n th value of DV for each damageable group of the facility for a given value of intensity measure under the condition that global collapse does not occur, $P(\text{DV}_j^n | \text{NC}, \text{IM}_m)$, with Equation 2-5b by considering the POE calculated in step 2, $P(\text{DV}_j^n | \text{EDP}_j^i)$, and probability of each value of EDP utilized in the fragility function of the group when subjected to the ground motions compatible with the considered intensity measure, $p(\text{EDP}_j^i | \text{IM}_m)$.

- Determine the probability of exceedance of the n th value of DV for the facility for a given value of intensity measure under the condition that global collapse does not occur, $P(DV^n|NC, IM_m)$, by summing up the POE of DV for each damageable group, $P(DV_j^n|NC, IM_m)$, using Equation 2-5c.
- Determine the probability of exceedance of the n th value of DV for the facility for a given value of intensity measure, $P(DV^n|IM_m)$, by summing up the POE of DV for non-collapse and collapse cases weighted with the probabilities of these cases, using Equation 2-5d.
- Finally, determine the probability of exceedance of the n th value of DV for the facility, $P(DV^n)$, by summing up the POE of DV for different intensity measures, $P(DV^n|IM_m)$, multiplied by the probabilities of these intensity measures, $p(IM_m)$, using Equation 2-5e. $P(DV^n)$ represents the POE in “ t ” years, which represents the duration for which the POE values are calculated for the IM in the hazard analysis.

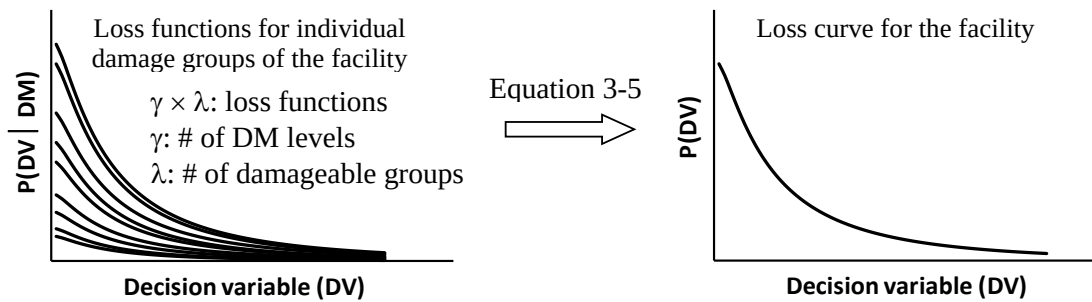


Fig. 2-6: Loss curve from the loss functions for individual damage groups

$$P(DV_j^n | EDP_j^i) = \sum_k P(DV_j^n | DM_k) p(DM_k | EDP_j^i) \quad (2-5a)$$

$$P(DV_j^n | NC, IM_m) = \sum_i P(DV_j^n | EDP_j^i) p(EDP_j^i | IM_m) \quad (2-5b)$$

$$P(DV^n | NC, IM_m) = \sum_j P(DV_j^n | NC, IM_m) \quad (2-5c)$$

$$P(DV^n | IM_m) = P(DV^n | NC, IM_m) p(NC | IM_m) + P(DV^n | C) p(C | IM_m) \quad (2-5d)$$

$$P(DV^n) = \sum_m P(DV^n | IM_m) p(IM_m) \quad (2-5e)$$

In Equation 2-5, $P(DV_j^n | DM_k)$ is the POE of the n^{th} value of the decision variable for the j^{th} damageable group of the facility when damage level k takes place (loss functions in Fig. 2-6), $p(DM_k | EDP_j^i)$ is the probability of the damage level k when it is subjected to the i^{th} value of the EDP utilized for the fragility function of the j^{th} damageable group (fragility function in Fig. 2-5), $p(EDP_j^i | IM_m)$ is the probability for the i^{th} value of the j^{th} EDP (EDP utilized for the fragility function of the j^{th} damageable group) for the m^{th} value of IM (structural analysis outcome in Fig. 2-4), and $p(IM_m)$ is the probability of the m^{th} value of IM (hazard analysis outcome in Fig. 2-3). Moreover, $p(C | IM_m)$ and $p(NC | IM_m)$ are the probabilities of having and not having global collapse, respectively, under the ground motion intensity IM_m as explained in the structural analysis sub-section. Finally, $P(DV^n | C)$ and $P(DV^n | NC)$ are the POE of the n^{th} value of DV in cases of global collapse and no global collapse, respectively. Krawinkler (2005) assumed a lognormal distribution for $P(DV^n | C)$ when the DV is the economic loss.

Additional comments about Equation 2-5 and loss analysis can be stated as follows:

- Equation 2-5 consists of summations instead of integrals since the probabilities for each damage level when subjected to EDP_j^i , $p(DM_k|EDP_j^i)$, in Equation 3-5a are discrete values. Therefore, all above equations are based on discrete values and summations.
- The loss curve defined with Equation 2-5e considers all the possible scenarios for hazard, whereas in some of the applications of PEER PBEE methodology, few IM values (e.g. IM for 2%, 10%, and 50% POE in 50 years) are considered separately. In this case, Equation 2-5e is not used and the loss curves for individual IM's (different scenarios) are defined with Equation 2-5d.
- From Equation 2-5d, the POE of the decision variable in case of collapse, $P(DV_n|C)$, is not conditioned on the intensity measure, whereas the POE of the decision variable in case of no collapse, $P(DV_n|NC, IM_m)$, is conditioned on the intensity measure, since "no collapse" case consists of different damage states and the contribution of each of these damage states to the "no collapse" case changes for different intensity measures. For example, loss function for slight damage will have the highest contribution for a small value of the intensity measure; whereas the loss function for severe damage will have the highest contribution for a large value of the intensity measure.
- It should be mentioned that Equation 2-5c is not exact since it neglects the multiplication terms that result as a convolution of the probability of different damageable groups. However, for the practical range of resulting probabilities, these terms are small enough to be neglected, which validates the use of Equation 2-5c as a very close approximation to the exact formulation.
- Variations in the above formulation are possible by using different decision variables and methods to express the outcome. As an example, DV's can be expressed with POE as shown in Equation 2-5 or with expected values, simply by replacing the POE values in Equation 2-5 with the expected values, e.g. $E[DV_j|EDP_j^i]$ instead of $P(DV_j^n|EDP_j^i)$.

2.2.1.4 Application of PEER formulation

In this section, application of PEER PBEE methodology is presented for University of California Science (UCS) building located at UC-Berkeley campus. A MATLAB (Mathworks 2008) script, excerpts from which are provided in the Appendix, is developed to combine results from hazard, structural, damage, and loss analyses as defined by Equation 2-5. Considered building is a modern RC shear-wall building which provides high technology research laboratories for organismal biology. Besides the research laboratories, the building contains animal facilities, offices and related support spaces arranged in six stories and a basement. The building is rectangular in plan with overall dimensions of approximately 93.27 m (306 ft) in the longitudinal (north-south) direction and 32 m (105 ft) in the transverse (east-west) direction (Comerio 2005). A RC space frame carries the gravity loads of the building, and coupled shear-walls and perforated shear-walls support the lateral loads in the transverse and the longitudinal directions, respectively, as shown in Fig. 2-7. The floors consist of waffle slab systems with solid parts acting as integral beams between the columns. The waffle slab is composed of a 114 mm (4.5 in.) thick RC slab supported on 508 mm (20 in.) deep joists in each direction. The foundation consists of a 965 mm (38 in.) thick mat.

This building is an example for which the non-structural components contribute to the PBEE methodology in addition to the structural components due to the valuable building contents, i.e. the laboratory equipment and research activities. Detailed information about the contents inventory and their importance can be found in (Comerio 2005).

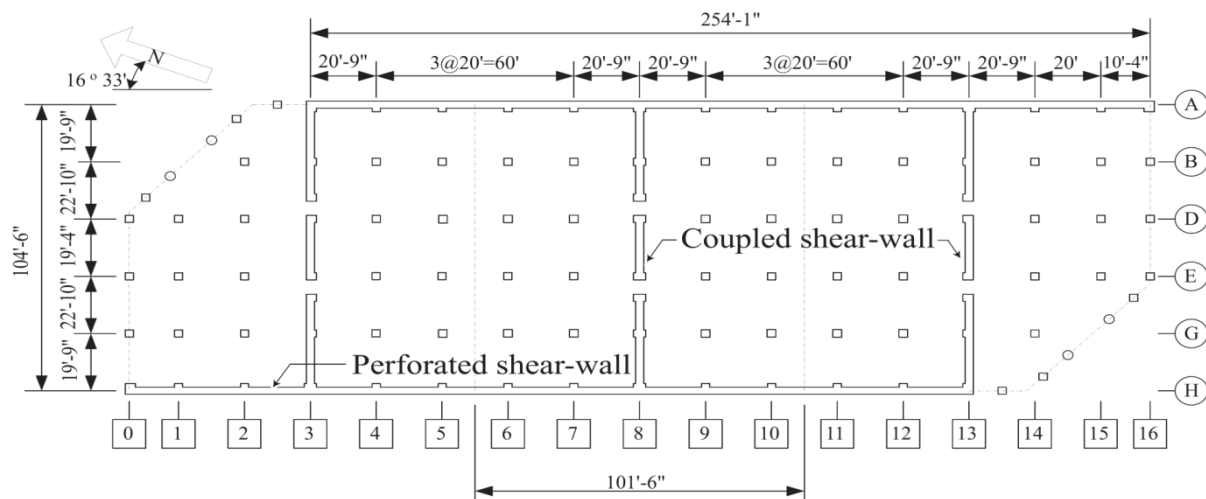


Fig. 2-7: Plan view of the UCS building (Lee and Mosalam 2006)

2.2.1.4.1 Hazard analysis

The UCS building, which is located in the southwest quadrant of the campus of UC-Berkeley, is within a 2 km (1.243 mile) distance from the Hayward fault (Comerio 2005), an active strike-slip fault with an average slip rate of 9 mm/yr (0.354 in/yr). The latest rupture of its southern segment (Fremont to somewhere between San Leandro and Berkeley) occurred on 21 October 1868, producing a magnitude 7 earthquake. Frankel and Leyendecker (2001) provide the mean annual exceedance frequency of spectral acceleration (S_a) at periods of 0.2, 0.3 and 0.5 seconds and B-C soil boundary as defined by the International Building Code (International Code Council 2000) for the latitude and longitude of the site of the building. Lee and Mosalam (2006) assumed a lognormal distribution of S_a with the mean of 0.633 g and standard deviation of 0.526 g, which is a good fit for the POE of S_a for $t = 50$ years obtained from Equation 2-1 using the mean annual exceedance frequency suitable for the period ($T = 0.38$ sec) and local site class (C) of the building. Considered mean annual frequency is plotted in Fig. 2-8. Probability $p(IM_m)$ in Equation 2-5e and POE values for discrete values of S_a between 0.1 g and 4.0 g with 0.1 g increments are shown in Fig. 2-9.

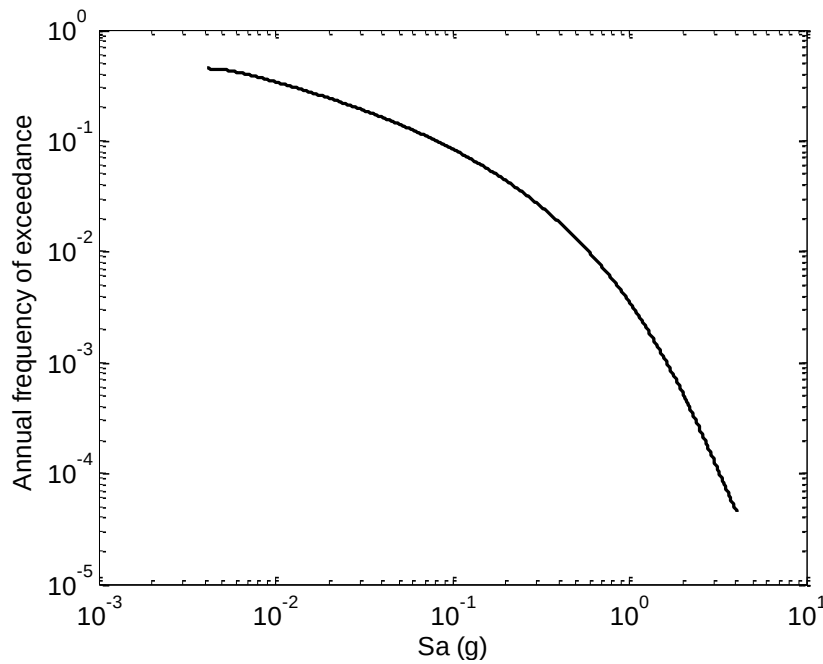


Fig. 2-8 Mean annual frequency of exceedance of S_a for the site of UCS building

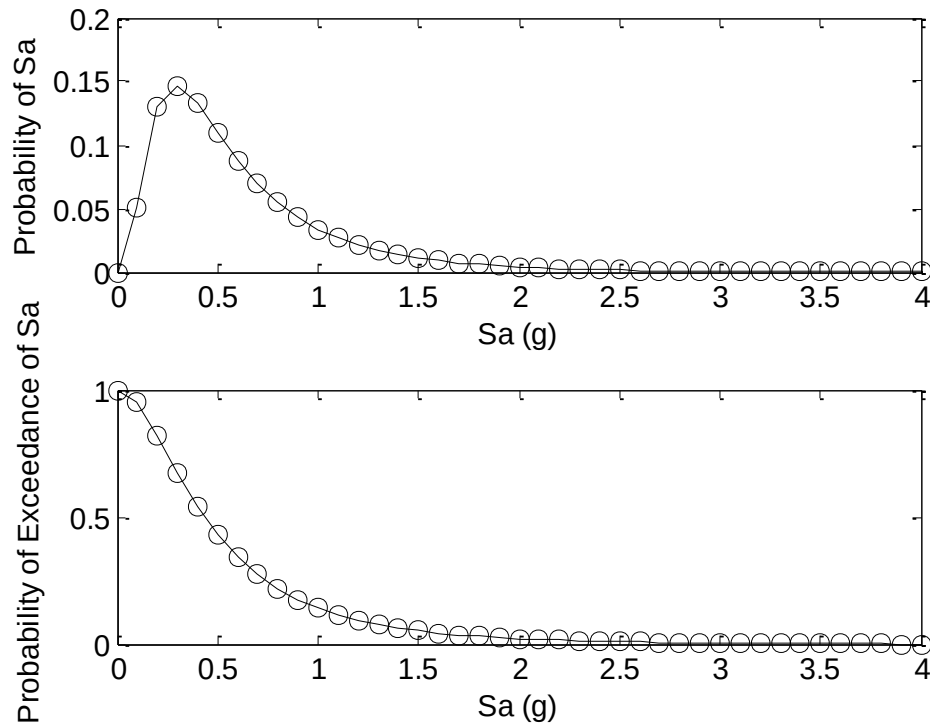


Fig. 2-9: Probability $p(IM_m)$ and probability of exceedance $P(IM_m)$ of S_a in 50 years for UCS building site

2.2.1.4.2 Structural analysis

Although it is possible to select more than two damageable groups, for brevity of the discussion, only two damageable groups are considered for the UCS building, namely (1) structural components and (2) non-structural components. Maximum peak interstorey drift ratio along the height (MIDR) and peak roof acceleration (PRA) are considered as the EDPs. Lee and Mosalam (2006) conducted nonlinear analyses of the building using 20 ground motions, which are selected as ground motions that have the same site class as the building site and distance to a strike-slip fault similar to the distance of the UCS building to Hayward fault. Ten different scales of these ground motions are considered. These scales are modified

for each ground motion to match S_a at the period of the UCS building first mode, $S_a(T_1)$, to S_a corresponding to POE of 10% to 90% with 10% increments from the hazard analysis (Fig. 2-9) as presented in Table 2-1. Lee and Mosalam (2006) calculated median and coefficient of variation (COV) of the selected EDPs for each S_a value. These data are fitted by quadratic or linear relationships as shown in Fig. 2-10 and extrapolated for S_a values for which data were not present. Capping values are considered for PRA considering that it would be limited by the base shear capacity of the structure. For each value of S_a , lognormal distribution is assumed for both of the EDPs with the median and COV obtained from the fitted relationships in Fig. 2-10. Probability for the discrete values of MIDR between 0.0001 and 0.04 with 0.0001 increments and for the discrete values of PRA between 0.001 g and 4.0 g with 0.001 g increments are plotted in Fig. 2-11 for example values of $S_a=0.5$ g, 2.0 g, and 3.0 g. These probabilities correspond to $p(\text{EDP}_j | \text{IM}_m)$ in Equation 2-5b. Cumulative distributions of MIDR and PRA for the same S_a values, which are obtained by the cumulative summation of the probabilities, are plotted in Fig. 2-12.

Table 2-1: S_a corresponding to different probability of exceedance values

Probability of exceedance (%)	90	80	70	60	50	40	30	20	10
S_a (g)	0.18	0.25	0.32	0.39	0.47	0.57	0.71	0.90	1.39

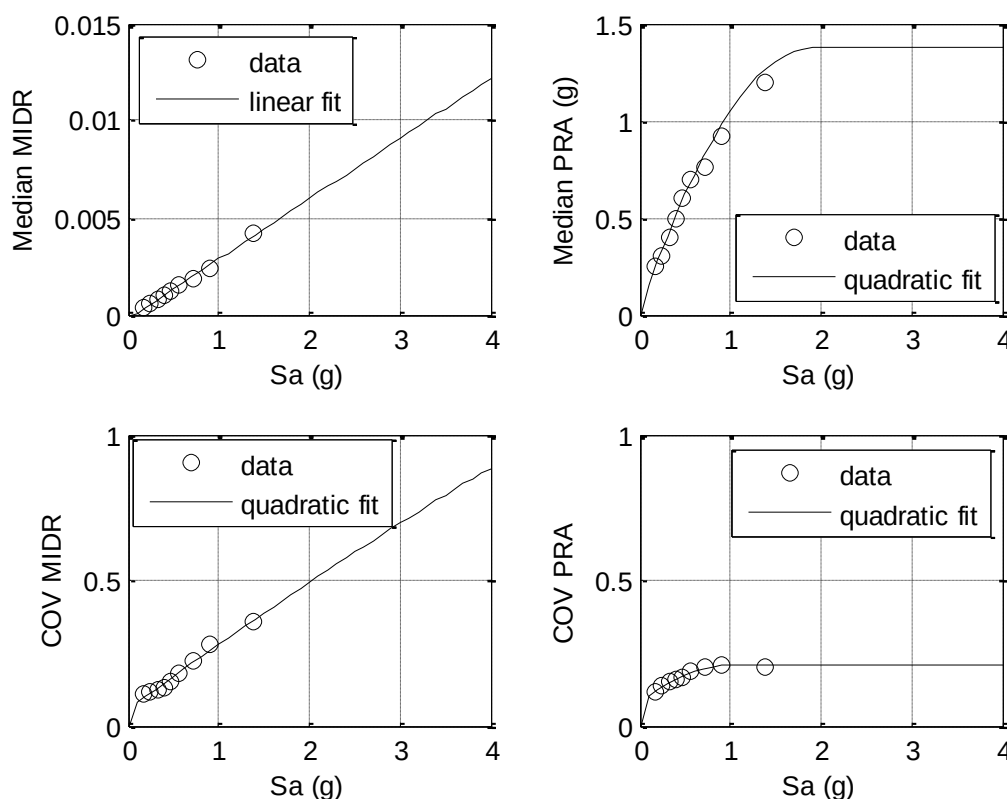


Fig. 2-10: Regression of median and COV data from (Lee and Mosalam 2006): MIDR (left) and PRA (right).

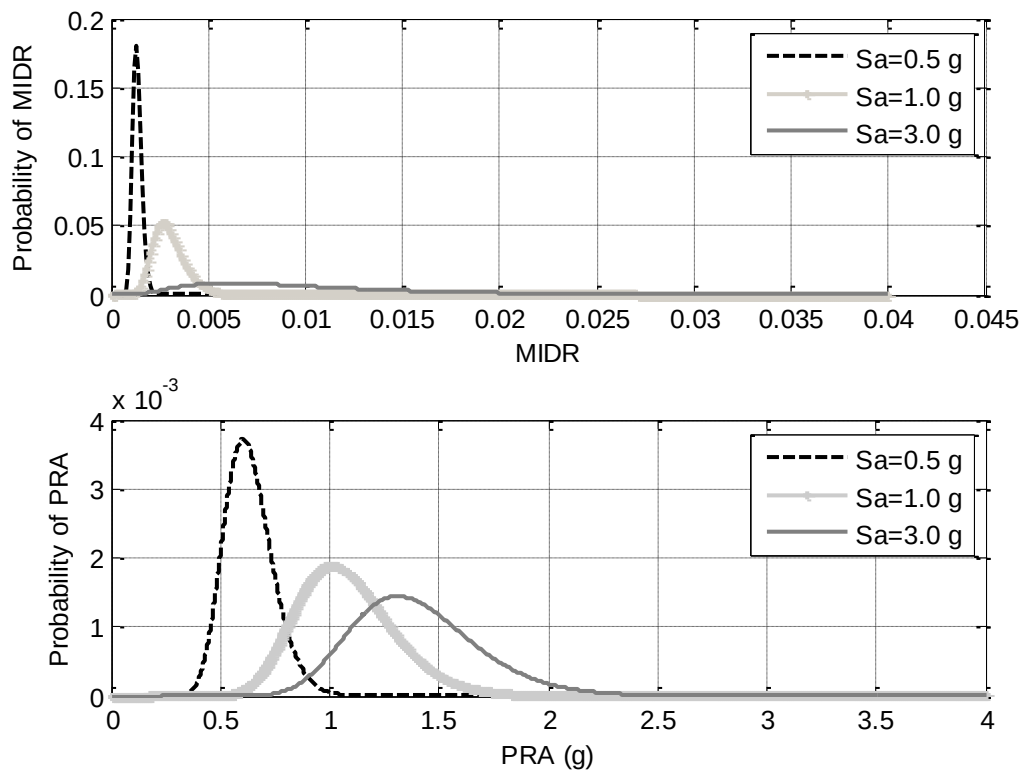


Fig. 2-11: Probability distributions of MIDR and PRA for different values of S_a , $p(\text{EDP}_{ji}|\text{IMm})$

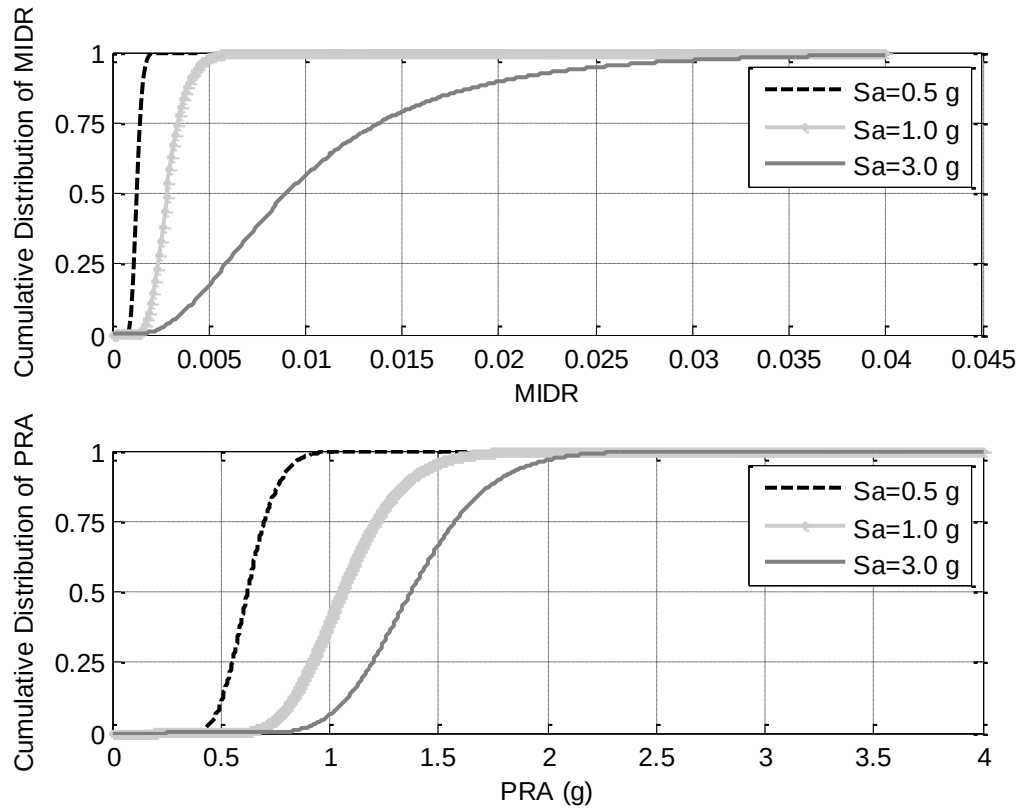


Fig. 2-12: Cumulative distributions of MIDR and PRA for different values of S_a

2.2.1.4.3 Damage analysis

Regarding damage analysis, fragility functions are obtained for the two damageable groups. Two and three damage levels are considered for non-structural and structural components, respectively. Damage levels considered for the structural components are slight, moderate, and severe damages. On the other hand, damage levels of the non-structural components are based on the maximum sliding displacement experienced by scientific equipment relative to the bench-top surface (Chaudhuri and Hutchinson 2005). Sliding displacement levels of 5 cm (0.2 in.) and 10 cm (0.4 in.) are considered as the two damage levels for the non-structural components.

The probability of a damage level given a value of engineering demand parameter, $p(DM_k|EDP_j^i)$, is assumed to be lognormal. Median and logarithmic standard deviation values for the damage levels of structural and non-structural components of the UCS building are shown Table 2-2. Values for the structural components are based on the work of (Hwang and Jaw 1990) and those for the non-structural components are obtained from the study by (Chaudhuri and Hutchinson 2005). Corresponding fragility curves for structural and non-structural components are shown in Fig. 2-13 and Fig. 2-14, respectively. It should be noted that, $p(DM_k|EDP_j^i)$ is used in Equation 2-5a, rather than $P(DM_k|EDP_j^i)$ defined by the fragility curve. However, fragility curve is plotted here, rather than $p(DM_k|EDP_j^i)$, since it is a commonly used representation in literature.

Table 2-2: Median and logarithmic standard deviation of EDP's for different damage levels

Component	Damage level	EDP	Median	Standard deviation
Structural	Slight	MIDR	0.005	0.30
	Moderate	MIDR	0.010	0.30
	Severe	MIDR	0.015	0.30
Non-structural	DM = 5 cm	PRA (g)	0.005	0.35
	DM = 10 cm	PRA (g)	0.010	0.28

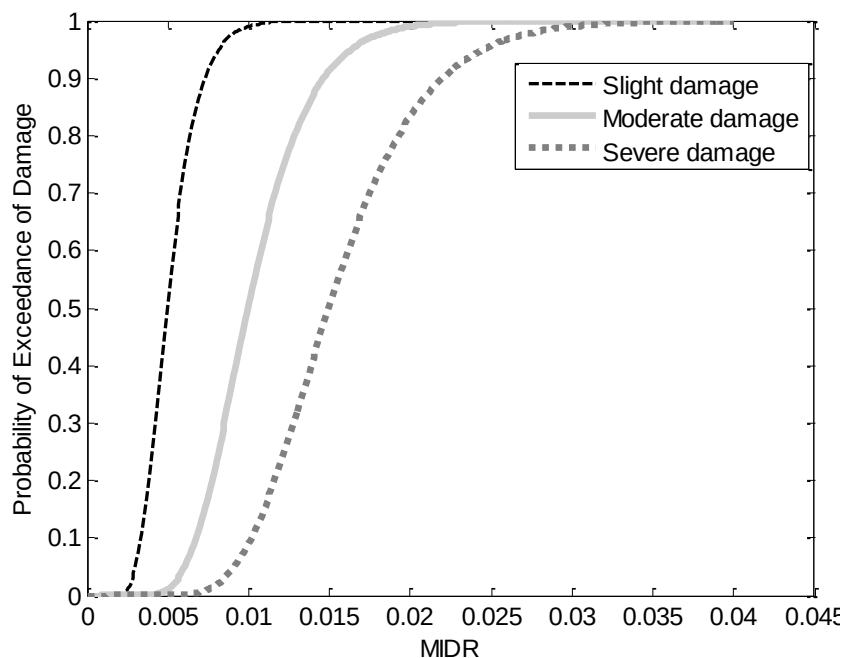


Fig. 2-13: Fragility curves for structural components, $P(DM_k|EDP_1^i)$

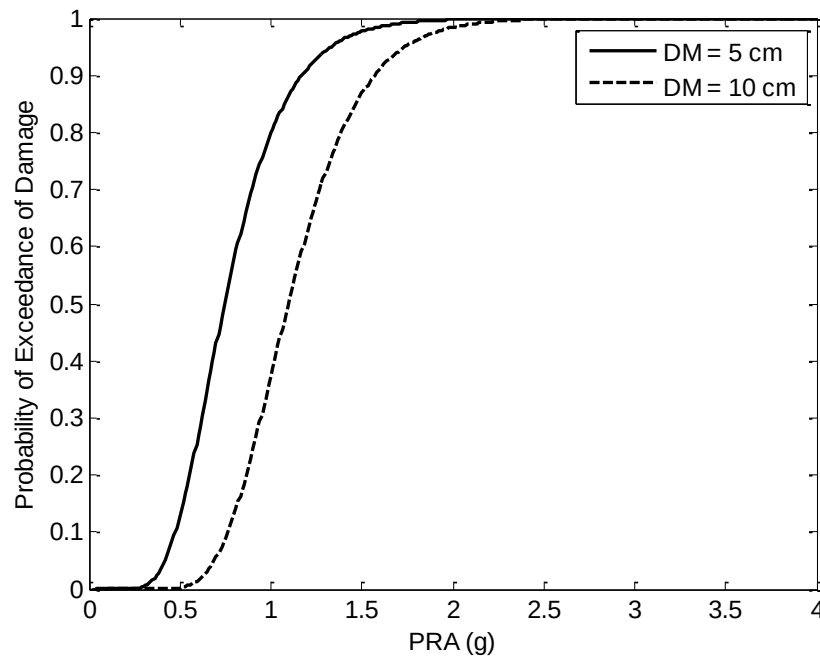


Fig. 2-14: Fragility curves for nonstructural components, $P(DM_k|EDP_2^i)$

2.2.1.4.4 Loss analysis

Monetary loss is chosen as the decision variable for the case study building. The loss functions are derived from the available reports on the case study building assuming that the probability distribution of monetary loss for a damage level is lognormal. Assumptions about the lognormal parameters were necessary, since fewer amounts of data are available in literature related to loss analysis than to the other analysis stages of PEER PBEE methodology.

Total value of the scientific equipment is estimated to be \$23 million (Comerio, 2003). Median values corresponding to the damage levels of 5 cm and 10 cm of sliding displacements are assumed to be \$6.90 million (30% of the total value) and \$16.10 million (70% of the total value), respectively. Standard deviation is assumed to be 0.2 for both of these non-structural component damage levels.

There is no available information about the monetary losses related to the structural components. However, since the contents damage has more significance for the building relative to the structural damage, median monetary losses for the slight, moderate and severe damage levels are assumed to be \$1.15 million, \$3.45 million and \$6.90 million, respectively, which correspond to 5%, 15% and 30% of the total value of the non-structural components. Standard deviation is assumed to be 0.4 for all the structural damage levels, which is larger than the standard deviation value for the non-structural components since a larger variation can be expected due to the lack of information. Resulting loss functions for structural and non-structural components are shown in Fig. 2-15 and Fig. 2-16, respectively.

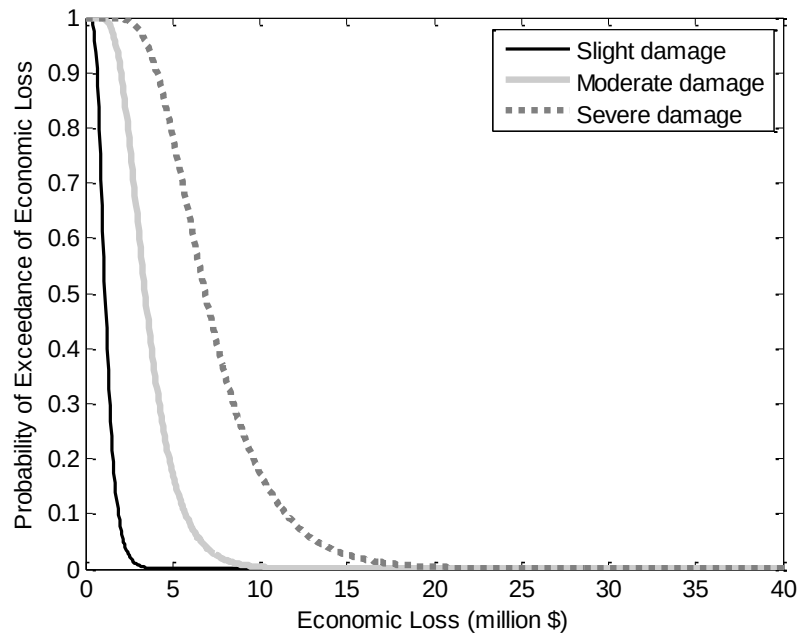


Fig. 2-15: Loss functions for structural components, $P(DV_1^n|DM_k)$

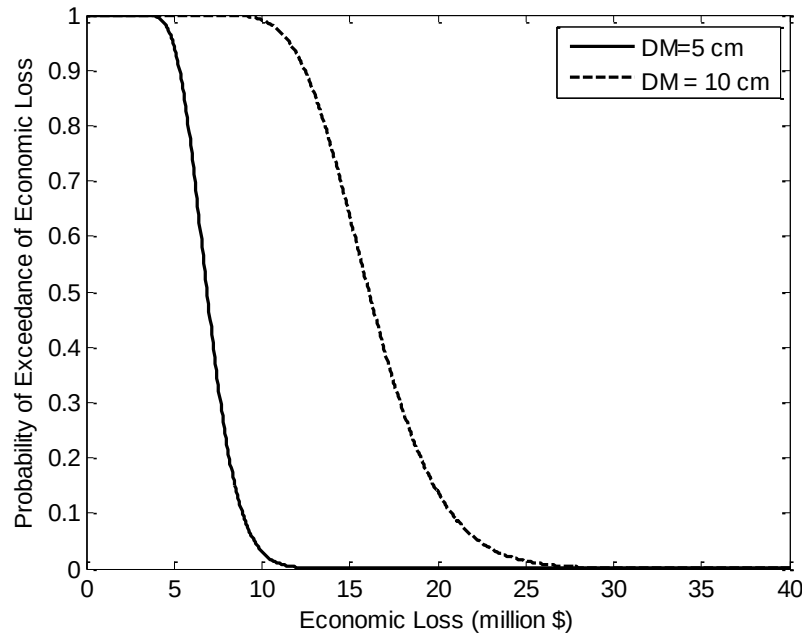


Fig. 2-16: Loss functions for non-structural components, $P(DV_2^n|DM_k)$

2.2.1.4.4.1 Collapse analysis

Determination of the loss curve requires the knowledge of the POE of monetary loss in case of global collapse, $P(DV|C)$, and the probability of global collapse, $p(C|IM_m)$ as shown in Equation 2-5d. The probability of the monetary loss in case of global collapse is assumed to be lognormal with the median of \$30 million, which corresponds to the total value of structural and non-structural components, and standard deviation of 0.2. The resulting loss function is shown in Fig. 2-17 with the loss functions for the damage levels of the structural damageable group, given previously in Fig. 2-15. The difference between the loss function for collapse and that for other damage levels emphasizes the importance of the non-structural building contents.

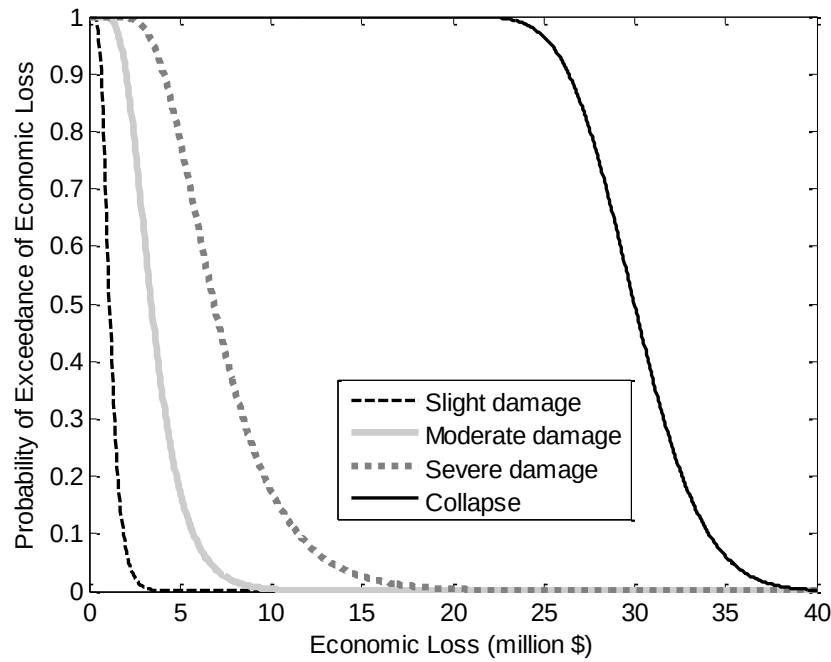


Fig. 2-17: Loss functions for collapse, $P(DV^n|C)$, and damage levels of the structural damageable group, $P(DV^n|DM_k)$

The probability of global collapse is determined in this application in a simplified manner by using the probability distribution of MIDR obtained from the structural analysis for each intensity measure. The median global collapse MIDR is accepted as 0.018 based on the study of Hwang and Jaw (1990). Probability of global collapse for each intensity measure is calculated by summing up the probabilities of MIDR values greater than the median collapse MIDR (shaded area in Fig. 2-18) for that intensity level. The resulting probability of global collapse and no global collapse data are plotted in Fig. 2-19.

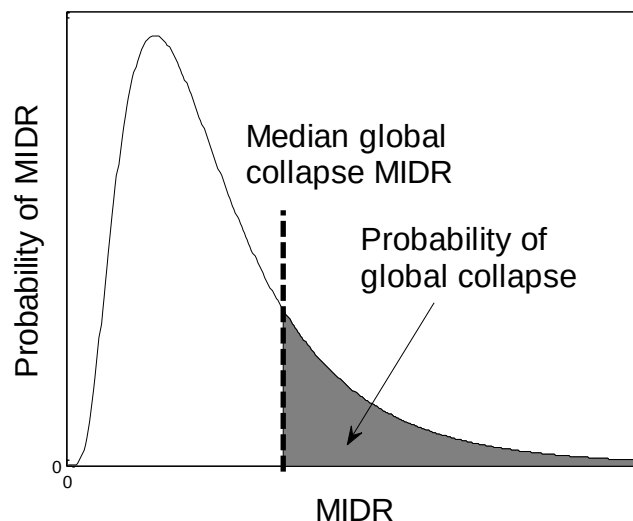


Fig. 2-18: Probability of global collapse from the MIDR probability distribution

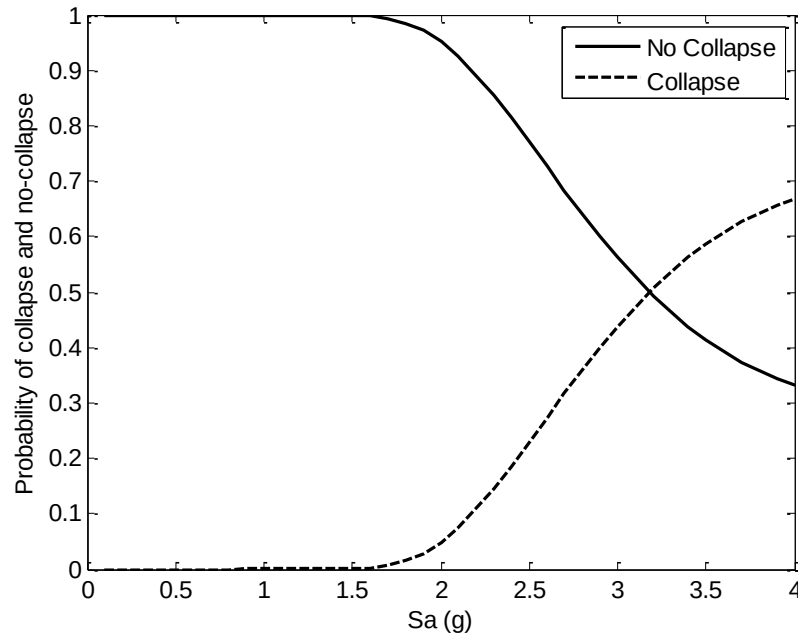


Fig. 2-19: Probability of global collapse, $p(C|IM_m)$, and no global collapse, $p(NC|IM_m)$, cases as a function of intensity measure

2.2.1.4.4.2 Determination of loss curves

The resulting loss curves obtained using Equation 2-5 are plotted in Fig. 2-20 and Fig. 2-21, where the vertical axes are POE in 50 years and the annual frequency of exceedance (calculated with Equation 2-1 by replacing IM with DV^n) respectively. The POE of the monetary loss is deaggregated to the POE due to global collapse and no global collapse in Fig. 2-22. It is observed that no global collapse case is more dominant on the loss curve for monetary losses less than \$10 million, where all the loss comes from global collapse for monetary losses greater than \$25 million.

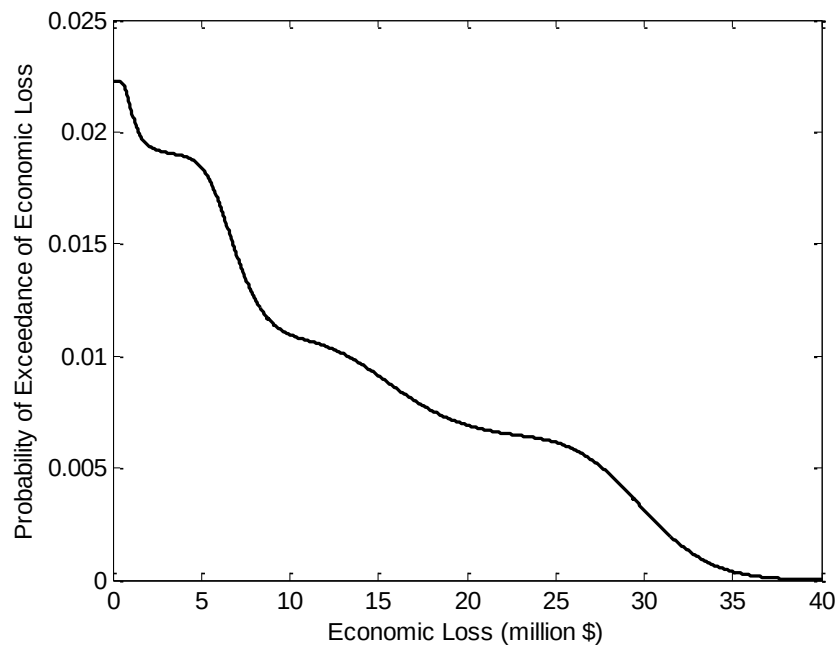


Fig. 2-20: Loss curve in terms of the probability of exceedance, $P(DV^n)$

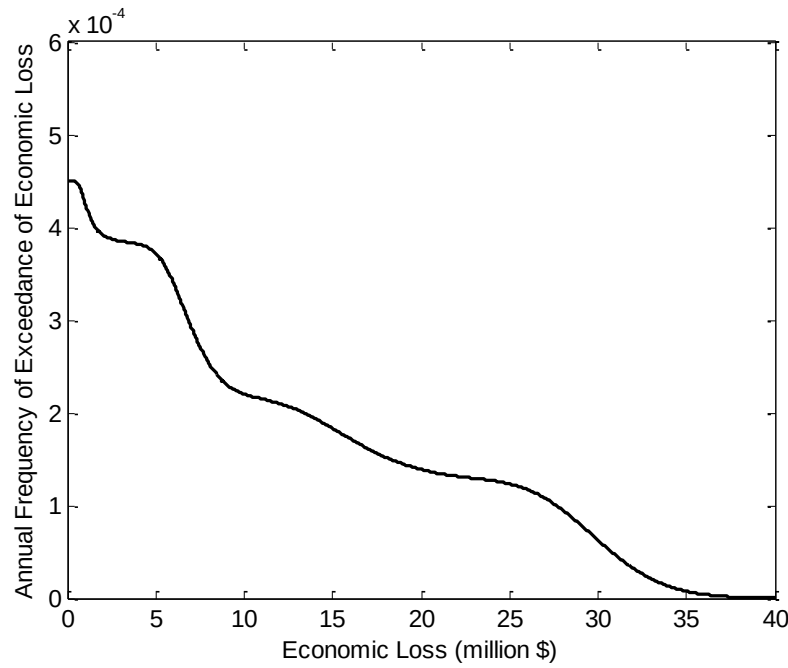


Fig. 2-21: Loss curve in terms of the annual frequency of exceedance, $\lambda(DV^n)$

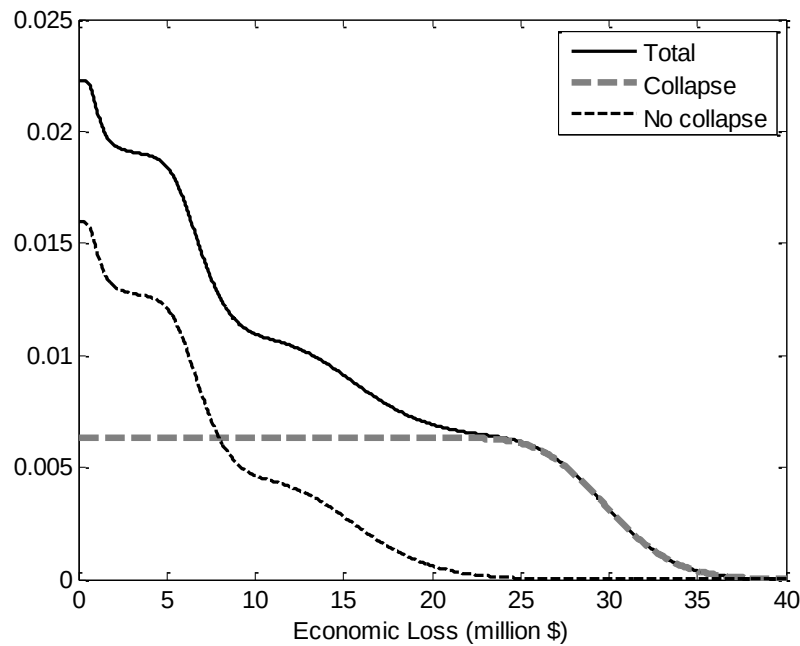


Fig. 2-22: Contribution of global collapse and no global collapse cases to the loss curve

2.2.1.5 Closure

From a design point of view, a designer has control on the results of the structural analysis stage. Hence, a designer can improve the loss curve by improving the response of the building with a different design. Fig. 2-23 shows the improvement of the loss curve for a hypothetical case where collapse is prevented for all intensity levels. In this regard, innovative and sustainable design and retrofit methods such as base isolation, rocking foundations, and self-centering systems are suitable candidates to be evaluated with PEER PBEE methodology as well as some conventional and existing structural types, such as moment resisting frames with unreinforced masonry infill walls.

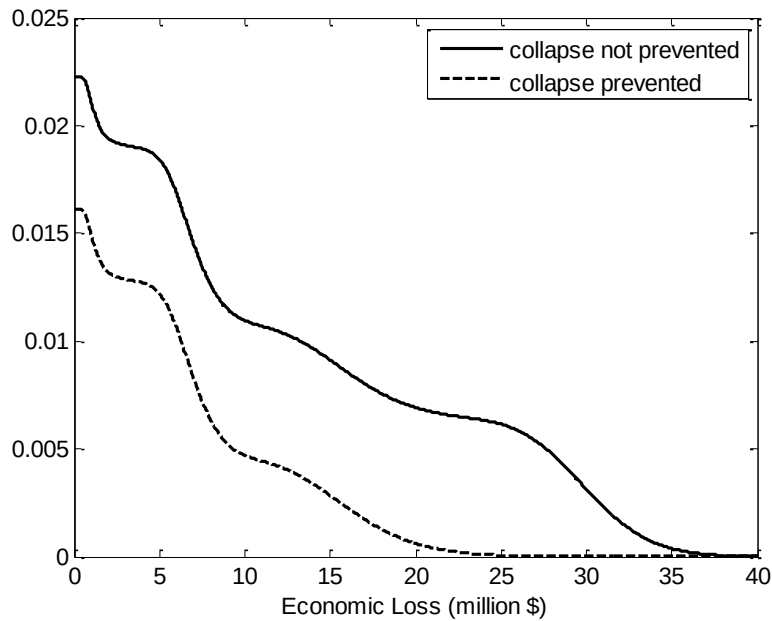


Fig. 2-23: Improvement of the loss curve due to collapse prevention

2.2.2 SAC/FEMA formulation

The SAC/FEMA Method (Cornell et al. 2002) was developed in the late nineties when a large body of research studies triggered by the 1994 Northridge earthquake coalesced into guidelines for the performance assessment of both existing and new buildings (SAC 2000a,b). This method is fully probabilistic. It was originally developed for assessing the seismic performance of steel-moment-resisting-frame (SMRF) buildings such as those that underperformed when they experienced the Northridge ground shaking by showing fractures in many beam-column joints when they should have been still in the elastic domain. This method, however, is general and can be applied (and has) to other types of buildings with only minor conceptual adjustments. It is an empirical method based on the assumption that an engineer could use earthquake ground motion records (either real or synthetic) and a “suitable” computer representation of a building to test the likelihood that the building will perform as intended over the period of time of interest. Suitable here means that the computer model is able to adequately capture the response of a building way past its elastic regime up to global collapse (or to the onset of numerical instability) and that the computed response is realistic.

The SAC/FEMA method, which was developed much earlier than the PEER formulation discussed in Subsection 2.2.1, can be thought as a special application of the more general PEER framework (Vamvatsikos & Cornell 2004). The SAC/FEMA method simplifies the treatment of two out of the four PEER variables, namely the Decision Variable (DV) and the Damage Measure (DM). Thus, the structural performance is explicitly assessed using only the Engineering Demand Parameter (EDP) and the Intensity Measure (IM). More precisely, in the SAC/FEMA method, the Decision Variable (DV) is a binary indicator (namely, a variable that can take on only values of 0 or 1) of possible violation of the performance level (0 means no violation, 1 means violation). In other words, this method is not aimed at assessing repair cost, downtime or casualties, but only at whether an engineering-level limit-state has been violated or not. This will essentially remove DV from the formulation. In addition, the state of a certain damage, as represented by the Damage Measure (DM, e.g. spalling of concrete from columns and beams), is assumed to be fully specified based on the value attained by the engineering demand parameter (EDP) adopted to gauge the structural performance (e.g. maximum interstorey drift of, say, 0.5%). Given this assumption that a value of a single EDP

is sufficient to identify with relative certainty and without bias the damage state that a structure is in (which, of course, is not entirely true) then the variable DM becomes redundant and can be removed from the discussion. This conceptual simplification has allowed the SAC/FEMA method to be closer to a practical implementation in real life applications. The SAC/FEMA method and its application are explained in simple steps below.

2.2.2.1 Motivation for the SAC/FEMA method

As it has been already mentioned in previous sections (e.g. §2.2.1.2), the conventional way of assessing whether a building³ is fit for purpose is to check whether its performance is acceptable for one or more levels of ground motions expected to be exceeded at the building site with specified annual probabilities. The criteria on the acceptable performance levels were usually fairly loosely stated but, in short, the building was supposed not to collapse when subject to a very rare level of shaking (e.g. 2% POE one or more times in 50 years) and to remain operable with negligible damage and, therefore, negligible probability to injure or kill its occupants, when subject to a frequent shaking level (e.g. 10% POE in 50 years). Specifications included in some codes for designing buildings and other structures are based only on the former requirement, namely a check that the ultimate state is not reached for the specified level of shaking, while more commonly codes include a dual-level design and enforce requirements for serviceability state as well.

If an engineer were to accept that the world is deterministic, then if he/she observes a structure (or better a suitable computer representation of it) not collapsing for the 2% in 50yrs level of shaking then he/she could conclude that the annual probability of global collapse, P_{GC} , of that building would certainly be less than 2% in 50yrs (i.e. about 1/2,500 chance every year or an annual POE of 4×10^{-4}). Unfortunately, there are many sources of uncertainty in this problem that need to be taken into account for a realistic assessment of the collapse probability of this building. What we do not know about the actual building behaviour makes the estimates of its true but unknown annual probability of collapse much higher than 4×10^{-4} . Also, what if the structure does collapse for some but not all the ground motions consistent with the 2% in 50yrs level of shaking?

2.2.2.2 The formal aspects of the SAC/FEMA method and its limitations

The SAC/FEMA project makes an attempt to systematically consider all the sources of uncertainty and, by making use of simplifications based on tenable assumptions, to present the computations needed to estimate P_{GC} , or the probability of any other damage state, in a more tractable manner. The sources of uncertainty that need be considered are numerous and pervasive. Historically, the nomenclature related to uncertainty has been ambiguous and, often, misused. A useful although rather obscure way of classifying uncertainty is to divide it into two classes: *aleatory* uncertainty and *epistemic* uncertainty⁴. The former, sometimes called randomness, refers here to natural variability that, at this time, is beyond our control, such as the location and the magnitude of the next earthquake and the intensity of the ground shaking generated at a given site. The latter, often simply called uncertainty, is due to the limited knowledge of the profession and it could potentially be reduced by collecting more

³ In this document we make no difference between checking the performance of an existing building or of a design of a new one. We will also not differentiate among buildings and other engineering structures, such as bridges, power plants, offshore platforms or dams. We will simply refer to them as buildings.

⁴ Note that this division is purely theoretical and the borders between these two classes of uncertainty are blurry. Also, strictly speaking, there is no need to divide uncertainty into two categories but doing so is often helpful.

data or doing more research. We will distinguish the nature of the uncertainty below when needed.

As alluded to earlier, conceptually to estimate whether a building will perform according to expectations one could build a computer model of it and subject it to a large amount of ground motions similar to those that the building could experience at the site. One could simply monitor which ground motions would fail the building⁵ and, knowing their likelihood of occurrence in the considered time period, one could estimate the desired probability, P_{PL} , that the performance level is not met. This brute-force method requires an unmanageable computational burden. The SAC/FEMA method still uses ground motions and computer models of a building to establish its performance but uses these tools more judiciously.

The SAC/FEMA method consists of two steps. More formally, although somewhat loosely⁶, the first step in its more basic form can be summarized as follows

$$P[D \geq d] = \sum_{\text{all } x_i} P[D \geq d \mid IM = x_i] P[IM = x_i] \quad (2-4)$$

The probability P that an EDP demand variable D equals or exceeds a specified level d (e.g. 3% maximum interstorey drift) is simply the sum of the probabilities that the same EDP demand level is equalled or exceeded when the building experiences ground motions (denoted here by the intensity measure IM) of different intensities (denoted here by x_i) times the probabilities that those levels of IM are observed at the site. Note that “demand” here may mean, for example, any measure of deformation imposed by the ground shaking that is meaningful for assessing its performance (e.g. maximum interstorey drift to estimate collapse probability of a SMRF building). What is implicit in this equation is the following:

One does not know what level of shaking will the building experience in the period of interest and, therefore, many intensity levels $IM = x_i$ need be considered and appropriately weighted by their probability of occurrence $P[IM = x_i]$ at the building site. These probabilities are given to engineers by earth scientists who perform Probabilistic Seismic Hazard Assessment (PSHA) studies. The format more frequently used is that of a hazard curve, $H(im)$ (Fig. 2-24) which gives the annual rate $IM \geq x_i$ from which the desired probability of “equaling” can be easily derived as $P[IM = x_i] = P[IM \geq x_{i-1}] - P[IM \geq x_{i+1}]$ where $x_{i-1} \leq x_i \leq x_{i+1}$.

The building can fail (or survive) if subject to ground motions of very different intensity levels. Unlike the traditional deterministic viewpoint, there is a finite likelihood that a fairly weak ground motion will cause failure of the building and, conversely, that an extremely violent shaking supposedly exceeding the building capacity will not. These chances need be accounted for.

The characteristics of a ground motion, namely its frequency content, sequence of cycles and phases, are condensed into only one scalar measure of its intensity, namely the IM. In the applications of the SAC/FEMA method, this quantity is typically $S_a(T_1)$, the 5%-damped, linear elastic, pseudo-spectral acceleration at the fundamental period of vibration of the building, T_1 . This simplification is needed to make the method more mathematically tractable, as it will become apparent later, but has some undesirable consequences:

Given that a building is not a single-degree-of-freedom (SDOF) oscillator, two different accelerograms with the same value of $IM = S_a(T_1)$ will cause different responses in a building. Therefore, this implies that many records with $S_a = x_i$ need to be run through the building computer model to estimate $P[D \geq d \mid S_a = x_i]$.

⁵ Failure here should be interpreted as failure of meeting the specified performance level, PL , which could refer to a ultimate limit state (global collapse, called GC above) or to a serviceability limit state (e.g. onset of damage). Unless specified in the text, we make no distinction here about the severity of the damage state.

⁶ The probability $P[S_a = x_i]$ should be thought as the probability that S_a is in the neighborhood of x_i .

Eq. (2-4) is based on a probabilistic concept called Theorem of Total Probability. The conditioning on only one variable, $IM=S_a(T_1)$, implies that the records with that value of $S_a(T_1)$ are chosen in such a way that all the other characteristics of the time histories selected, which may potentially affect the building response (e.g. frequency content) are statistically consistent with those that may be experienced at the site. If they are not (e.g. an average spectral shape not consistent with that of ground motions generated by earthquakes in the region around the site) then the estimate of $P[D \geq d | S_a = x_i]$ may be tainted.

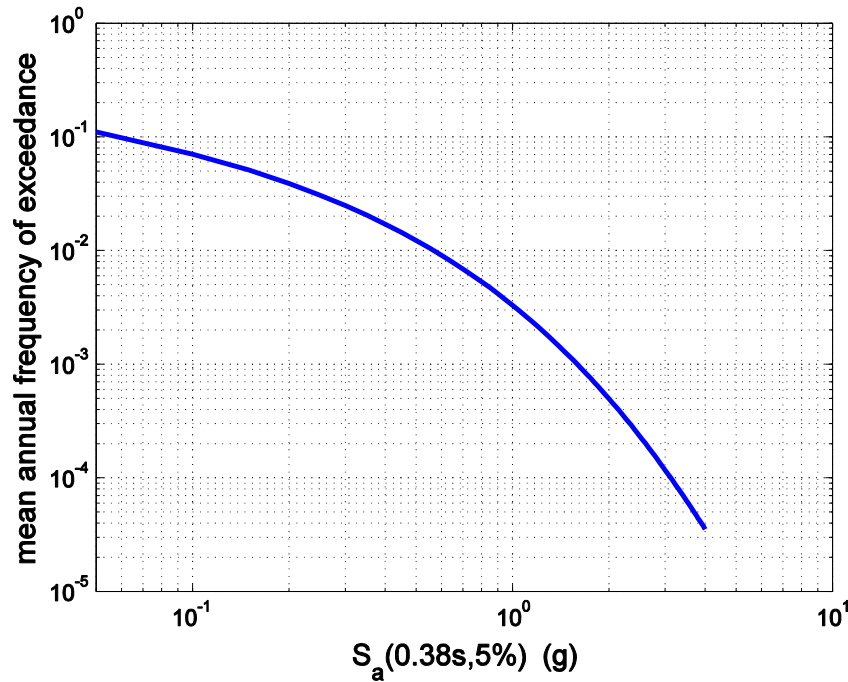


Fig. 2-24: Seismic hazard curve for the intensity measure $S_a(T_1)$ for $T_1=0.38$ s

If the engineer were to be sure that the building under consideration would fail its performance level when it reached an EDP demand D equal to d or larger, then the probability P_{PL} he sought would be provided by Eq. (2-4), that is $P_{PL} \approx P[D \geq d]$. Unfortunately, Eq. (2-4) takes the engineer only mid-way to where he/she needs to go because there is uncertainty in what the EDP capacity of the building, expressed in terms of the same parameter, really is. As before, there is a finite likelihood that the building will not meet its performance level even if the demand is lower than the expected capacity of the building and likelihood that the performance level will be met even if the demand is larger than the expected EDP capacity. These probabilities will need to be quantified and accounted for.

Therefore, the SAC/FEMA method has a second step:

$$P_{PL} = P[C \leq D] = \sum_{\text{all } d_i} P[C \leq D | D = d_i] P[D = d_i], \quad (2-5)$$

which in mathematical terms states what was said above, namely that one needs to account for the probability that the EDP capacity will be smaller than the EDP demand for any given level of demand, d_i , that the building may experience (i.e. $P[C \leq D | D = d_i]$). These probabilities need to be weighted by the probabilities that the demand will be reached by the building at that given site (i.e. $P[D = d_i]$).

The framework above systematically includes all the sources of randomness into the three ingredients that lead to the estimation of P_{PL} : the IM hazard, the EDP demand, and the EDP capacity. However, it is intuitive to understand that our estimates of these three quantities are

only as accurate as the tools that are used to estimate them. More formally, the $P[IM = x_i]$; the $P[D \geq d | IM = x_i]$; and $P[C]$ are themselves random variables and what we can compute are only estimates of their true, but unknown, values. For example, different legitimate seismotectonic models may lead to different estimates of $P[IM = x_i]$. Different computer models of the building with more or less sophistication (e.g. model with and without P-Delta effect) or with different choices of stress-strain backbone curves or hysteresis cycle rules for their elements may lead to different estimates of building demand induced by the same ground motion records. Also, for any computer model adopted to describe the building, the estimates of the building response for a given level of ground motion are done using a finite set of accelerograms. A larger suite or a different suite of accelerograms may lead to a different value of $P[D \geq d | IM = x_i]$. Similarly, different amount of knowledge about material properties, member dimensions, etc., may lead to different estimates of the building capacity. This epistemic uncertainty can be reduced but never eliminated.

The direct consequence of including the epistemic uncertainty into the picture is that the engineer will only be able to state with a certain statistical confidence whether his/her building will meet the performance level but he/she will never be 100% sure of it. The SAC/FEMA method formalizes all these aspects and puts them in a tractable and simplified format that is easy to implement in practical applications such as those shown later in §2.2.2.6 and §2.2.2.7.

The SAC/FEMA method can be applied using either one of two theoretically equivalent formats, namely the Mean Annual Frequency (MAF) format and the Demand and Capacity Factored Design (DCFD) format. The attractiveness of both formats is that they use a set of relatively non-restrictive assumptions to avoid the numerical computation of the integrals (although simplified into summations) present in the previous equations. Thus, they allow for a simple, closed-form evaluation of the seismic risk (for the MAF format) or for a check of whether the building satisfies or not the requested limit-state requirements (DCFD).

2.2.2.3 MAF format

The MAF format is useful when we want to estimate the actual mean annual frequency λ_{PL} of violating a certain performance level PL. This is the inverse of the mean return period of exceedance and it is also intimately tied to the probability P_{PL} of violating PL within a certain time period t (see also Section 2.2.1.3.1), as

$$P_{PL} = 1 - \exp(-\lambda_{PL} t) \approx \lambda_{PL} t \quad (2-6)$$

where the approximation holds for rare events, i.e. small values of $\lambda_{PL} \cdot t$, such as those that we are interested in for engineering purposes. For example, by inverting Eq. (2-6), the familiar requirement for $P = 10\%$ probability of exceedance in $t = 50$ yrs, translates to a MAF of $\lambda = -\ln(1 - 0.10)/50 = 0.00211$ or roughly 1/475 years, i.e. a mean return period of 475 years. Equivalently, for $t = 1$, it also corresponds to an almost equal value of annual probability of $1 - \exp(-0.00211) = 0.00211$.

According to SAC/FEMA, we can estimate the value of λ_{PL} as:

$$\lambda_{PL} = H(\hat{IM}_C) \exp \left[\frac{k^2}{2b^2} (\beta_{DT}^2 + \beta_{CT}^2) \right], \quad (2-7)$$

where IM is the intensity measure for which we have the seismic hazard curve $H(IM)$ (e.g. Fig. 2-24) for the building site of interest. $\hat{IM}_C$ is the specific value of IM that causes the building to reach, on average, the given value of the EDP capacity that is associated with the onset of the limit-state corresponding to the performance level PL. β_{CT} and β_{DT} are the total

dispersions of the demand and capacity, respectively, due to both aleatory randomness and epistemic uncertainty (see Figure 2-25):

$$\beta_{DT} = \sqrt{\beta_{DR}^2 + \beta_{DU}^2} , \quad (2-8)$$

$$\beta_{CT} = \sqrt{\beta_{CR}^2 + \beta_{CU}^2} . \quad (2-9)$$

where β_{DR} and β_{CR} are the EDP dispersions due to aleatory randomness in demand and capacity, respectively. The first is mainly attributed to the variability observed in structural response from record-to-record, while the second may be derived from the natural variability that can be observed in tests to determine the EDP capacity of the relevant structural or non-structural component. Similarly, β_{DU} and β_{CU} are the additional dispersions in EDP demand and capacity, introduced by the epistemic uncertainty, i.e. due to our incomplete knowledge of the structure and our less than perfect modeling and analysis methods.

The positive constant k represents the slope of the mean hazard curve $H(IM)$ (see Figure 2-24 and Figure 2-27a to come) in the vicinity of $\hat{IM}_C$, thus providing information about the frequency of rarer or more frequent earthquakes close to the intensity of interest. It is derived from a power law approximation, or, equivalently, a straight line fit in log-log coordinates around $\hat{IM}_C$:

$$H(IM) \approx k_0(IM)^{-k} \quad (2-10)$$

where $k_0 > 0$, $k > 0$ are the fitting constants.

Similarly, the positive constant b characterizes the relationship of the (median) structural response EDP versus the intensity IM as described by the results of the nonlinear dynamic analyses in the vicinity of $\hat{IM}_C$. It is also derived from a power law fit:

$$EDP \approx a(IM)^b \quad (2-11)$$

where $a > 0$ and $b > 0$ are constants. Thus, the (median) value of IM that induces in the building the (median) EDP capacity becomes

$$\hat{IM}_C = \left(\frac{EDP_C}{a} \right)^{1/b} , \quad (2-12)$$

which offers a convenient way to estimate $\hat{IM}_C$ and obtain all the necessary values to successfully apply Eq. (2-7).

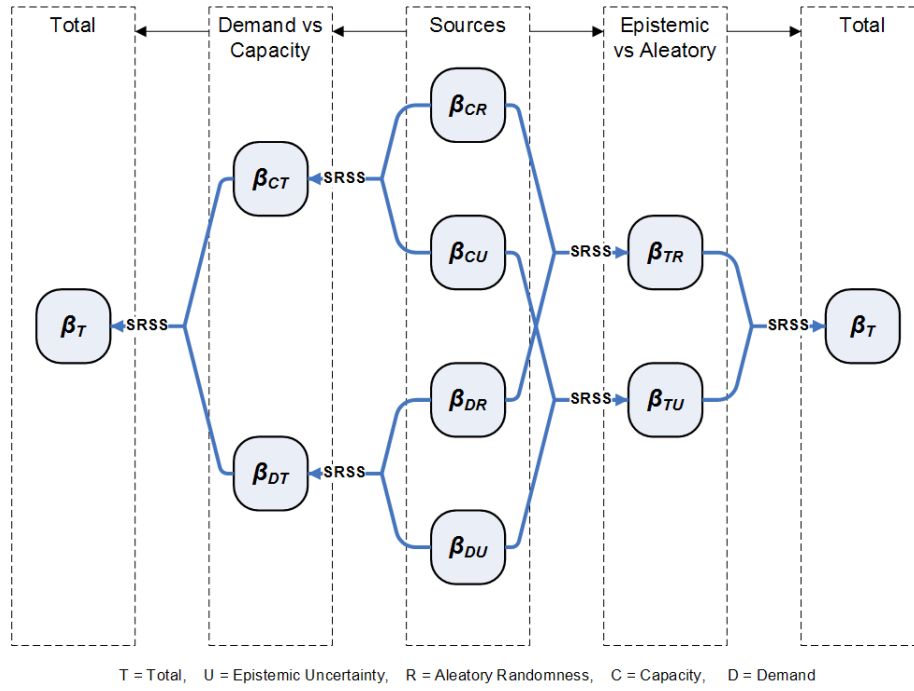


Fig. 2-25: The four sources of variability considered in the SAC/FEMA framework, combined in two different ways via a square-root-sum-of-squares rule to estimate the total dispersion.

2.2.2.4 DCFD methodology

The Demand and Capacity Factored Design (DCFD) format is meant to be used as a check of whether a certain performance level has been violated. Unlike the MAF format presented earlier, it cannot provide an estimate of the mean annual frequency of exceeding a given performance level. Instead, it was designed to be a checking format that conforms to the familiar Load and Resistance Factor Design (LRFD) format used in all modern design codes to check, e.g. member or section compliance. It can be represented by the following inequality:

$$FC \geq FD_{P_o} \Leftrightarrow \phi \cdot \hat{EDP}_C \geq \gamma \cdot \hat{EDP}_{P_o}, \quad (2-13)$$

where FC is the factored capacity and FD_{P_o} is the factored demand evaluated at the probability P_o associated to the selected performance objective. Correspondingly, $\hat{EDP}_C$ is the median EDP capacity defining the performance level (for example, the 1% maximum interstorey drift suggested in Table 2-2 of §2.2.1.4.3 for moderate damage of the UCS building) and $\hat{EDP}_{P_o}$ is the median demand evaluated at the IM level that has a probability of exceedance equal to P_o . For example, $P_o = 0.0021 \approx 1/475$ for a typical 10% in 50 years Life Safety performance level, as discussed above. The capacity and demand factors ϕ and γ are similar to the safety factors of LRFD formats and they are defined as:

$$\phi = \exp \left[-\frac{1}{2} \frac{k}{b} (\beta_{CR}^2 + \beta_{CU}^2) \right] \quad (2-14)$$

$$\gamma = \exp \left[\frac{1}{2} \frac{k}{b} (\beta_{DR}^2 + \beta_{DU}^2) \right] \quad (2-15)$$

where the parameters k , b and all the β -dispersions are defined exactly as in the previously presented MAF format. It should be noted here that the DCFD format has been derived

directly from the MAF format. Therefore, the same approximations discussed earlier have been employed and the same limitations also apply.

By satisfying Eq. (2-13) a structure is guaranteed, within the range of applicability of the underlying assumptions, to have a *mean* probability that the specified performance level is violated that is less or equal to the probability P_o associated to the selected performance objective. The word *mean* in front of probability is due to the explicit consideration of the epistemic uncertainty that plagues the structural problem. Hence, it can be said that we have a certain confidence⁷ in the above result that is somewhere above the 50% level. In other words, given our incomplete knowledge about our structural system we are more than 50% sure that what Eq. (2-13) implies is actually true.

Obviously, the above statement may not be satisfactory for all applications. There may be situations where we may wish for a higher confidence level in the results, having a tunable level of safety that can be commensurate, for example, with the implications of the examined failure mode. Thus, an alternative and enhanced DCFD format has also been proposed that differs only by including explicitly the desired confidence level given the uncertainty present in the evaluation:

$$\begin{aligned} FC \geq FD_{P_o} \cdot \exp\left(K_x \beta_{TU} - \frac{1}{2} \frac{k}{b} \beta_{TU}^2\right) &\Leftrightarrow \\ \phi \cdot E\hat{D}P_C \geq \gamma \cdot E\hat{D}P_{P_o} \cdot \exp\left(K_x \beta_{TU} - \frac{1}{2} \frac{k}{b} \beta_{TU}^2\right). \end{aligned} \quad (2-16)$$

Equivalently, this can be simplified to become:

$$FC_R \geq FD_{R P_o} \cdot \exp(K_x \beta_{TU}) \Leftrightarrow \phi_R \cdot E\hat{D}P_C \geq \gamma_R \cdot E\hat{D}P_{P_o} \cdot \exp(K_x \beta_{TU}), \quad (2-17)$$

where the factored demand and capacity, and equivalently the ϕ_R and γ_R factors, only include the aleatory randomness

$$\phi_R = \exp\left[-\frac{1}{2} \frac{k}{b} \beta_{CR}^2\right], \quad (2-18)$$

$$\gamma_R = \exp\left[\frac{1}{2} \frac{k}{b} \beta_{DR}^2\right], \quad (2-19)$$

while the epistemic uncertainty in demand and capacity is introduced by the total uncertainty dispersion (see, again, Figure 2-25),

$$\beta_{TU} = \sqrt{\beta_{DU}^2 + \beta_{CU}^2}, \quad (2-20)$$

assuming no correlation exists between EDP demand and capacity.

Finally, to ensure the factored capacity, FC , exceeds the factored demand, FD_{P_o} , with the designated MAF at a confidence level of α , we include K_x . This is the standard normal variate corresponding to the desired level α . Values of K_x are widely tabulated, and can also be easily

⁷ The confidence level in the probability estimate is the probability with which the actual true probability of PL will be lower or equal to the estimate. It is here assumed that the actual true but unknown probability is distributed lognormally and hence its mean is larger than its median. It follows that if the estimate coincides with the mean we have a larger than 50% confidence level.

calculated by the NORMINV function in Excel. For example NORMINV(0.9,0,1) yields $K_x = 1.28$ for $\alpha = 90\%$ ⁸, while NORMINV(0.5,0,1) produces $K_x = 0$ for $\alpha = 50\%$ confidence.

Note that Eq. (2-13) is the same as Eq. (2-16) for a confidence level higher than $\alpha = 50\%$, or, equivalently, K_x being greater than zero. The exact level depends on the details of the problem at hand, especially on the value of β_{UT} . In other words, whereas Eq. (2-13) is a checking format with a fixed level of confidence, Eq. (2-16) allows a user-defined level of confidence to be incorporated in the assessment. This is a quality that may prove to be very useful since different required levels of confidence can be associated with ductile versus brittle modes of failure or local versus global collapse mechanisms. The significant consequences of a brittle or of a global failure often necessitate a higher level of safety and can be accommodated with an appropriate higher value of K_x . This is fundamental in the practical application of the FEMA-350/351 guidelines where different suggested values of the confidence level are tabulated for a variety of checking situations.

2.2.2.5 Theoretical background on SAC/FEMA assumptions

In essence, Eq. (2-7) is a closed-form approximation of the PEER integral that specializes in estimating the mean annual frequency of violating a certain performance level, rather than of exceeding a certain value of a decision variable DV. The numerical integration implied by the PEER formulation, or equivalently by Eq. (2-5) of the SAC/FEMA method, is well represented by the closed-form solution of Eq. (2-7) if several assumptions hold in addition to those detailed in Section 2.2.2.2:

1. The hazard curve function $H(IM)$ can be approximated in the vicinity of $\hat{IM}_C$ with a power law, or equivalently a straight line in log-log coordinates. Formally this is described by Eq. (2-10).
2. The median structural response EDP given the intensity IM obtained using statistical regression of the results of the nonlinear dynamic analysis results can be approximated as a power law function, or equivalently a straight line in log-log coordinates. Formally this is expressed by Eq. (2-11).
3. In the region of interest, i.e. around $\hat{IM}_C$, the distribution of EDP given IM can be adequately represented as a lognormal random variable. This has a standard deviation of β_{DR} for the logarithm of EDP|IM that is *independent* of the intensity IM.
4. Epistemic uncertainty in the model does not introduce any bias in the response, i.e. it does not change its median value. It simply causes the response to be lognormally distributed around the median EDP with an inflated dispersion of β_{DT} . This is estimated from its constituents β_{DR} and β_{DU} by the square-root-sum-of-squares rule of Eq. (2-8). This is also referred to as the “first order assumption” for epistemic uncertainty.
5. The EDP capacity used to define the performance level PL is lognormally distributed with a median of $\hat{EDP}_C$ and a total dispersion of β_{CT} , estimated from its constituents β_{CR} and β_{CU} according to the square-root-sum-of-squares rule of Eq. (2-9).

Despite the large number of assumptions made, recent studies have shown that most are actually quite accurate. Only one or two may, under certain circumstances, influence significantly the accuracy of the results.

⁸ In simple words, this means that a normal standard variable has a $1-0.9 = 0.1$ probability of assuming values larger than 1.28.

The lognormal distribution assumption of the demand EDP given the IM (item # 3 in the list above) is generally quite accurate (e.g. NIST, 2010) besides in those cases when the response variable has an upper bound and thus it saturates. Typically, this is the norm for force-based responses, where the maximum strength of the elements constitutes a natural upper bound to the moment, axial or shear response. On the other hand, a lognormal distribution has no upper limit, thus becoming inaccurate if the median EDP is close to its upper bound. Such problems may be easily resolved by using instead the corresponding displacement, rotation or strain EDP to define the performance level. Quantities such as the axial strain, shear deformation or moment rotation are essentially unbounded and they can serve the same purpose as the force-based EDPs.

Another case where problems may arise is when certain accelerograms cause global dynamic instability of the building model. On a realistic structural model and a robust analysis platform the instability directly manifests itself as a numerical non-convergence during the dynamic analysis (Vamvatsikos & Cornell, 2002). In general, if more than 10% of the dynamic analyses at any level of intensity do not converge, this is a strong indication that the closed form solution of SAC/FEMA should not be employed. More complex closed-form formulations are available (Jalayer, 2003) that are similar in spirit to Eq. (2-7) but properly take into account the probability of collapse. Given their complexity, these methods are omitted here.

Homoscedasticity, i.e. constant dispersion of the EDP response given the IM in the regression mentioned in item # 2 above, is also not always an accurate assumption. The dispersion generally tends to increase with the intensity level when the structure enters its inelastic response region. However, the changes are not steep but rather gradual in nature. Since Eq. (2-7) needs only local homoscedasticity, rather than global, the impact of imperfect conformance tends to be of secondary importance. Still, Aslani & Miranda (2005) have shown that there are cases where the changes in the dispersion can hurt the accuracy of the SAC/FEMA format.

Properly fitting the hazard curve via the power law function of Eq. (2-10) has been shown to be the greatest potential source of inaccuracy (e.g. Vamvatsikos & Cornell 2004, Aslani & Miranda 2005) at least when $S_a(T_1)$ is used as the IM. These studies, however, have considered the original suggestion of Cornell et al. (2002) to use a tangent fit at the $\hat{IM}_C$ point of the hazard curve for the computation of the value of k . Dolsek and Fajfar (2008) have suggested that a left-weighted (or right-biased) fit can actually achieve superior accuracy. Since for large IM values of engineering significance the hazard curve descends very rapidly, it is more important to capture well the mean annual frequency values of IM that are to the left of $\hat{IM}_C$ (i.e. IM values lower than $\hat{IM}_C$), thus accepting a conservative bias on the right of $\hat{IM}_C$. One may achieve such a fit by considering only the portion of the hazard curve that lies within $[0.25\hat{IM}_C, 1.5\hat{IM}_C]$ and use this segment to draw a straight line in log-log coordinates that provides a best fit (see later for a numerical example). Thus, the most important potential source of error in the SAC/FEMA approximation can be easily removed.

The final point of concern is the fitting via statistical regression of a power law function of the IM – EDP pairs obtained via nonlinear dynamic analyses. This can potentially become the most problematic aspect in the entire application of the SAC/FEMA format, as the “ b ” parameter can make a large difference in Eq. (2-7). The original FEMA-350/351 documents dealt with this issue by assuming a $b = 1$ (Cornell et al. 2002). This is a relatively conservative assumption for many but not all situations. Ideally, b should be estimated by locally fitting the results of a number of nonlinear dynamic analyses that have been performed using accelerograms with IM values in the vicinity of $\hat{IM}_C$. While seemingly easy, defining $\hat{IM}_C$ itself depends on the knowledge of the IM – EDP relationship. This means that the parameters

a and b from Eq. (2-11) should be known. This is a circular argument that makes a practical implementation doable but complicated as it will be discussed in the application section for the MAF methodology.

2.2.2.6 Illustrative assessment example of the DCFD methodology

The proposed DCFD evaluation methodology is applied here to evaluate whether a modern 6-storey reinforced concrete shear wall building complies with a $EDP_C = \hat{\theta}_{\max,C} = 1\%$ interstorey drift limit at a 10% in 50 years exceedance frequency, consistent with a moderate damage limit-state (Table 2-2). In other words, we are checking whether we can be at least 50% confident that the exceedance of the 1% drift limit has a probability lower than 10% in 50 years to occur. The structure, shown in Fig. 2-7 is the University of California Science (UCS) building located at UC-Berkeley campus in California and it is described in detail in §2.2.1.4. The seismic hazard is assumed to be represented by the mean seismic hazard curve of Fig. 2-24, which defines the hazard in terms of spectral acceleration at the first-mode period ($T_1 = 0.38$ s) for viscous damping equal to 5% of critical. Similar information about hazard curves for sites within the United States can be obtained from the USGS website (www.usgs.gov) or by carrying out site-specific probabilistic seismic hazard analysis (PSHA).

The exceedance of the interstorey drift capacity limit of 1% is subject to both aleatory randomness and epistemic uncertainty. The aleatory randomness is associated with natural variability of earthquake occurrence while epistemic uncertainty is associated with our incomplete knowledge of the seismotectonic setting (i.e. the building block of the hazard estimation), and of characteristics of the building that affect its dynamic behavior. Uncertainty in the hazard is addressed by using mean rather than median hazard information (Cornell et al., 2002). Based on past studies (e.g. Liel et al., 2009; Dolsek, 2009; and Vamvatsikos & Fragiadakis, 2010), we set $\beta_{DU} = 20\%$ as a possible estimate of dispersion due to epistemic uncertainty, associated mainly with the modeling parameters. Note that other sources of uncertainty that have not been accounted for, such as structural damping, storey mass, storey stiffness, or the effect of cladding and interior walls, may increase such estimates. The capacity limit is assumed to be lognormally distributed with total dispersion (standard deviation of the natural log of the data) assumed here to be equal to $\beta_{CT} = 0.3$. It is estimated as the square root of the sum of the squares of the dispersions due to epistemic uncertainty and aleatory variability, $\beta_{CU} = 23\%$ and $\beta_{CR} = 20\%$, respectively.

The example assessment illustrates a methodology based on the use of S_a as the intensity measure. In this example we utilized the same suite of 20 ground motion records introduced in §2.2.1.4.2. As already mentioned, these motions are consistent with the site class of the UCS structure and have been recorded at distances that roughly correspond to the distance from the site to the Hayward fault that causes most of the hazard.

Assessment based on $S_a(T_1)$ follows the basic steps of the DCFD methodology, as discussed, for example, in Jalayer & Cornell (2009). The first step is to determine the median value of the $S_a(T_1)$ corresponding to a probability of exceedance of 10% in 50 years. This translates to a mean annual frequency (MAF) of $\lambda = 0.00211$ or 1/475 years, as explained in Section 2.2.2.3. Thus, the 475-year mean return period IM value where the check is going to be performed is $S_{aPo} = 1.21$ g, a value that is obtained from the hazard curve in Figure 2-26a. Now, a straight line is fitted to the hazard curve in log-log coordinates within the “region of interest”, defined by $\lambda_{Sa} = k_0 \cdot S_a^{-k}$. This region, according to the suggestions in Dolsek & Fajfar (2008), should be defined over the interval $[0.25S_{aPo}, 1.5S_{aPo}]$. As mentioned earlier, the region over which the hazard curve approximation is performed is not centered at S_{aPo} but includes more values lower than S_{aPo} since these are the values with probabilities of

exceedance that are higher than those for values on the right of S_{aPo} . The resulting fit appears as a red dashed line in Figure 2-26a, corresponding to a slope of $k = 2.12$. By comparing the straight line with the hazard curve we can immediately tell that this simplification will lead to overestimating the hazard for most values of S_a . This observation implies that this implementation of the S_a -based approach will result in a conservative evaluation of the load versus capacity check implied by this approach.

In the first set of nonlinear dynamic analyses, ground motion records are individually scaled to the S_{aPo} level. The maximum interstorey drift response $\theta_{\max}$ determined in each nonlinear dynamic analysis is the only engineering demand parameter (EDP) of interest in this example. The median of the maximum interstorey drift values obtained by exciting this building with the 20 records is $\theta_{\max,50} = 0.0035$. Since dynamic instability (i.e. numerical non-convergence for a well-executed nonlinear dynamic analysis) was not registered for any of the 20 records, we may estimate the EDP dispersion as the standard deviation of the logarithm of the 20 values of $\theta_{\max}$ response obtained. If some of the analyses had instead failed to converge, they would effectively correspond to an infinite value of EDP response, a fact that needs to be incorporated into the analysis. As long as collapse occurs infrequently, i.e. for less than 10% of the analyses, EDP dispersion can be safely estimated as

$$\beta_{DR} = \beta_{\theta_{\max}|S_a} = \ln \theta_{\max,84} - \ln \theta_{\max,50} = \ln(0.0045) - \ln(0.0035) = 25\%,$$

where $\theta_{\max,84}$ is the 84% percentile of the 20 $\theta_{\max}$ values (collapses included), easily estimated, for example, using the PERCENTILE function in Excel. If, on the other hand, collapse has been observed for more than 10% of the records (i.e. more than 2 out of 20), the probability of collapse should be considered explicitly with an alternative format (Jalayer, 2003).

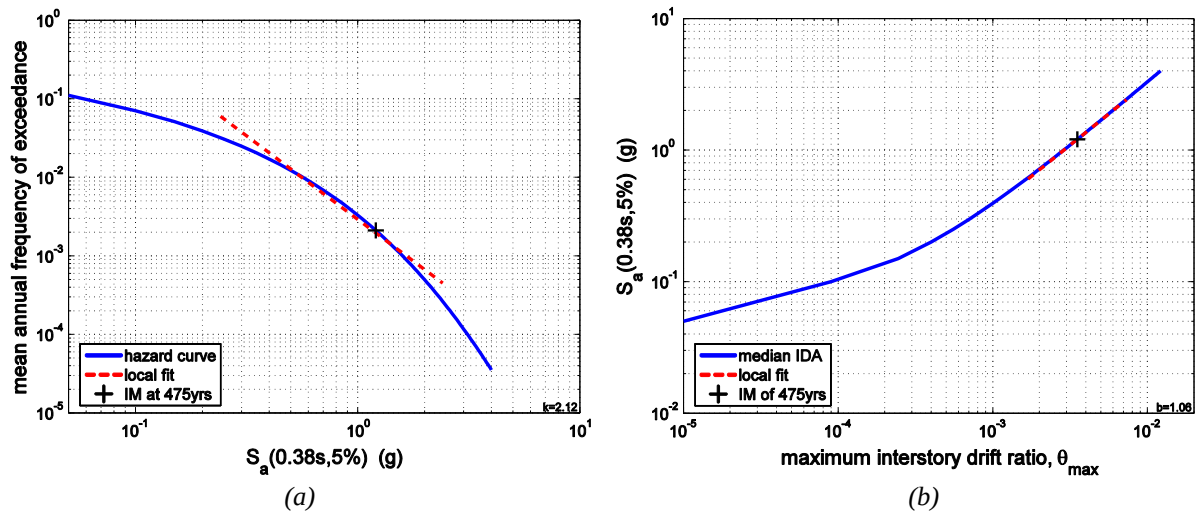


Fig. 2-26: The two power-law fits needed for the DCFD approach: (a) The mean S_a -hazard curve and (b) the median EDP-IM fit, both in the region of the 475 year intensity level

To estimate the slope of the median $\theta_{\max}$ versus S_a diagram, a second set of nonlinear dynamic analyses were performed using ground motion records scaled to $1.20 \cdot S_{aPo} = 1.45 \text{ g}$ to determine the median value $\theta_{\max,50(1.20)}$. Based on the full set of 20 records, the median value of $\theta_{\max,50(1.20)}$ was found to be 0.0043. These two median values allow the slope of the median EDP curve, as shown in Figure 2-26b, to be estimated as

$$b = \frac{\ln(\theta_{\max,50(1.20)}) - \ln(\theta_{\max,50})}{\ln(1.45/1.21)} = \frac{\ln(0.0043) - \ln(0.0035)}{\ln(1.20)} = 1.06. \quad (2-21)$$

In this case, an assumption of $b = 1$, as originally suggested by FEMA-350/351, would be only slightly conservative.

Finally, using the full set of 20 records, the factored demand and factored capacity values are estimated:

$$FD_{RPo} = \theta_{\max,50} \exp \left[\frac{k}{2b} \beta_{\theta_{\max}|Sa}^2 \right] = 0.0035 \exp \left[\frac{2.12}{2 \cdot 1.06} \cdot 0.25^2 \right] = 0.0037, \quad (2-22)$$

$$FC_R = \hat{\theta}_{\max,C} \exp \left[-\frac{k}{2b} \beta_{CR}^2 \right] = 0.01 \cdot \exp \left[-\frac{2.12}{2 \cdot 1.06} 0.2^2 \right] = 0.0096. \quad (2-23)$$

If the exceedance of the 1% maximum interstorey drift for this building is assumed to involve a ductile mechanism that may only produce local damage, the probability of exceedance can be evaluated (for the purposes of this example) at a 50% confidence level. This corresponds to a lognormal standard variate of $K_x = 0$, effectively discounting in its entirety the detrimental effect of epistemic uncertainty. Thus, the evaluation inequality becomes:

$$FC_R > FD_{RPo} \cdot 1, \quad (2-24)$$

or, equivalently, $0.0096 > 0.0037$, an inequality that is satisfied. A result of $FC = FD \cdot 1$, would have indicated that the demand is equal to the capacity of the building, on average, once every 475 years, at a 50% level of confidence. However, the result showing a factored capacity larger than the factored demand, indicates that, on average, it would take longer than 475 years for the demand to exceed the capacity of the building.

In some circumstances a level of confidence higher than 50% for a given recurrence interval may be desired in evaluating factored capacities, especially if involving a brittle or a global collapse mechanism that may have severe consequences on the building occupants. For illustration purposes, let us repeat the calculations under this assumption. For a confidence level of 90%, the lognormal standard variate is $K_x = 1.28$ and the evaluation inequality becomes:

$$FC_R > FD_{RPo} \exp(K_x \beta_{TU}) = 0.0035 \cdot \exp(1.28 \sqrt{0.23^2 + 0.2^2}) = 0.0055 \quad (2-25)$$

Since the factored capacity of 0.0096 is higher than the factored demand of 0.0055 the building is deemed to be safe also at the 90% confidence level.

For the sake of discussion, how shall an engineer proceed in the case the 90% confidence check failed but the 50% confidence check succeeded? As mentioned earlier, the decision may depend on the mode of failure that is being checked. If the check concerns a ductile failure mode, for which a sudden, catastrophic failure can be reasonably ruled out, then even a 50% level of confidence may be adequate to declare the structure safe at the 10% in 50yrs level. However, if the failure mode were to involve a brittle collapse then the failed 90% confidence check may prompt the engineer to exercise more caution and consider the structure unfit for purpose since the occurrence of this failure mode (e.g. beam or column shear failure) may lead to a catastrophic collapse with potentially deadly consequences.

2.2.2.7 Illustrative assessment example of the MAF methodology

We will now use the MAF assessment methodology to estimate the MAF of violating the moderate damage performance level for the UCS building, which based on the results of the previous section is known to be lower than $1/475 = 0.0021$. For this limit-state, similarly to the previous subsection, the maximum interstorey drift ratio $\theta_{\max}$ capacity is deemed to follow a lognormal distribution with a median of 1% (Table 2-2, Section 2.2.1.4.3). Randomness and epistemic uncertainty in the capacity are accounted for by the dispersions of $\beta_{CR} = 0.2$ and $\beta_{CU} = 0.23$ respectively, with a combined value of $\beta_{CT} = 0.3$.

The building performance is again tested using the same set of 20 ground motions. Finding the $\hat{IM}_C$ value corresponding to the $\hat{EDP}_C = \hat{\theta}_{\max,C} = 1\%$ capacity, as discussed earlier, can become difficult, as it is an inverse problem that requires knowledge of the median curve of EDP versus IM. Usually, simple iterative calculations can be used to select two IM-levels that would produce median EDP values that, ideally, would closely bracket $\hat{EDP}_C$. This trial and error process is not necessary here since the entire structural analysis results shown in Fig. 2-10 are at our disposal. First, we choose a trial IM value at the design level of 10% in 50yrs for this limit-state, i.e. $S_a = 1.21$ g. The resulting median EDP response is 0.0035, as shown in the previous subsection. Linear extrapolation (assuming the equal displacement rule is accurate enough) suggests that by tripling this value to $S_a = 3.63$ g we should effectively bracket the median EDP capacity. Indeed the median response comes out to be 0.0110 which is sufficiently close to $\hat{EDP}_C$ to allow for an effective estimation. Additional steps can further improve the accuracy of our bracketing values. Thus, either by linearly interpolating in log-log space the structural analysis results from the closest two IM-levels determined through this iterative approach, or by taking advantage of the EDP-IM curve of Fig. 2-10 we arrive at an estimate of $\hat{IM}_C = 3.3$ g for 1% drift capacity. The corresponding EDP dispersion at this IM-level was found to be $\beta_{DR} = 0.64$ from structural analysis, while, similarly to the previous section, uncertainty is assumed to be $\beta_{DU} = 0.20$, which leads to a combined value of $\beta_{DT} = 0.67$.

The corresponding hazard for $\hat{IM}_C = 3.3$ g is 0.00008. Local fitting in this region of the hazard curve yields a hazard slope of $k = 3.25$ as seen in Fig. 2-27a. Taking advantage of the trial runs we already performed, we can use their results to estimate the value of b :

$$b = \frac{\ln(\theta_{\max,50(S_{a2})}) - \ln(\theta_{\max,50(S_{a1})})}{\ln(S_{a2}/S_{a1})} = \frac{\ln(0.0110) - \ln(0.0035)}{\ln(3.6/1.2)} = 1.04 \quad (2-26)$$

A closer approximation based on locally fitting the actual IM-EDP curve (rather than just the two trial runs), actually yields $b = 1.06$, as seen in Fig. 2-27b, a value that we are going to utilize here. With these results at hand, the MAF estimate λ_{MD} for the moderate damage limit-state according to Eq.(2-7) becomes:

$$\begin{aligned} \lambda_{MD} &= H(\hat{IM}_C) \exp \left[\frac{k^2}{2b^2} (\beta_{DT}^2 + \beta_{CT}^2) \right] = \\ &= 0.00008 \cdot \exp \left[\frac{3.25^2}{2 \cdot 1.06^2} (0.67^2 + 0.3^2) \right] = 0.00126 \end{aligned} \quad (2-27)$$

As expected, the UCS structure violates the MD performance level less frequently than the maximum allowed annual value of $0.0021 = 1/475$ (10% in 50yrs). Therefore, the structure passes this check and it is considered safe for the MD level.

It should be noted here that although the application of the MAF estimation is considerably more difficult (due to the need for establishing the value of $\hat{IM}_C$), it is also numerically more accurate than the DCFD format. The improved accuracy stems from the local fitting of the hazard curve and of the median IM-EDP curve that is performed at the region of interest (i.e. at the structure's capacity), rather than at an arbitrary limit-state MAF value. This can be observed by comparing the fits in Figure 2-26 against their counterparts in Figure 2-27. Therefore, although the two methods generally produce compatible results, in the few cases where they might disagree it is advisable to favor the MAF format results.

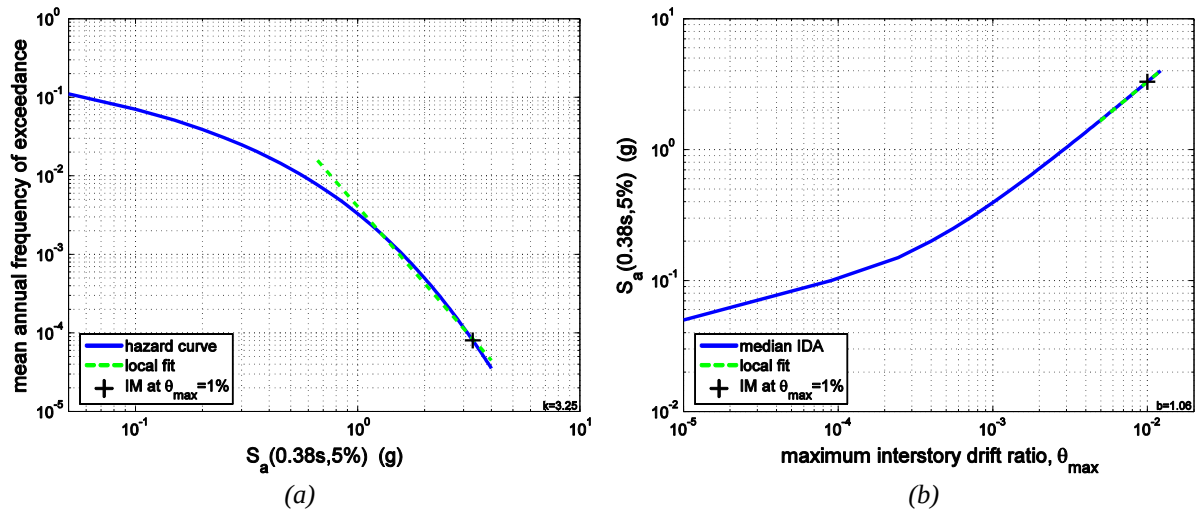


Fig. 2-27: The two power-law fits needed for the MAF estimation: (a) The mean S_a -hazard curve fit and (b) the median IM-EDP fit, both in the region of the IM corresponding to median EDP capacity

2.2.2.8 Future applications and concluding remarks

Despite their limitations, the SAC/FEMA formats are arguably the simplest available methods for sound performance-based seismic design. They offer relatively easy-to-use formulas that allow the integration of seismic hazard, the results of nonlinear dynamic analysis and the associated epistemic and aleatory uncertainty. One could argue that their use in safety assessment of structures has been superseded by the PEER format, which offers superior accuracy and a wealth of options for communicating the results to a non-technical audience. However, in the realm of design, the unparalleled simplicity and, familiarity to engineers makes the SAC/FEMA in the DCFD format a very strong candidate for future guidelines. It can only be expected that the forthcoming applications will draw heavily on the presented framework which will enjoy continued use in the future.

2.3 Unconditional probabilistic approach

2.3.1 Introduction

This section illustrates an approach to the determination of the mean annual frequency of negative structural performances, which does not make recourse to an intermediate conditioning variable, such as the local seismic intensity measure. The main difference with the previously described practice of IM-conditioning rests in the models employed to describe the seismic motion at the site. The hazard-curve/recorded-motions pair is replaced by stochastic models that describe the random time-series of seismic motion directly in terms of macro-seismic parameters, such as the magnitude and distance, etc.

All methods that belong to this approach require that the randomness in the problem be described by a vector of random variables, denoted by $\mathbf{x}$ in the following. This vector should ideally collect randomness relating to the earthquake source, propagation path, site geology/geotechnics, frequency content of the time-series, structural response and capacity. Most methods then resort to simulation, i.e. the frequency of negative performances is evaluated by taking the ratio of the latter to the total number of realizations of $\mathbf{x}$ sampled from the probability distribution that describes it, $f(\mathbf{x})$.

The following sections present simulation methods (§2.3.2), two representative models of the seismic motion in terms of random variables (§2.3.3), a summary of these simulation procedures with flow-charts (§2.3.4) and an example application (§2.3.5), respectively.

2.3.2 Simulation methods

Simulation is a robust way to explore the behaviour of systems of any complexity. It is based on the observation of system response to input. Simulation of a set of inputs from $f(\mathbf{x})$ and evaluation of corresponding outputs allows to determine through statistical post-processing the distribution of the output (in this respect, the *IM*-based methods presented earlier can be seen as “small-sample simulations”, more on this later). This section sketches the basics of simulations.

2.3.2.1 Monte Carlo simulation methods

According to the axioms of probability, the probability of an event E , union of N mutually exclusive (ME) events e_i , equals the sum of their respective probabilities: $p_E = \sum p_{ei}$. If the event of interest is failure in meeting specified performance requirements, then it is common to denote the corresponding probability as p_f , the “failure probability”. Further, if randomness is collected in a random vector $\mathbf{x}$, then each elementary failure event corresponds to a single value of $\mathbf{x}$ and its probability is $f(\mathbf{x})d\mathbf{x}$. It follows that:

$$p_f = \int_F f(\mathbf{x})d\mathbf{x} \quad (2-28)$$

where F is the portion of the sample space (the space where $\mathbf{x}$ is defined) collecting all $\mathbf{x}$ values leading to failure. Eq. (2-28) is called “reliability integral”.

Simulation methods start from Eq. (2-28) by introducing the so-called “indicator function” $I_f(\mathbf{x})$, which equals one if $\mathbf{x}$ belongs to F , and zero otherwise. It is apparent that p_f is the expected value of I_f :

$$p_f = \int_F f(\mathbf{x})d\mathbf{x} = \int I_f(\mathbf{x})f(\mathbf{x})d\mathbf{x} = E[I_f(\mathbf{x})] \quad (2-29)$$

Monte Carlo (MC) simulation (Rubinstein, 1981) is the crudest possible way of estimating p_f , in that it amounts to estimating the expectation of I_f as an arithmetic average $\hat{p}_f$ over a sufficiently large number N samples of $\mathbf{x}$:

$$p_f = E[I_f(\mathbf{x})] \cong \frac{1}{N} \sum_{i=1}^N I_f(\mathbf{x}_i) = \frac{N_f}{N} = \hat{p}_f \quad (2-30)$$

The problem is then reduced to that of sampling realizations $\mathbf{x}_i$ of $\mathbf{x}$ from the distribution $f(\mathbf{x})$, and of evaluating the performance of the structure for each realization in order to assign a value to the indicator function.

It can be shown that $\hat{p}_f$ is an unbiased estimator of p_f , and that its variability (variance) around p_f is proportional to p_f itself and decreases with increasing number N of samples. A basic result that follows is that the *minimum* number of samples required for a specified confidence in the estimate (in particular to have 30% probability that $\hat{p}_f \in [0.67, 1.33] \cdot p_f$) is given by:

$$N \geq 10 \frac{1-p_f}{p_f} \cong \frac{10}{p_f} \quad (2-31)$$

The above result (which holds for sufficiently small p_f 's, say, in the order of 10^{-3} or lower) is of immediate qualitative justification: since $\hat{p}_f = N_f/N$, if p_f and hence $\hat{p}_f$ is very small, we are looking at an extremely rare event and it takes an exorbitant number N of trials in order to get a few outcomes in F . It also follows that in order to reduce the minimum required N one must act on the variance of $\hat{p}_f$. This is why the wide range of enhanced simulation methods that have been advanced in the last decades fall under the name of “variance reduction techniques”.

One such technique is *Importance sampling* (IS), a form of simulation based on the idea that, when values of $\mathbf{x}$ that fall into F are rare and difficult to sample, they can be conveniently sampled according to a more favourable distribution, somehow shifted towards F , as shown in Fig. 2-28. Of course the different way $\mathbf{x}$ values are sampled must be accounted for in estimating p_f according to:

$$p_f = \int I_f(\mathbf{x})f(\mathbf{x})d\mathbf{x} = \int I_f(\mathbf{x})\frac{f(\mathbf{x})}{h(\mathbf{x})}h(\mathbf{x})d\mathbf{x} = E_h\left[I_f(\mathbf{x})\frac{f(\mathbf{x})}{h(\mathbf{x})}\right] \cong \frac{1}{N} \sum_{i=1}^N I_f(\mathbf{x}_i)\frac{f(\mathbf{x}_i)}{h(\mathbf{x}_i)} \quad (2-32)$$

where now p_f is expressed as the expectation of the quantity $I_f(\mathbf{x})f(\mathbf{x})/h(\mathbf{x})$ with respect to the distribution $h(\mathbf{x})$, called *sampling density*. The quantity $\alpha(\mathbf{x}) = f(\mathbf{x})/h(\mathbf{x})$ is called IS weight. The difficulty associated with the IS method is to devise a good sampling density $h(\mathbf{x})$, since it requires some knowledge of the failure domain F . An example of the construction of the sampling density based on problem-specific information is illustrated in the next section 2.3.2.3.

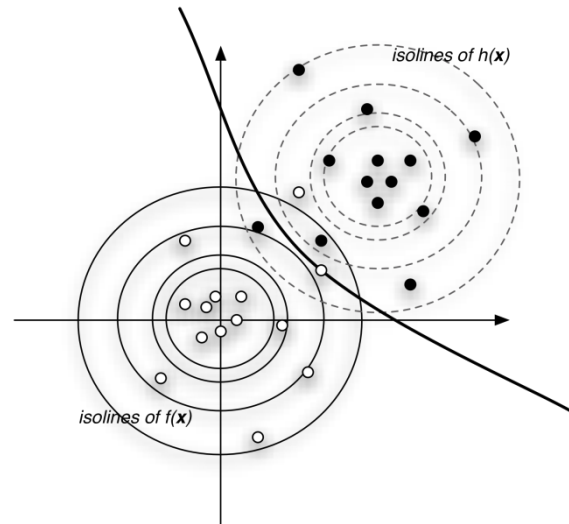


Fig. 2-28: Monte Carlo simulation samples (white dots) and Importance Sampling samples (black dots)

2.3.2.2 Application to the estimation of a structural MAF

The application of the above simulation methods to the problem of estimating the mean annual frequency of exceedance of a structural limit state, λ_{LS} , proceeds as follows. A probabilistic model (i.e. a joint distribution) is set up for the seismogenetic sources affecting the site of interest, from which events in terms of magnitude, location and other source parameters such as, e.g. faulting style etc. can be sampled. This model usually encompasses several sources that can be spanned by an index i . If each source has an activity described by the mean annual rate of generated events λ_i , then one can write:

$$\lambda_{LS} = \sum_{i=1}^N \lambda_i p_{LS|i} = \lambda_0 \sum_{i=1}^N (\lambda_i / \lambda_0) p_{LS|i} = \lambda_0 \sum_{i=1}^N p_{LS|i} \cdot p_i = \lambda_0 p_{LS} \quad (2-33)$$

where $\lambda_0 = \sum_{i=1}^N \lambda_i$ is the rate of all events generated in the area affecting the site, $p_i = \lambda_i / \lambda_0$ is the probability that the event is generated in source i and p_{LS} the probability that the limit state is exceeded, given that an event occurs.

Simulation is employed to evaluate p_{LS} . To this purpose, the probabilistic model for the event generation must be complemented with a model for the ground motion at the site, and a model for structural randomness in capacity and response. The next sections describe an enhanced simulation method, models for generating random time-series of ground motion as a function of source and site parameters, and flow-charts of the typical simulation run, while Section 2.3.5 reports an example with a comparison of simulation and IM-based results.

2.3.2.3 Importance sampling with K-means clustering

This section describes an effective variance reduction technique, which exploits the importance sampling method and enhances it with a statistical technique called clustering in order to further decrease the required number N of simulations. The method has been recently proposed by Jayaram and Baker (2010) for developing a small but stochastically representative catalogue of earthquake ground-motion intensity maps, i.e. events, that can be used for risk assessment of spatially distributed systems. The method uses Importance Sampling to preferentially sample “important” events, and K-Means Clustering to identify and combine redundant events in order to obtain a small catalogue. The effects of sampling and clustering are accounted for through a weighting on each remaining event, so that the resulting catalogue is still a probabilistically correct representation.

Even though the method has been devised for risk assessment of distributed systems, nothing prevents it from being employed for the risk assessment at a single site. The required modification is minor and concerns mainly the criterion for clustering events. The remainder of this section describes the modified single-site version of the method, and the reader interested in the details of the differences can refer to Jayaram and Baker (2010).

The method uses an importance sampling density h on the random magnitude M . The original density for M is defined as a weighted average of the densities $f_i(m)$ specified for each of the n_f active faults/sources, weighted through their corresponding activation frequencies λ_i (the mean annual rate of all events on the source, i.e. events with magnitude larger than the lower bound magnitude for that source):

$$f(m) = \frac{\sum_{i=1}^{n_f} \lambda_i f_i(m)}{\sum_{i=1}^{n_f} \lambda_i} \quad (2-34)$$

Given that an earthquake with magnitude $M = m$ has occurred, the probability that the event was generated in the i -th source is:

$$p(i|M = m) = \frac{\lambda_i f_i(m)}{\sum_{j=1}^{n_f} \lambda_j f_j(m)} \quad (2-35)$$

If m_{min} is the minimum magnitude of events on all sources, i.e. the minimum of the lower bound magnitudes of all considered sources, and m_{max} is the corresponding maximum magnitude, the range $[m_{min}, m_{max}]$ contains all possible magnitudes of events affecting the site. The original probability density in Eq.(2-34) is much larger near m_{min} than towards m_{max} . The range $[m_{min}, m_{max}]$ can be partitioned (stratified) into n_m intervals:

$$[m_{min}, m_{max}] = [m_{min}, m_2] \cup [m_2, m_3] \cup \dots \cup [m_{n_m}, m_{max}] \quad (2-36)$$

where the partitions are chosen so as to be small at large magnitudes and large at smaller magnitudes. The procedure, also referred to as *stratified sampling*, then requires sampling a magnitude value from each partition using within each partition the original density. This leads to a sample of n_m magnitude values that span the range of interest, and adequately cover important large magnitude values. The IS density $h(m)$ for m lying in the k -th partition is then:

$$h(m) = \frac{1}{n_m} \frac{f(m)}{\int_{m_k}^{m_{k+1}} f(m) dm} \quad (2-37)$$

Once the magnitudes are sampled using IS, the rupture locations can be obtained by sampling faults using fault probabilities $p(i|M=m)$, which will be non-zero only if the maximum allowable magnitude on fault i exceeds m . Fig. 2-29 shows the sampling density $h(m)$.

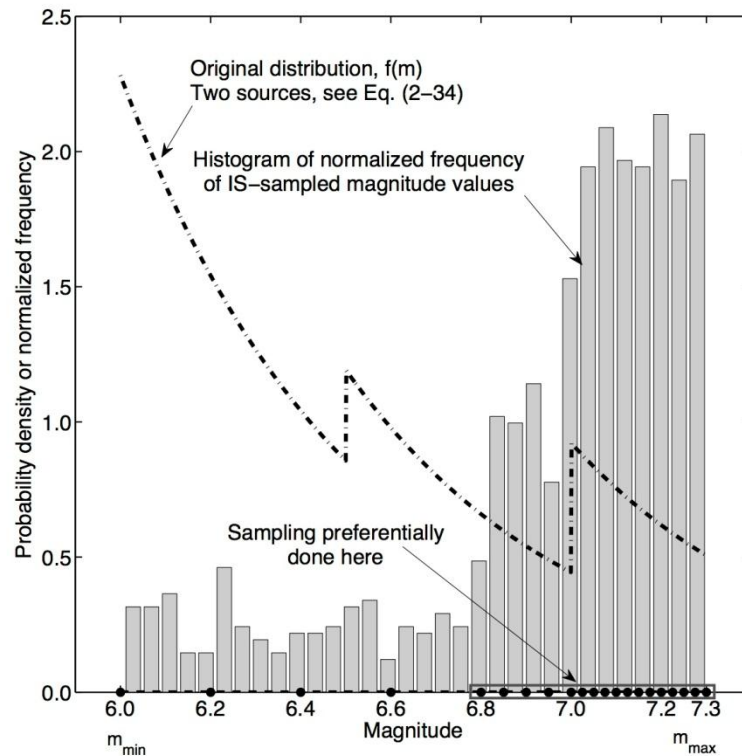


Fig. 2-29: Sampling density for the magnitude (adapted from Jayaram and Baker, 2010). Dots on the M axis indicate magnitude interval boundaries.

Once magnitude M and location, and hence source to site distance R , have been sampled for an event, a ground motion time series model (see §2.3.3) can be used to generate an input motion to be used for structural performance assessment. Given the uncertainty affecting the motion at a site for a given (M, R) pair, repeated calls to the ground motion time series model will yield different input motions. Often time series coming from different events present similar spectral content. Repeating structural performance evaluation for such similar motions is not going to add much valuable additional information for the risk assessment. This is where the statistical technique of clustering enters into the picture.

K-means clustering groups a set of observations into K clusters such that the dissimilarity between the observations within a cluster is minimized (McQueen, 1967).

Let $\mathbf{S}_1, \dots, \mathbf{S}_r$ denote the response spectra of r motions, generated using IS, to be clustered. Each spectrum $\mathbf{S}_j = [s_{1j}, \dots, s_{ij}, \dots, s_{pj}]$ is a p -dimensional vector (p being the number of considered vibration periods), where $s_{ij} = s_j(T_i)$ is the spectral ordinate at the i -th period for the j -th motion. The K -means method groups these events into clusters by minimizing V , which is defined as follows:

$$V = \sum_{i=1}^K \sum_{\mathbf{S}_j \in S_i} \|\mathbf{S}_j - \mathbf{C}_i\|^2 = \sum_{i=1}^K \sum_{\mathbf{S}_j \in S_i} \sum_{q=1}^p (s_{qj} - C_{qi})^2 \quad (2-38)$$

where K denotes the number of clusters, S_i denotes the set of events in cluster i , $\mathbf{C}_i = [C_{1i}, \dots, C_{qi}, \dots, C_{pi}]$ is the cluster *centroid* obtained as the mean of all the spectra in cluster i , and $\|\mathbf{S}_j - \mathbf{C}_i\|^2 = \sum_{q=1}^p (s_{1qj} - C_{qi})^2$ denotes the distance between the j -th event and the cluster centroid, evaluated as the Euclidean distance, and adopted to measure dissimilarity

In its simplest version, the K -means algorithm is composed of the following four steps:

- Step 1: Pick (randomly) K events to denote the initial cluster centroids.
- Step 2: Assign each event to the cluster with the closest centroid.
- Step 3: Recalculate the centroid of each cluster after the assignments.
- Step 4: Repeat steps 2 and 3 until no more reassignments take place.

Once all the events are clustered, the final catalogue can be developed by randomly selecting a single event from each cluster (accounting for the relative weight of each event), which is used to represent all events in that cluster on account of the similarity of the events within a cluster. In other words, if the event selected from a cluster produces a given structural response value, it is assumed that all other events in the cluster produce the same value by virtue of similarity. The events in this smaller catalogue can then be used in place of those generated using IS for the risk assessment, which results in a dramatic improvement in the computational efficiency. This procedure allows selecting K strongly dissimilar input motions as part of the catalogue, but will ensure that the catalogue is *stochastically representative*. Because only one event from each cluster is now used, the total weight associated with the event should be equal to the sum of the weights of all the events in that cluster:

$$\alpha_i = \sum_{\mathbf{S}_j \in S_i} \alpha(\mathbf{S}_j) = \sum_{\mathbf{S}_j \in S_i} \frac{f(m)}{h(m)} \frac{p(i)}{p(i|M=m)} = \sum_{\mathbf{S}_j \in S_i} \frac{f(m)}{h(m)} \frac{\lambda_i/\lambda_0}{p(i|M=m)} \quad (2-39)$$

2.3.3 Synthetic ground motion models

2.3.3.1 Seismologically-based models

The development of models for ground motion at the soil surface that are based on the physical process of earthquake generation and propagation, and are fit for practical use, is a success story whose beginnings date back not earlier than the late sixties. Today, these models have reached a stage of maturity whereby nature and consequences of the underlying assumptions are well understood, and hence they have started to be applied systematically in geographical regions where data are not sufficient for a statistical approach to seismic hazard, as, for example, in some North-American regions (Atkinson and Boore, 1997) (Toro *et al.*, 1997) (Wen and Wu, 2000), or Australia (Lam *et al.*, 2000), but also in several regions of the world whose seismic activity is well-known, for the double purpose of checking their field of validity and supplementing existing information. A list of applications of this latter type is contained in (Boore, 2003).

The number of different models that have been proposed in the literature in the last two decades is vast: a recent survey with about two hundreds references is contained in (Papageorgiou, 1997); additional references can be found in (Boore, 2003). For the purpose of this section, the so-called *stochastic ground motion model* described by Atkinson and Silva (2000), whose origin is due to Brune (1971), Hanks and McGuire (1981) and Boore (1983), and is widely used in applications, is described in some detail. The choice of this particular model is subjective, and intended only to provide one example, without implications of merit. Its presentation can be conveniently separated into two parts. The first part is devoted to describing the expected Fourier amplitude spectrum of the motion at the surface, based on the gross characteristics of the source and of the travel path (closely following Au and Beck 2003 and Pinto *et al.* 2004), while the second one deals with the procedure for generating synthetic acceleration time-series from the former spectrum.

(1) The acceleration-amplitude Fourier spectrum (or Radiation spectrum)

The frequency content, or spectral characteristics, of the motion at the site, as a function of the event magnitude and source-to-site distance are described by the so-called radiation spectrum, which is the expected Fourier amplitude spectrum of the site motion. This spectrum consists of several factors which account for the spectral effects from the source as well as the propagation path through the earth crust:

$$A(f; M, R) = A_0(f) \frac{1}{R'} \exp(-\gamma(f)R') \cdot \exp(-\pi f \kappa) V(f) \quad (2-40)$$

In Eq.(2-40) $A_0(f)$ is the equivalent point-source spectrum, or simply *source spectrum*, based on two magnitude-dependent corner frequencies $f_a = 10^{2.18-0.496M}$ and $f_b = 10^{2.41-0.408M}$:

$$A_0(f) = CM_0 (2\pi f)^2 \left[\frac{1-\varepsilon}{1+(f/f_a)^2} + \frac{\varepsilon}{1+(f/f_b)^2} \right] \quad (2-41)$$

where $M_0 = 10^{1.5(M+10.7)}$ is the seismic moment, and $C = C_R C_P C_{FS} / (4\pi\rho\beta^3)$, with $C_R = 0.55$ the average radiation pattern for shear waves; $C_P = 2^{-0.5}$ accounts for the partition of waves in two horizontal components; $C_{FS} = 2$ is the free-surface amplification, while ρ and β are the density and shear-wave velocity in the vicinity of the source. The corner frequencies are weighted through parameter $\varepsilon = 10^{0.605-0.255M}$.

Further terms in Eq.(2-40) are as follows: the term $1/R'$ is the *geometric spreading factor* for direct waves (the general form being $1/(R')^n$, with $n = 1$ being valid for direct waves that dominate the surface motions up to a distance of 50 km), with $R' = \sqrt{h^2 + R^2}$ the radial distance between source and site, R the epicentral distance and $h = 10^{-0.05+0.15M}$ the nominal depth of fault (in km) ranging from about 5 km for $M = 5$ to 14 km for $M = 8$; the term $\exp(-\gamma(f)R')$ accounts for *anelastic attenuation*, with $\gamma(f) = \pi f / (Qb)$ and $Q = 180f^{0.45}$ is a regional quality factor; the term $\exp(-\pi f \kappa)$ accounts for the *upper-crust or near-surface attenuation* of high-frequency amplitudes; finally, $V(f)$ describes the *amplification* through the crustal velocity gradient (the passage in the last portion of the travel path from stiffer rock layers, the bedrock, to surface soil layers) as well as soil layers.

It can be observed how starting from the source spectrum, the attenuation of waves in the model is described by three terms, the geometric one which considers only the decrease in energy density while the radiated energy spreads to fill an increasingly larger volume, the anelastic term which accounts for dissipation taking place within this volume and the upper-crust term. The last two terms account for the same phenomenon, i.e. anelastic energy

dissipation, though in different portion of the travel path. Their relative importance depends on the region of interest: in general the correction due to the anelastic attenuation can be disregarded, while the upper crust factor is more important, especially when the rocks in the closer 3 to 4 km to the surface are old and weathered, as is the case e.g. for California, where this model has been developed.

The radiation spectrum is shown for $R = 20$ km and three different magnitudes in Fig. 2-30(a). It can be seen that, as the magnitude increases, the spectral amplitude increases at all frequencies, with a shift of the dominant frequencies towards the lower-frequency regime, as expected.

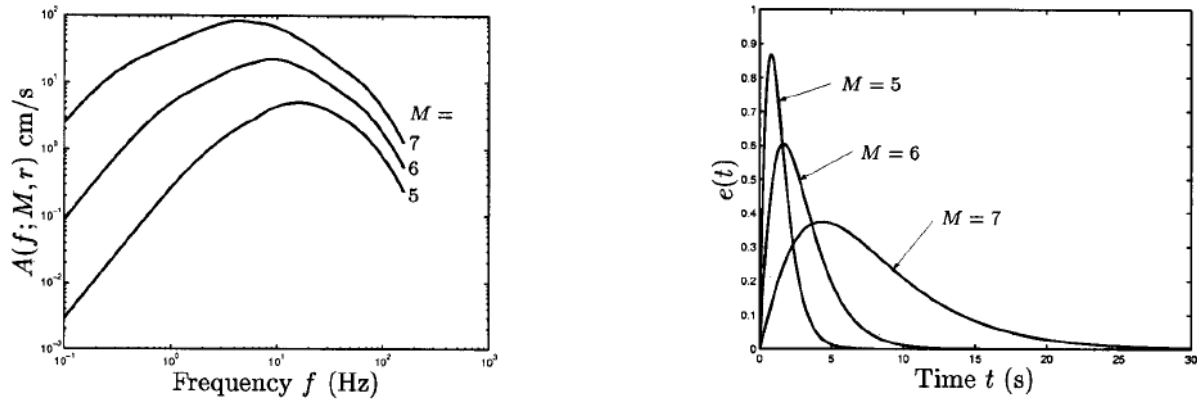


Fig. 2-30: (a) Radiation spectrum as a function of magnitude; (b) envelope function as a function of magnitude, for a source to site distance of 20 km (from Au and Beck, 2003; reprinted with permission from ASCE)

(2) Generation of time-series

Generation of a realization of ground motion acceleration with this model starts with the sampling of a sequence $\mathbf{w}$ of independent identically distributed (i.i.d.) variables representing a train of discrete acceleration values in time $w(t)$ (what is called a *white noise* sequence). This signal is then multiplied by an envelope function $e(t; M, R)$ that modulates its amplitude in time and is a function of the event magnitude M and source to site distance R (Iwan and Hou, 1989):

$$e(t; M, R) = \alpha_1 t^{\alpha_2 - 1} \exp(-\alpha_3 t) U(t) \quad (2-42)$$

where the dependence on M and R is introduced through parameter α_3 , and parameter α_1 is a normalizing factor in order for the envelope to have unit energy $\int_0^\infty e(t; M, R)^2 dt = 1$, and $U(t)$ is the unit-step function. The envelope function is shown for $R=20$ km and three different magnitudes in Fig. 2-30 (b). It can be seen that, as the magnitude increases, the duration increases, as expected.

A discrete Fourier Transform (DFT) is then applied to the modulated signal and the resulting spectrum is multiplied with the Radiation spectrum from the previous section. Inverse Fourier Transform yields back in the time-domain the generated non-stationary non-white sample of acceleration $a(t; M, R, \mathbf{w})$. The generation process is schematically represented in Fig. 2-31.

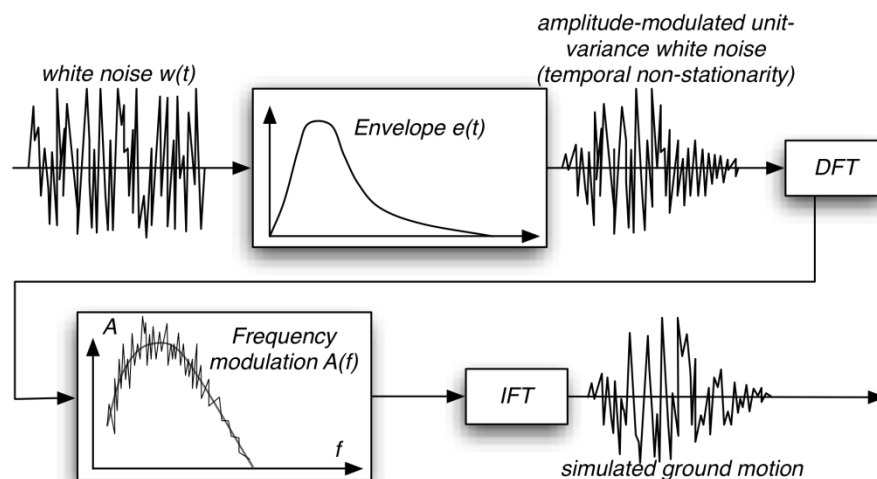


Fig. 2-31: Procedure for simulating a single ground motion realization according to the presented seismological model

(3) Criticism

A comment is in order. In a study directed towards risk assessment, the uncertainties introduced at each step of the analysis need to be quantified. The variability introduced by multiplying the spectrum of the semi-empirical seismological model by that of a windowed white noise, although certainly reasonable, is just one component of the total variability. It is quite obvious that all parameters entering the factors in Eq.(2-40) must be affected by uncertainty, but this kind of epistemic uncertainty is disregarded altogether in the presented model. In this respect, the attribute *stochastic* attached to the model name is misleading. The only randomness is that introduced by the random white noise sequence $\mathbf{w}$. The resulting synthetic ground motions are thus expected to exhibit a lower variability than that characterizing recorded ground motions. This is confirmed by the comparison reported in §2.3.5, where a random correction term needs to be introduced that multiplies the radiation spectrum $A(f)$. Besides, the DFT-multiplication-IFT procedure inevitably introduces some distortion in the spectral content of the simulated samples.

2.3.3.2 Empirical models

These models consist of parameterized stochastic or *random process models*. A basic need of the earthquake engineering community has always been that of defining realistic models of the seismic action for design purposes. Without seismological models available to help in this task, engineers started to look at the records that were rapidly accumulating, in search of characteristics of the ground motion possessing a stable statistical nature (given earthquake and site characteristics such as magnitude, distance and site soil type). This empirical approach has focussed mainly on the frequency content of the motion, with due attention also paid to the modulation in time of the motion and, to a much lesser extent, to the modulation with time of the frequency content, the latter phenomenon stemming from the obvious complexity of the radiation of seismic waves from the source to the site. The observed statistical stability of the frequency content of the motions under similar conditions of M , R and site conditions is at the base of the idea of considering the ground motion acceleration time-series as samples of random processes. Several stochastic models of varying degrees of sophistication have been proposed in the past; in this section one very recent and powerful model is presented due to Rezaeian and Der Kiureghian (2010). The model overcomes the

criticism expressed in the previous section about the correct quantification of all uncertainties contributing to the total variability of the ground motion histories.

A *random process* or *field* is a random scalar- or vector-valued function of a scalar- or vector-valued parameter. One component of ground motion acceleration can be modelled as a random scalar function of the scalar parameter t . The simplest way to obtain a random function of time is as a linear combination of deterministic basis functions of time $h_i(t)$ with random coefficients $\mathbf{x}$:

$$a(t) = \sum_{i=1}^n x_i h_i(t) \quad (2-43)$$

One class of such processes is that of *filtered white noise processes*, where the independent identically distributed random coefficients represent a train of discrete values in time $w(t)$ (the already introduced *white noise* sequence), and the functions $h_i(t)$ represent the impulse response function (IRF) of a linear filter. The well-known Kanai-Tajimi (Kanai, 1957) (Tajimi, 1960) process is one such process and the filter IRF is the acceleration IRF of a linear SDOF oscillator of natural frequency ω_g and damping ratio ξ_g :

$$h_i(t) = h(t - \tau_i) = \frac{\omega_g}{\sqrt{1 - \xi_g^2}} \exp[-\xi_g \omega_g (t - \tau_i)] \sin[\omega_g \sqrt{1 - \xi_g^2} (t - \tau_i)] \quad (2-44)$$

The model by Rezaeian and Der Kiureghian (2010) employs the above expression but, in order to introduce frequency non-stationarity makes the filter parameters time-dependent: $\omega_g(t)$ and $\xi_g(t)$, collectively denoted as $\lambda(t)$.

The output of the filter (the filtered white noise) is then normalized to make it unit-variance and modulated in time with the three-parameters $\alpha = (\alpha_1, \alpha_2, \alpha_3)$ envelope function in Eq. (2-42). Finally, the process is high-pass filtered to ensure zero residual velocity and displacement and accuracy for long-period spectral ordinates of the synthetically generated motions. The generation procedure is schematically represented in Fig. 2-32.

The strength of the model, however, rests in the predictive equations that the authors have developed, through statistical regression, for the parameters in $\lambda(t)$ and α as functions of earthquake and site characteristics such as magnitude M , distance R , faulting style F and average shear wave velocity V_{s30} .

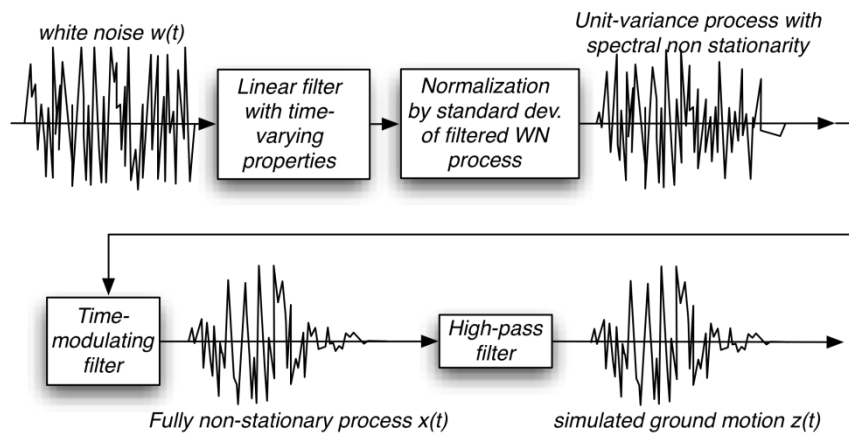


Fig. 2-32: Procedure for simulating a single ground motion realization according to the presented empirical model (adapted from Rezaeian and Der Kiureghian, 2010)

The procedure employed to derive the predictive equations is only briefly recalled here, a detailed description can be found in Rezaeian and Der Kiureghian (2010).

An advantage of this model is that the temporal and spectral non-stationarity are completely separated and hence the corresponding parameters can be estimated in two distinct steps.

As far as the time-modulation is concerned the three parameters in α have been related to three quantities that can be easily identified in any record: the Arias intensity I_a , the effective duration $D_{5-95} = t_{95} - t_5$, and the time at the middle of the strong motion phase t_{mid} , identified in t_{45} (t_{xx} is the time where $xx\%$ of the Arias intensity is attained).

For what concerns the frequency content evolution with time, the damping ratio ξ_g is considered constant and the natural frequency of the filter is modelled as linear in t : $\omega_g(t) = \omega_{mid} + \omega'(t - t_{mid})$, where ω_{mid} and ω' are the frequency and its derivative at t_{mid} . In summary the physically based parameters $\theta = (I_a, D_{5-95}, t_{mid}, \omega_{mid}, \omega', \xi_g)$ completely define the time modulation and the evolutionary frequency content of the non-stationary ground motion model. The simulation procedure is based on generating samples of these parameters for given earthquake and site characteristics.

These parameters have been identified within the selected set of recorded motions, which is targeted at “strong” shaking, and includes only $M \geq 6$ and $R \geq 10$ km records, specifically excluding motions with near-fault features. The authors have worked with a reduced set of recorded ground motions taken from the so-called NGA (Next Generation Attenuation) data base (PEER-NGA). Fig. 2-33 shows the histograms of the identified parameters within the set. In order to perform regression analysis (where Gaussianity of the residual or error term is assumed) the values of the six parameters in θ are transformed into standard normal through marginal transformations with the appropriate distribution (a generalization of the usual logarithmic transformation, necessary due to the non-lognormal distributions exhibited by the parameters, see Fig. 2-33):

$$\nu_i = \Phi^{-1}[F_{\theta_i}(\theta_i)] \quad i = 1, \dots, 6 \quad (2-45)$$

Then a random-effect regression model (see, e.g. Pinto *et al.* 2004) is used in order to account for the clustering of the employed records in sub-sets from the same event, an effect that introduces correlation amongst same-event records and leads to a block-diagonal correlation matrix of the experiments:

$$\nu_{i,jk} = \mu_i(F_j, M_j, R_{jk}, V_k, \beta_i) + \eta_{ij} + \varepsilon_{ijk} \quad (2-46)$$

where $i = 1, \dots, 6$ spans the parameters, j the events, and k the records from each event. The function μ_i is the conditional (on the earthquake and site characteristics) mean of the i -th model parameter, with coefficients β_i , and η_i and ε_i are the inter-event and intra-event model errors, which are zero mean Gaussian variables with variances τ_i^2 and σ_i^2 , respectively. The functional forms together with parameters values for the μ_i can be found in Rezaeian and Der Kiureghian (2010).

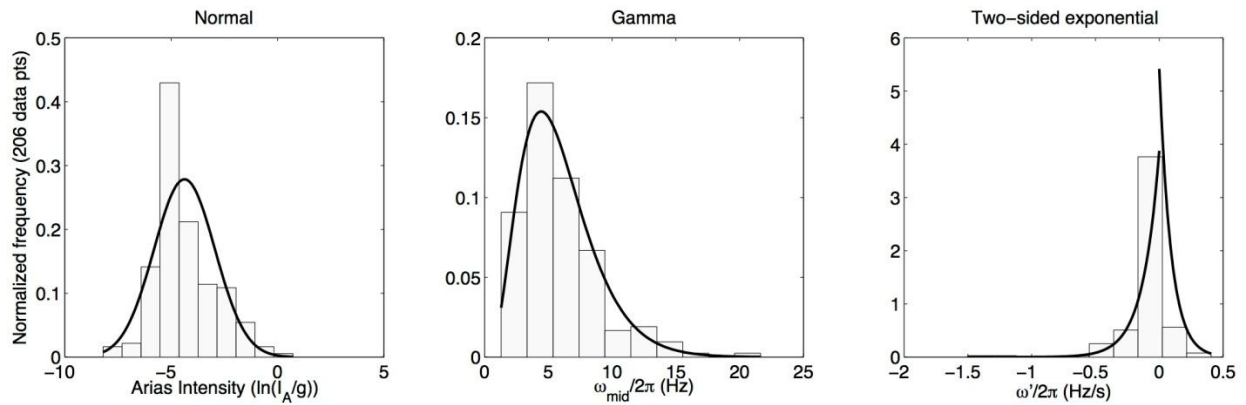


Fig. 2-33: Sample probability density functions superimposed on observed normalized frequency diagrams for three of the six model parameters (raw data courtesy of Sanaz Rezaeian and Armen Der Kiureghian)

It is important to observe how the functions μ_i in a (loose) way, conceptually correspond to the radiation spectrum and the envelope of the seismological model presented earlier, in that they give the frequency content and time modulation for assigned earthquake and site characteristics. The difference of this model rests in the inter- and intra-event errors that, with their variances, model the additional variability missing in the previous model.

The effectiveness of this model can be appreciated from Fig. 2-34, where median and median $\pm$ one log-standard deviation of the 5% damped elastic response spectra of 500 samples from the model are compared with the corresponding spectra from the NGA ground-motion prediction equations, showing how the model describes very well the total variability of the ground motion. The performance of the model is consistently good for other M, R pairs, but for the $M=6$ case (not shown) where, however, the model is close to its range of validity and data were relatively few.

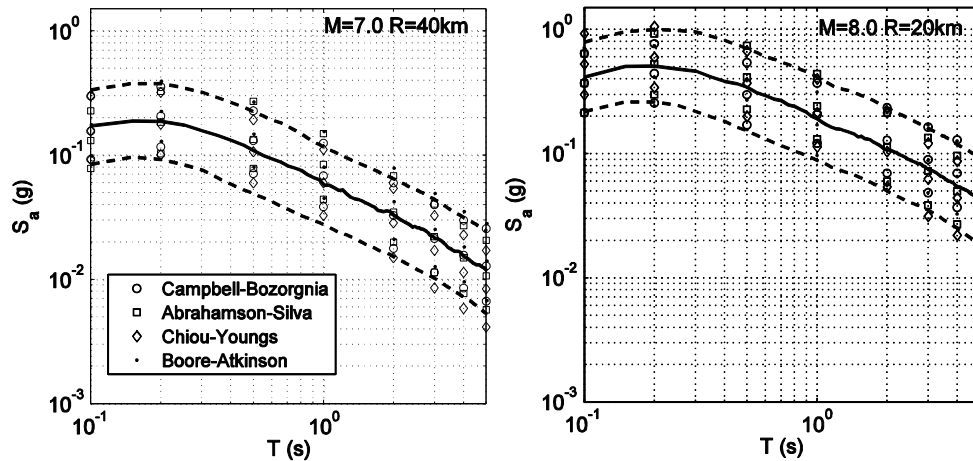


Fig. 2-34: Median and median $\pm$ one log-standard deviation of the 5% damped elastic response spectra of 500 samples from the model versus the corresponding spectra from the NGA ground-motion prediction equations for two scenario events (adapted from Rezaeian and Der Kiureghian, 2010). Markers indicate different GMPE employed. Solid lines are the fractiles of synthetic motions, while dashed lines the corresponding fractiles of GMPE spectral ordinates.

2.3.4 Flow-chart of a seismic assessment by complete simulation

Fig. 2-35 and Fig. 2-36 describe the flow of operations carried out within a single simulation run, either as part of a plain MCS or of a IS-K, for the simpler Atkinson-Silva (AS2000) and the more recent Rezaeian-Der Kiureghian (R-ADK2010) model, respectively.

As stated in §2.3.1, the vector $\mathbf{x}$ of random variables should collect randomness relating to the earthquake source, propagation path, site geology/geotechnics, frequency content of the time-series, structural response and capacity. Correspondingly, as illustrated in Fig. 2-35 the vector is partitioned as follows:

$$\mathbf{x} = \left\{ \begin{matrix} x_1 \\ x_2 \\ x_3 \\ \mathbf{x}_4 \\ \mathbf{x}_5 \\ \mathbf{x}_6 \end{matrix} \right\} = \left\{ \begin{matrix} M \\ Z \\ E \\ \mathbf{w} \\ \mathbf{x}_5 \\ \mathbf{x}_6 \end{matrix} \right\} \quad (2-47)$$

where the first three variables describe the randomness in the source (event magnitude M , active fault/zone Z and epicentre location E in the simulation run) and are part of the *seismicity model*, the fourth component of $\mathbf{x}$ is a vector and contains the stationary white noise time series $\mathbf{w}$, the fifth component is a vector describing randomness in the site geotechnical characterization and in the site-response model, while the sixth and last component is a vector describing randomness in the structure and its model.

The figure shows how the first variable to be sampled is the magnitude, from its distribution, given either by Eq.(2-34) for MCS or Eq.(2-37) for IS-K. Conditional on magnitude, the active zone is sampled from its discrete probability distribution Eq.(2-35). Once the zone is known, the epicentre location can be sampled, from which the distance R from the site S (whose position is deterministically known, as denoted by a lozenge symbol in the scheme, as opposed to circles/ovals denoting random quantities) can be evaluated.

Magnitude and distance enter into the ground motion model to determine the shapes of the time-envelope and of the amplitude spectrum. As described in Fig. 2-31, the ground motion time series (on rock/stiff soil) $a(t)$ is obtained by taking a sample of stationary white noise $\mathbf{w}$, modulating it in time by multiplication for the time-envelope $e(t)$, feeding this to the DFT, “colouring” the result with the amplitude spectrum $A(f)$, to inverse transform it back in the time domain by the IFT. This motion may enter into a site-response analysis module in order to obtain the input motion to the structure at the surface. This module implements a site-response model (e.g. a one-dimensional nonlinear, or equivalent linear, model) and takes as an input the soil strata and their stiffness/strength properties. The strata thicknesses and properties may all be affected by uncertainty, modelled by the sub-vector $\mathbf{x}_5$.

Finally, the surface motion enters into the finite element model which determines the response of the structure $\mathbf{r}(t)$. Both the structure itself, and the response-model implemented in the analysis software, are affected by uncertainty, modelled by the sub-vector $\mathbf{x}_6$. The end result of the run is the value of the performance indicator $I_f(\mathbf{x})$.

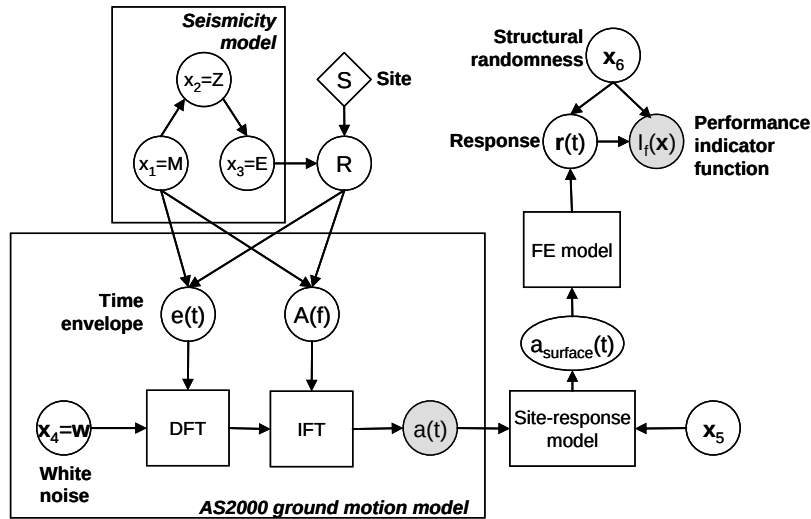


Fig. 2-35: Flow of simulation for a single run, employing the Atkinson and Silva model

In the case of the R-ADK2010 model the procedure is unchanged with the exception of the ground motion time-series generation. As shown in Fig. 2-36 and in Eq.(2-46), the ground motion model requires two additional inputs, with respect to the AS2000 model: the fault mechanism, which depends on the active fault/zone in the simulation run Z , and the shear-wave velocity V , which depends on the site (actually the model depends on the wave velocity in a velocity range that is commonly associated with rock/stiff-soil and hence can be regarded as a model to predict the motion at stiff sites only, to be complemented, as for the previous model, with a site-response analysis model when the local site conditions require it).

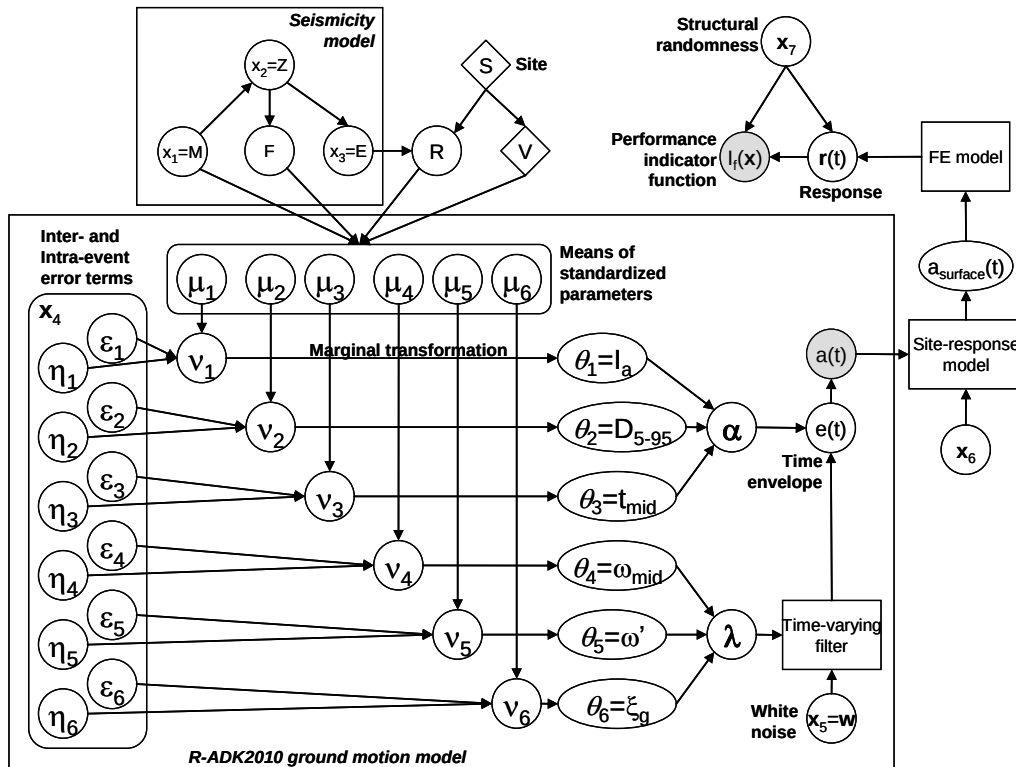


Fig. 2-36: Flow of simulation for a single run, employing the Rezaeian and Der Kiureghian model

Magnitude, distance, shear-wave velocity and faulting style concur to determine the means μ_i ($i=1,\dots,6$) of the six standardized model parameters v_i . These, summed with the corresponding inter- and intra-event error terms (collected in the random sub-vector $\mathbf{x}_4$), are transformed by marginal transformations as in Eq. (2-45) to the six physically meaningful parameters θ . Once the latter vector is known the stationary white noise time-series can be filtered and modulated to produce the acceleration time-series $a(t)$. From there on the procedure follows the same steps outlined for the case of the AS2000 model.

2.3.5 Example

In order to illustrate the application of the unconditional probabilistic approach, the MCS method and IS-K method described in §2.3.2.3, jointly with the R-ADK time series model described in §2.3.3.2 are applied to the determination of a structural MAF λ_{LS} for the fifteen storeys RC plane frame shown in Fig. 2-37. Results show how MAFs in the order of 10^{-3} can be obtained with a few hundreds of analyses.

Given that, subject to the quality of the models (namely, the ground-motion time-series model), this approach is more general than the IM-based or conditional one, the example is also used to offer a term of comparison for the results obtained with the conditional probability approach. Within the limits of the considered example, the outcome of this comparison provides a cross-validation, on one hand of the IM-based methods, and on the other of the employed synthetic ground motion model.

2.3.5.1 Illustration of MCS, ISS and IS-K methods

Fig. 2-37 shows the frame overall dimensions and the reinforcement (with layout shown in the same figure), in terms of geometric reinforcement ratio (percent), of the total longitudinal reinforcement for the columns and of the top longitudinal reinforcement for the beams. Beams have all the same cross-section dimensions, 0.30 m wide by 0.68 m deep, across all floors. Columns taper every five floors. Exterior columns, with a constant 0.50 m width, have 0.73 m height for the base and middle columns, 0.63 m for top columns. Interior columns, with a constant 0.40 m width, have 0.76 m, 0.73 m and 0.62 m height, for the base, middle and top columns, respectively.

The frame is located at a site affected by two active seismo-genetic sources, as shown in Fig. 2-38, left. The figure reports the parameters of the probabilistic model for the activity rate of each source. The model is the truncated Gutenberg-Richter one, which gives the mean annual rate of events with magnitude $M \geq m$ on source i as the product of the mean annual rate of all events λ_i on the source, times the probability that given an event, it has $M \geq m$:

$$\lambda_i(m) = \lambda_i \frac{e^{-\beta_i m} - e^{-\beta_i m_{iu}}}{e^{-\beta_i m_{il}} - e^{-\beta_i m_{iu}}} \quad (2-48)$$

The model is called “truncated” because the probability density for M is non-zero only within the interval $[m_{il}, m_{iu}]$ defined by the lower and upper magnitudes. Fig. 2-38, right, shows the discrete conditional probability distribution for the random variable Z used to sample the active zone in each simulation run. Three cases are shown, corresponding to three ranges of the conditioning variable M : $M < 6.5$, in which case the only zone that can generate the event is zone 1 ($p_1 = 1, p_2 = 0$); $6.5 \leq M < 7.0$, in which case $p_i = \lambda_i / \lambda_0$; $M > 7.0$, in which case the only zone that can generate the event is zone 2 ($p_1 = 0, p_2 = 1$).

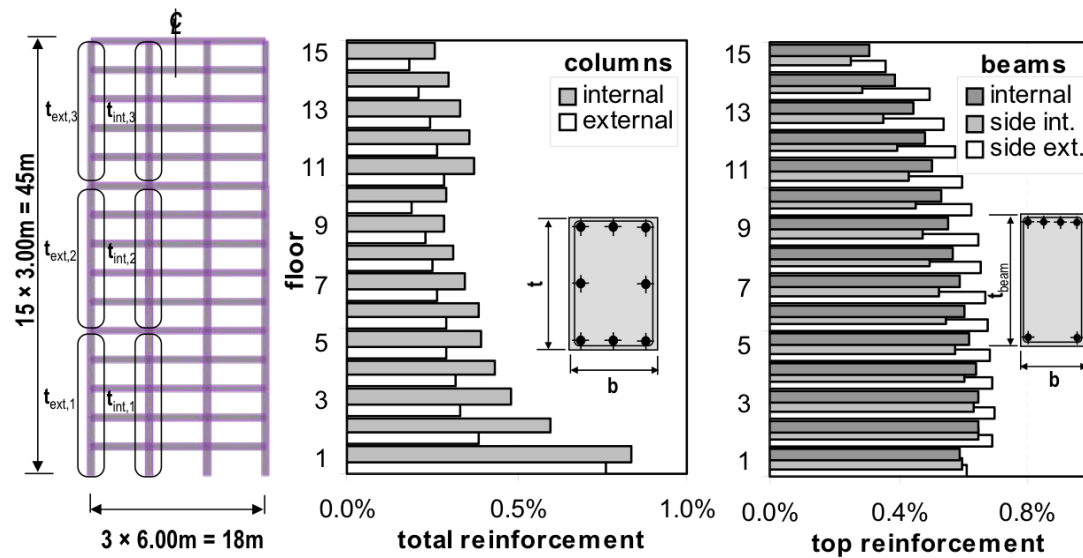


Fig. 2-37: The 15-storeys RC plane frame

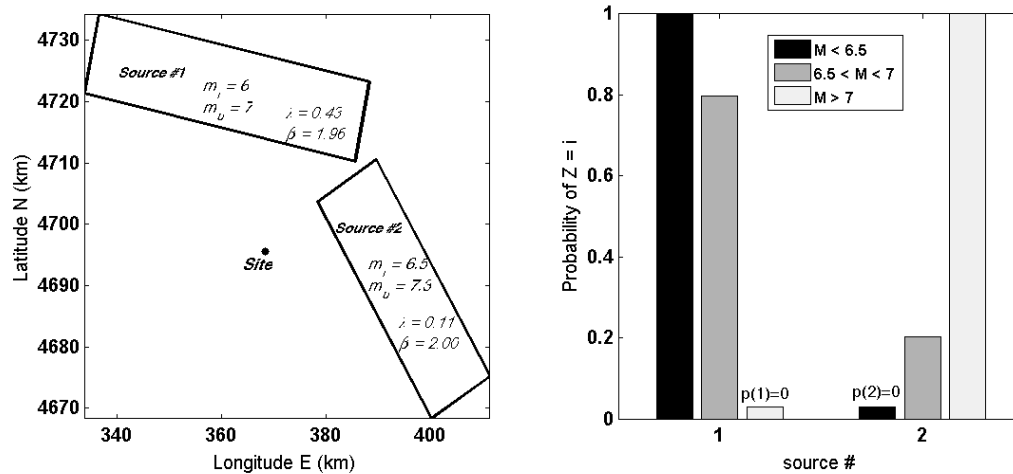


Fig. 2-38: The seismo-tectonic environment affecting the site of the frame: the two sources and the site (left), the discrete probability distribution for the random variable $x_2 = Z$ for $M < 6.5$, $6.5 \leq M \leq 7$ and $M > 7$

The results shown in the following are obtained by means of three independent simulations: a reference case consisting of a plain Monte Carlo simulation with 10,000 runs, an Importance sampling on magnitude with 1,000 runs, and the IS-K method where the previous 1,000 ground motions sampled for the IS are clustered into 150 events. In all cases, given a (M,R) pair, the R-ADK model is employed to produce an acceleration time series at the site of the frame. Fig. 2-39 shows two sample motions generated for the same (M,R) pair.

For the IS-K method, the clustering proceeds through the four steps described in §2.3.2.3. The final result is obtained in less than 10 iterations. For the sake of illustration, Fig. 2-40 shows the first nine clusters obtained from the procedure. The motions are represented by their displacement response spectra. The spectrum of the time series randomly sampled to represent the whole cluster is shown in solid black. Notice how the cluster size is not constant (e.g. compare clusters #2 and #5, the latter having only two time series).

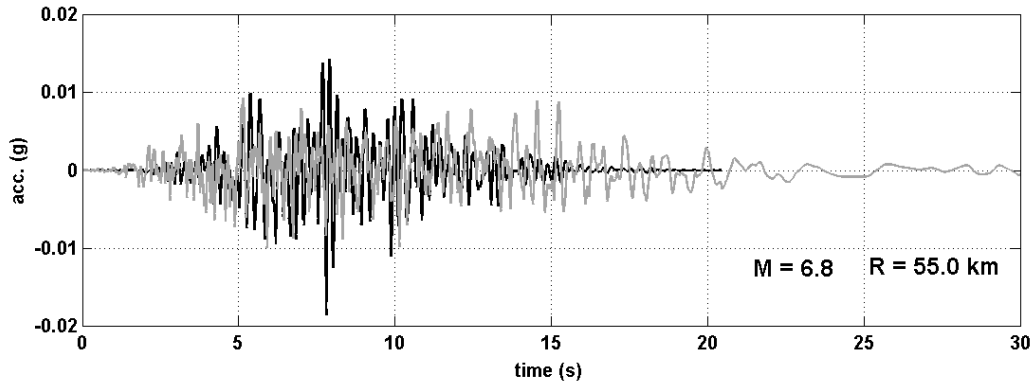


Fig. 2-39: Two acceleration time series obtained from the R-ADK2010 model for a $M = 6.8$ and $R = 55$ km event

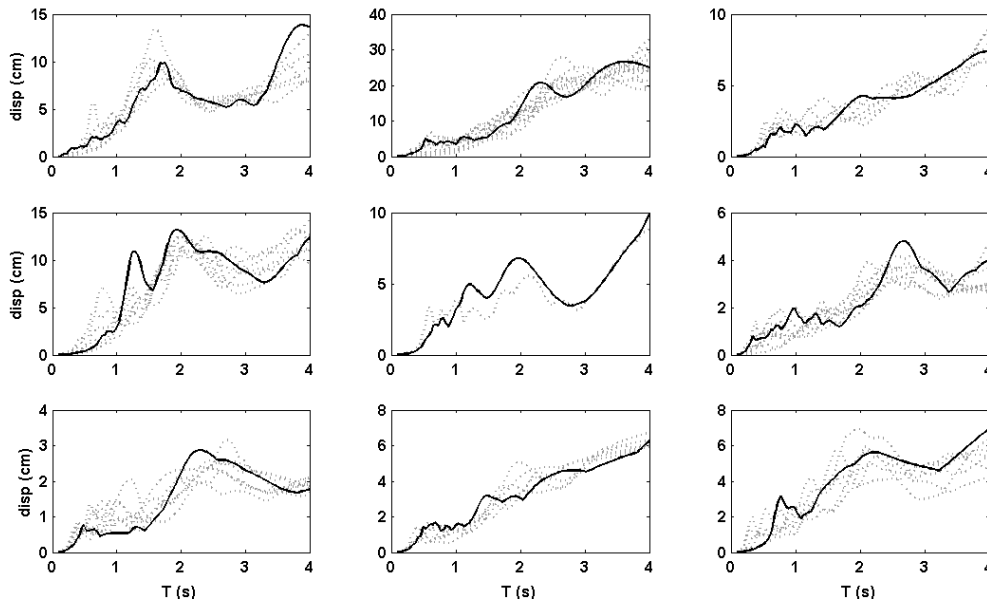


Fig. 2-40: Nine of the 150 clusters employed in the IS-K method: displacement response spectra of the time-series in each cluster (dashed grey line) and the spectrum of the randomly selected record representative of the entire cluster (solid black line)

Structural response has been evaluated with an inelastic model set up in the analysis package OpenSEES. The model consists of standard nonlinear beam-column elements with fibre discretized sections (Scott-Kent-Park concrete and Menegotto-Pinto steel models). Gravity loads are applied prior to time-history analysis. The structural performance measure adopted is the peak interstorey drift ratio $\theta_{\max}$.

Fig. 2-41, left, shows the histogram of the relative frequency of the $\theta_{\max}$ samples from the Monte Carlo simulation. Similar histograms, with a lower number of bins reflecting the smaller sample size, are obtained for the IS and IS-K methods. These histograms are used to obtain the cumulative distribution function $F_{\theta_{\max}}(x)$. Fig. 2-41, right, shows the MAF curves for $\theta_{\max}$ obtained by the three simulation methods according to the expression:

$$\lambda_{\theta_{\max}}(x) = \lambda_0 G_{\theta_{\max}}(x) = (\lambda_1 + \lambda_2)[1 - F_{\theta_{\max}}(x)] \quad (2-49)$$

The curves are remarkably close to each other, down to rates in the order of 10^{-3} , which is also as far as one can trust an MCS result with 10,000 runs. In the figure there are two curves for IS-K method. They correspond to two clustering criteria, the first (green line) being that presented in §2.3.2.3, i.e. similarity of the time-series is determined based on their full response spectrum. The second criterion judges similarity based only on the spectral ordinate at the fundamental period of the structure. The closeness of the two curves is expected due to the dynamic properties of the considered structure which has a weak second mode contribution to response.

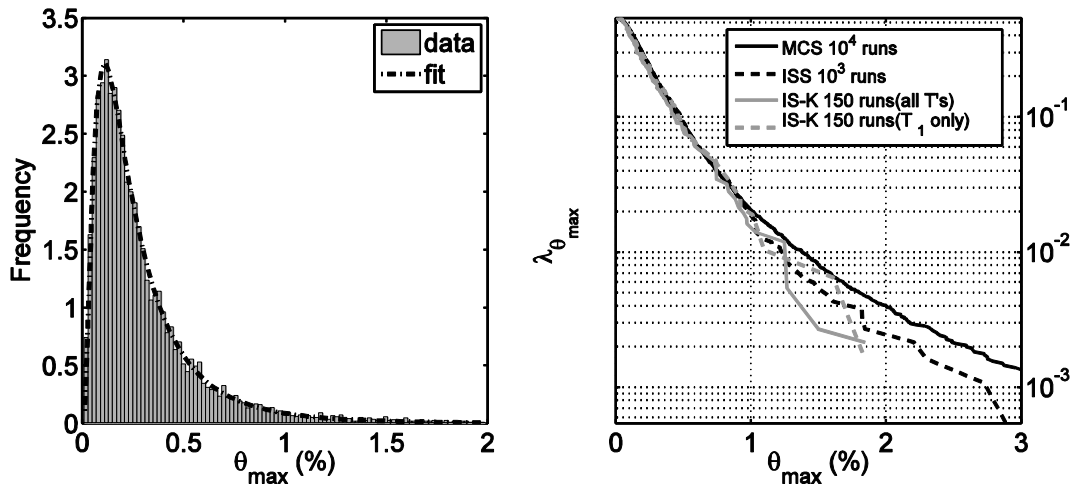


Fig. 2-41: Mean annual frequency of exceedance of the peak interstorey drift ratio

In conclusion, the IS-K method is shown to yield results equivalent to those obtained with plain Monte Carlo, for an effort which is two orders of magnitude lower and therefore makes the approach affordable in practice.

2.3.5.2 Comparison with the IM-based approach

The cornerstone of the conditional probability approach is the split between the work of the seismologist, that characterizes the seismic hazard at the site with a MAF of an intensity measure, and that of the structural engineer, whose task is to produce the conditional distribution of the limit-state given the IM.

In order to be able to compare the two probabilistic approaches, it is first necessary to investigate differences in the hazard, as obtained through attenuation laws during a PSHA, and as implied by the employed ground motion model in the unconditional simulation approach. Large differences in the intensity measure at the site, based on the same regional seismicity characterization, would directly translate into different MAFs of structural response.

Fig. 2-42 shows the MAF of the spectral acceleration, at the first mode period of the frame (left) and at $T=1.0$ s (right), evaluated through PSHA, employing six distinct attenuation laws, four of which developed as part of NGA effort (PEER, 2005) and, thus, sharing the same experimental base (recorded ground motions) as the Rezaeian and Der Kiureghian synthetic motion model. The figure shows also the MAF of S_a obtained from the synthetic motions sampled for the MCS and IS cases, evaluated as:

$$\lambda_{S_a}(x) = \lambda_0 G_{S_a}(x) = \lambda_0 [1 - F_{S_a}(x)] \quad (2-50)$$

As shown by the figure the hazard obtained employing the synthetic motions falls within the range of variability of the attenuation laws. Comparing Fig. 2-42, left and right, one can see how the performance of the synthetic ground motion model is not uniform over the range of vibration periods. In any case, the quality of its predictions should be judged in light of the differences exhibited by the GMPEs themselves. The GMPE by Idriss shows a closer match at both periods and is used in the comparison of MAFs of structural response.

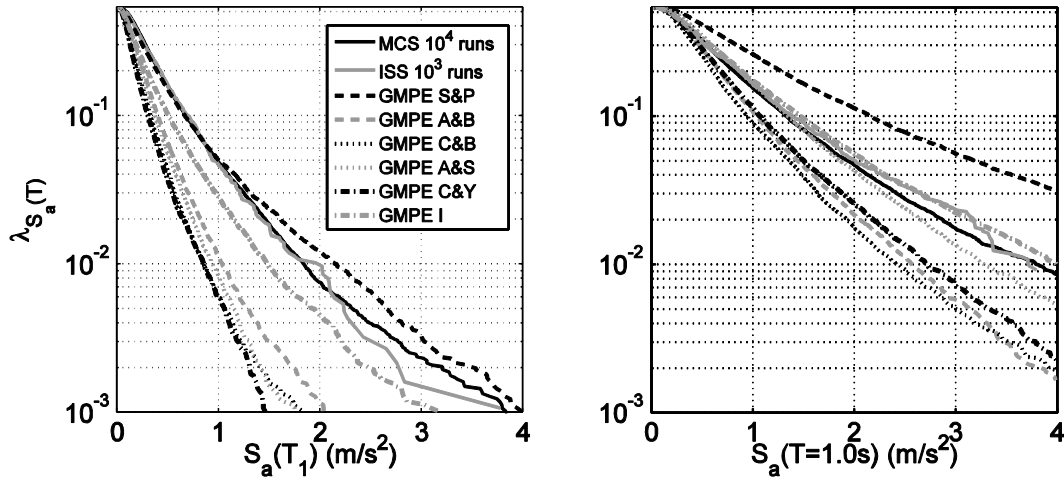


Fig. 2-42: Mean annual frequency of exceedance of the spectral acceleration (hazard curve), at the first mode period $T_1 = 2.68$ s (left) and at $T = 1.0$ s, as obtained by PSHA with different attenuation laws (S&P: Sabetta and Pugliese 1996, A&B: Atkinson and Boore 2003, C&B: Campbell and Bozorgnia, A&S: Atkinson and Silva, C&Y: Chiou and Youngs, I: Idriss) and by post processing the spectral ordinates of the synthetic motion samples.

In the conditional probability approach structural analysis for recorded ground motions are employed to establish the distribution of maximum response conditional on intensity measure. This can be done in essentially two different ways, as shown in Sections 2.2.1 and 2.2.2. In the former case motions are scaled to increasing levels of the IM to produce samples of the EDP from which a distribution, commonly assumed lognormal, is established at each level. In the latter case, as explained at length in §2.2.2, additional assumptions are made, two of which are related to the distribution of EDP given IM: the median EDP-IM relationship is approximated with a power-law, and the EDP dispersion is considered independent of the IM. Accordingly, the MAF of structural response, used here to compare with the results from MCS, can be expressed as:

$$\lambda_{\theta_{\max}}(x) = \int_0^\infty G_{\theta_{\max}}(x|S_a = y) d\lambda_{S_a}(y) = \int_0^\infty \left[1 - \Phi\left(\frac{\ln x - \mu_{\theta_{\max}}(y)}{\beta_{\theta_{\max}}(y)}\right) \right] d\lambda_{S_a}(y) \quad (2-51)$$

$$\lambda_{\theta_{\max}}(x) = \int_0^\infty G_{\theta_{\max}}(x|S_a = y) d\lambda_{S_a}(y) = \int_0^\infty \left[1 - \Phi\left(\frac{\ln x - (\ln a + b \ln y)}{\beta_{\theta_{\max}}}\right) \right] d\lambda_{S_a}(y) \quad (2-52)$$

where $d\lambda_{S_a}(y)$ is the absolute value derivative of the hazard curve $\lambda_{S_a}(y)$, while a , b and $\beta_{\theta_{\max}} = \beta_D$ are the parameters of the power-law fit to the intensity-demand points.

Fig. 2-43 shows the so-called incremental dynamic analysis (IDA) curves (Vamvatsikos and Cornell, 2002) that relate the chosen EDP = θ_{max} with the IM = $S_a(T_1)$. Two sets of curves are shown, obtained with 20 motions taken randomly from those sampled for the MCS using the R-ADK2010 model, labelled “Synthetic”, on the left, and with 20 recorded ground motions taken randomly from the same set of records (PEER, 2005) used as a basis to develop the R-ADK2010 model. The figure shows with black dots the structural analyses results (S_a - θ_{max} pairs) employed to draw the IDA curves, and with red dots the values of θ_{max} interpolated at one value of $S_a = y$, to establish the complementary distribution $G_{\theta_{max}}(x|S_a = y)$ (basically estimating $\mu_{\theta_{max}}(y)$) and $\beta_{\theta_{max}}(y)$ - the figure reports the estimates of median and dispersion at the same intensity for the two cases). Notice that the number of analyses is not the same for the synthetic and natural motions. This is due to the algorithm used to trace the curves which requires a number of analyses that is record-dependent (see Vamvatsikos and Cornell, 2002). In particular, the algorithm (called “Hunt and Fill”) first increases the intensity with a geometric progression until it reaches a pre-defined “collapse” response (e.g. global dynamic instability), then fills in the curve with a bi-section rule to better describe the region around the threshold. In this case termination was set at a threshold value of the θ_{max} equal to 2%.

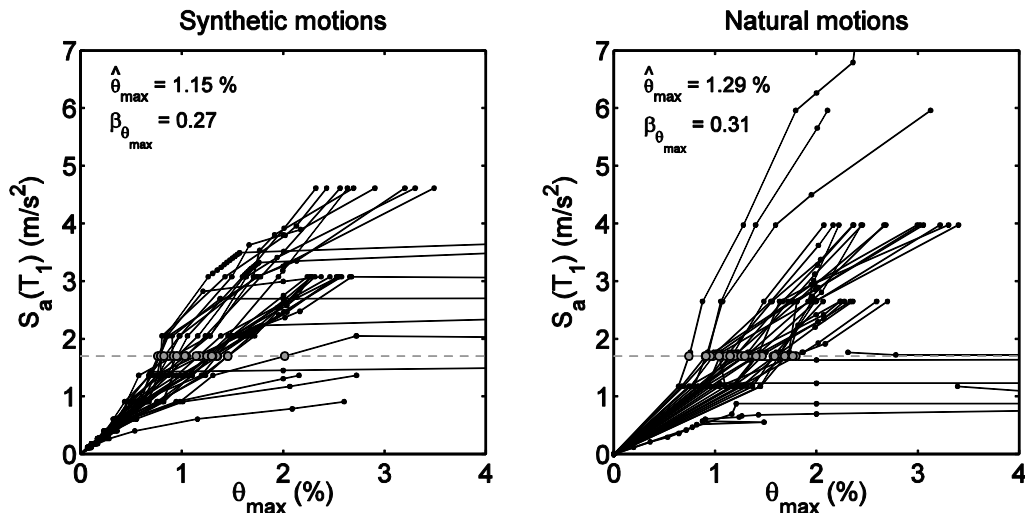


Fig. 2-43: IDA curves obtained with 20 motions (synthetic on the left, natural recorded motions on the right)

Fig. 2-44 shows the results of the second approach, where a power-law (line in log-log space) is employed to express the median θ_{max} as a function of S_a , based on the results (S_a - θ_{max} pairs) of 30 structural analyses carried out with unscaled records. This use of unscaled records to obtain a sample of structural responses is often called a “cloud” analysis. As in the previous case the response determination is repeated for synthetic and natural motions.

The final results can be condensed in the two MAF plots shown in Fig. 2-45, where the curves are evaluated according to Equations (2-44), (2-45) and (2-46). The left plot shows the MAF of structural response obtained using synthetic motions also for the conditional probability approach, i.e. for deriving the hazard curve and the distribution of response. This plot provides a comparison of the probabilistic approaches “all other factors being the same”, and the results show that, at least for the considered structure, the approximation associated with the IM-based method is completely acceptable.

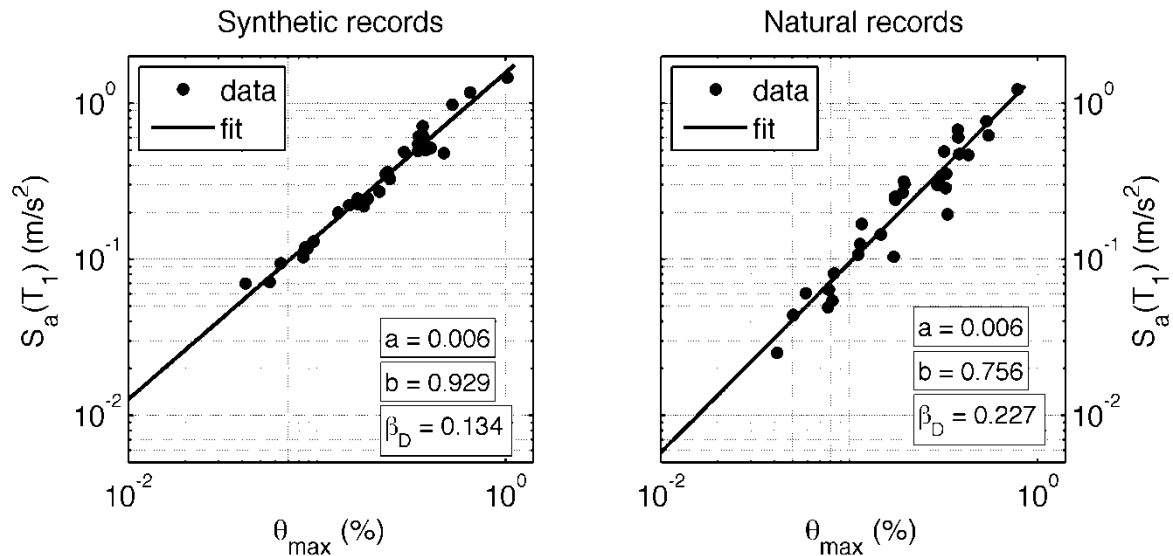


Fig. 2-44: Results of structural analyses carried out with 30 unscaled records (a “cloud” analysis)

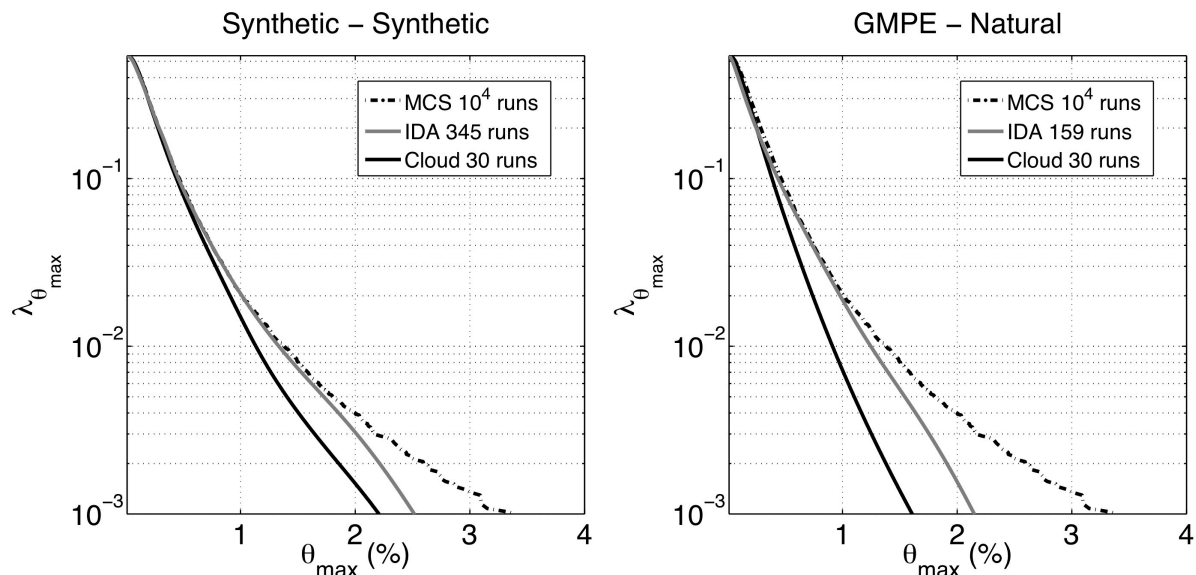


Fig. 2-45: Mean annual frequency of exceedance of the peak interstorey drift ratio

The right plot shows analogous results where now the IM-based curves are obtained with the Idriss hazard curve and recorded ground motions. The match is still quite good for the IDA-based case, while the cloud with its fewer runs and constrained median response predicts lower values ($b = 0.756$). The good match constitutes a measure of the quality of the synthetic ground motion model, which, as claimed by the authors, simulates with an acceptable accuracy both the median intensity of natural motions and their total variability. As a final comment, it appears from the above analyses that IS-K and IDA yield similar results with comparable efforts (150 vs. 159 runs in this particular case), suggesting that the choice between these methods may become in a close future a matter of personal preference.

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3 Probabilistic seismic design

3.1 Introduction

The state of development of fully probabilistic design methods of structures against seismic actions is much behind that of assessment methods. This is not surprising if one considers that a similar situation applies to deterministic methods. The ambition of the chapter cannot go beyond that of providing an overview of the available options, complemented by two simple examples that highlight the progress that still needs to be accomplished.

Section 3.2 deals with the use of optimization methods for probabilistic seismic design, while Section 3.3 presents non-optimization based options. In both cases the underlying probabilistic theory is basically the same as that introduced in IM-based methods in Chapter 3.

3.2 Optimization-based methods

Structural optimization problems can generally be expressed in the simple mathematical form:

$$\min \mathbf{f}(\mathbf{x}) \quad \text{subject to} \quad \mathbf{g}(\mathbf{x}) \leq 0 \quad (3-1)$$

where $\mathbf{f}$ and $\mathbf{g}$ represent the objectives and constraints, respectively, and $\mathbf{x}$ is a vector of decision variables. Although this is a common notation for almost all optimization problems, the structure being optimized, variables, constraints and the domain of optimization can be significantly different.

The problems can be separated into three classes: *sizing*, *shape* and *topology* optimization, which are illustrated in Fig. 3-1. In *sizing* optimization the locations and number of elements are fixed and known, and the dimensions are varied to obtain the optimal solutions [Fig. 3-1 (a)]. In *shape* optimization, on the other hand, the boundary of the structural domain is optimized while keeping the connectivity of the structure the same, i.e. no new boundaries are formed [Fig. 3-1 (b)]. In *topology* optimization, the most general class, both the size and location of structural members are determined and the formation of new boundaries is allowed [Fig. 3-1 (c)]. In this case the number of joints in the structure, the joint support locations, and the number of members connected to each joint are unknown.

Most studies on structural earthquake engineering deal with the class of sizing optimization, where the design variables are limited to member/section properties.

In the following sections, first, the basic terminology used in structural optimization is introduced, some of the most commonly used tools for solving optimization problems are briefly described and a review of structural optimization studies is provided. Then, a life-cycle cost (LCC) formulation is provided in which the uncertainty in structural capacity and earthquake demand is incorporated into the design process by converting the failure probabilities into monetary value. Finally, the section is closed by an illustrative example.

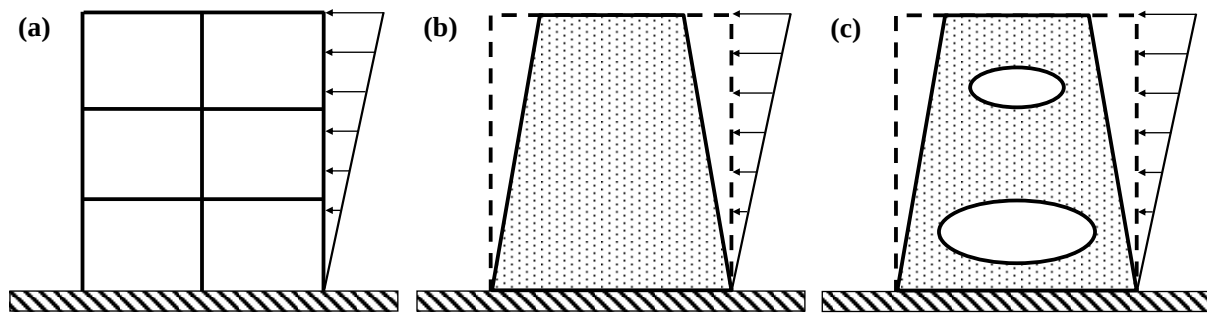


Fig. 3-1: Example classification of optimization problems (a) sizing, (b) shape, and (c) topology optimization

3.2.1 Terminology

Objective (merit) function: A function that measures the performance of a design. For every possible design, the objective function takes a different value. Examples include the maximum interstorey drift and initial cost.

Design (decision) variables: A vector that specifies design. Each element in the vector describes a different structural property that is relevant to the optimization problem. The design variables take different values throughout the optimization process. Examples include section dimensions and reinforcement ratios.

Performance levels (objectives or metrics): Predefined levels that describe the performance of the structure after an earthquake. Usually the following terminology is used to define the performance level (limit state) of a structure: immediate occupancy (IO), life safety (LS) and collapse prevention (CP). Exceedance of each limit state is determined based on the crossing of a threshold value in terms of structural capacity.

Hazard levels: Predefined probability levels used to describe the earthquake intensity that the structure might be subjected to. Hazard levels are usually assigned in terms of earthquake mean return periods (or mean annual frequency of exceedance), and represented by the corresponding spectral ordinates.

Space of design (decision) variables or search space: The boundaries of the search space are defined by the range of the design variables. The dimension k of the search space is equal to the number of design variables in the problem. Each dimension in the search space is either continuous or discrete depending on the nature of the corresponding design variable.

Solution (objective function) space: Usually the solution space is unbounded or semi-bounded. The dimension l of the solution space is equal to the number of objective functions in the optimization problem. The optimal solution(s) is (are) defined in the solution space. The set of optimal solutions in the solution space is referred to as a Pareto-front or Pareto-optimal set.

Pareto-optimality: To define Pareto-optimality, consider the function $f: \mathcal{R}^k \rightarrow \mathcal{R}^l$ which assigns each point, $\mathbf{x}$ in the space of decision variables to a point, $\mathbf{y} = \mathbf{f}(\mathbf{x})$ in the solution space. Here $\mathbf{f}$ represents the objective functions. The Pareto-optimal set of solutions is constructed by comparing the points in the solution space based on the following definition: a point $\mathbf{y}$ in the solution space *strictly dominates* another point $\bar{\mathbf{y}}$ if each element of $\mathbf{y}$ is less than or equal to the corresponding element of $\bar{\mathbf{y}}$, that is $y_i \leq \bar{y}_i$, and at least one element, i^* is strictly less, that is, $y_{i^*} < \bar{y}_{i^*}$ (assuming that this is a minimization problem). Thus, the Pareto-front is the subset of points in the set of $\mathbf{Y} = \mathbf{f}(\mathbf{X})$, that are not strictly dominated by another point in $\mathbf{Y}$. Pareto-optimality is illustrated in Fig. 3-2: the plot is in the two-dimensional solution space of the objective functions, f_1 and f_2 . Assuming that the objective is minimization of both f_1 and f_2 , the Pareto-front lies at the boundary that minimizes both objectives as shown in the figure.

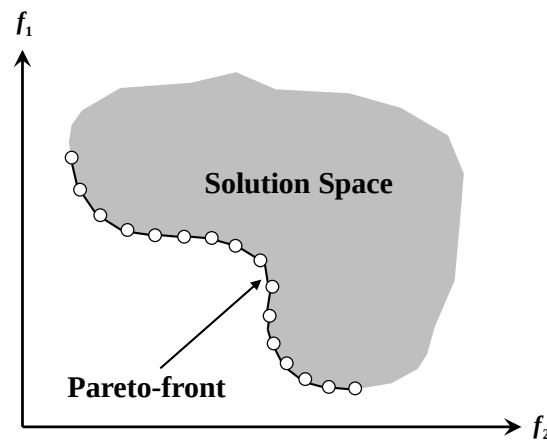


Fig. 3-2 Illustration of Pareto-optimality

3.2.2 Tools for solving optimization problems

Earlier studies in structural optimization focused on *single-objective* optimization using *gradient-based* algorithms. In simplest terms, these algorithms work to minimize or maximize a real function by systematically choosing variables from within an allowed search space. The most commonly used gradient-based algorithms in structural optimization include linear and nonlinear programming, optimality criteria and feasible directions. In these studies, the merit function was almost exclusively selected as the initial cost (or the material usage). Several constraints (most often based on code provisions) were applied to determine the validity of designs. Explicit formulations, which could be evaluated with little effort, were used for both the objective function and the constraints. The underlying reason for the selection of gradient-based algorithms was their relative computational efficiency due to rapid convergence rates. Gradient-based algorithms, however, require the existence of continuous objective functions and constraints in order to evaluate gradients and, in some cases, Hessians, thus limiting the range of problems that these algorithms can be applied to.

Most practical design problems in structural engineering entail the discrete representation of design variables (e.g. section sizes, reinforcement areas). Furthermore, the advent of (increasingly nonlinear) numerical analysis methods has led to discontinuous objective functions and/or constraints. Hence, researchers resorted to zero-order optimization algorithms that do not require existence of gradients or the continuity of merit functions or constraints.

A class of zero-order optimization algorithms is the *heuristic* methods. As the name indicates, these methods are experience-based and they depend on some improved version of basic trial and error. The main advantage of these approaches is that they can be adapted to solve any optimization problem with no requirements on the objectives and constraints. Furthermore, these approaches are very effective in terms of finding the global minimum of highly nonlinear and/or discontinuous problems where gradient-based algorithms can easily be trapped at a local minimum. The main criticism to heuristic approaches is that they are not based on a mathematical theory and that there is no single specific heuristic optimization algorithm that can be generalized to work for a wide class of optimization problems.

The most commonly used approaches include Genetic Algorithms (GA), Simulated Annealing (SA), Tabu Search (TS), and shuffled complex evolution (SCE). It is beyond the scope of this document to compare and contrast different optimization methods; however, TS is briefly described in the following because it is used in the illustrative example in Section 3.2.4.

Tabu Search is due to Glover (1989, 1990), and it is generally used to solve combinatorial optimization problems (i.e. a problem of finding an optimum solution within a finite set of feasible solutions, a subset of discrete optimization). TS employs a neighborhood search procedure to sequentially move from a combination of design variables x (e.g. section sizes, reinforcement ratios) that has a unique solution y (e.g. maximum interstorey drift, maximum strain), to another in the neighborhood of y until some termination criterion has been reached. To explore the search space, at each iteration TS selects a set of neighboring combinations of decision variables using some optimal solution as a seed point. Usually a portion of the neighboring points is selected randomly to prevent the algorithm being trapped at a local minimum. TS algorithm uses a number of memory structures to keep track of the previous evaluation of objective functions and constraints. The most important memory structure is called the *tabu list*, which temporarily or permanently stores the combinations that are visited in the past. TS excludes these solutions from the set of neighboring points that are determined at each iteration. The existence of the tabu list is crucial to optimization problems where the evaluation of objective functions and/or constraints are computationally costly. A flowchart of the algorithm is provided in Fig. 3-3 and a sample code is included in the Appendix. An advantage of the TS algorithm is that it naturally lends itself to parallel processing, which is often needed to solve problems when evaluating the objective functions or the constraints is computationally costly.

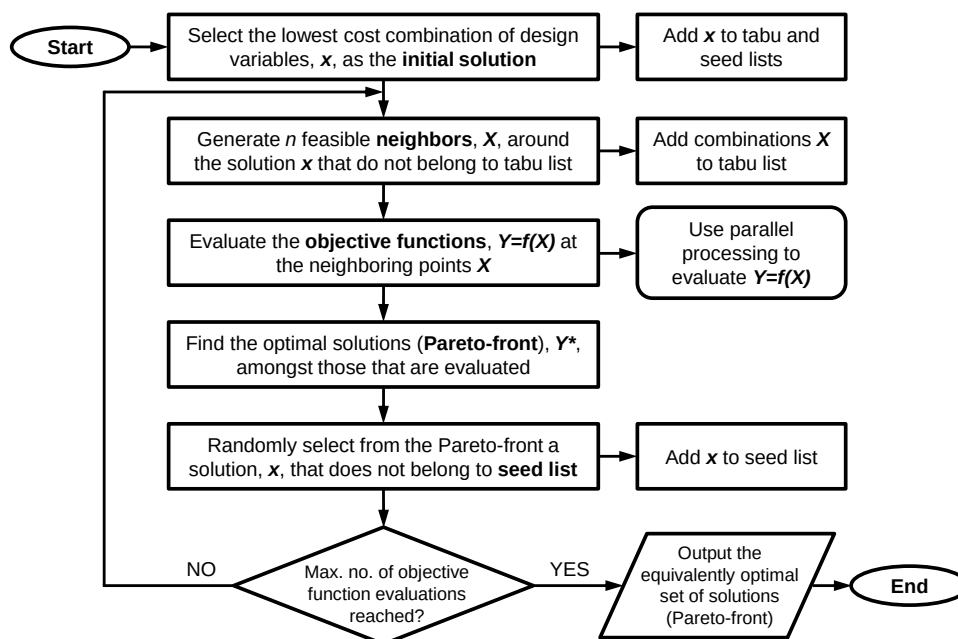


Fig. 3-3: Flowchart of the Tabu search algorithm

In addition to various other fields of optimization, TS algorithm has also been applied to structural optimization problems. Bland (1998) applied the TS algorithm to weight minimization of a space truss structure with various local minima and showed that TS algorithm is very effective in finding the global minimum when both reliability and displacement constraints are applied. Manoharan and Shanmuganathan (1999) investigated the efficiency of TS, SA, GA and branch-and-bound in solving the cost minimization problem of steel truss structures. It was concluded that TS produces solutions better than or as good as both SA and GA and it arrives at the optimal solution faster than both methods. In a more recent study, Ohsaki et al. (2007) explored the applicability of SA and TS algorithms for optimal seismic design of steel frames with standard sections. It was concluded that TS is advantageous over SA in terms of the diversity of the Pareto solutions and the ability of the algorithm to search the solutions near the Pareto front.

3.2.3 A review of structural optimization studies

The evolution of probabilistic performance-based seismic assessment has been covered in the preceding chapters. It was only after the probabilistic approaches have reached a mature state in seismic assessment that the studies on structural optimization have started using these tools. As a matter of fact, in earlier studies, for the most part, the (single-) objective function was selected as the minimum weight (for steel structures) or the minimum total cost. Example studies include those by Feng *et al.* (1977), Cheng and Truman (1985), and Pezeshk (1998).

As discussed earlier, due to the limitations resulting from the use of gradient-based optimization algorithms in addressing more practical design problems, the use of zero-order algorithms (mainly heuristic approaches) started to become popular in structural optimization (e.g. Jenkins, 1992; Pezeshk *et al.*, 2000; Lee and Ahn, 2003; Salajegheh *et al.*, 2008).

A further step in structural optimization was the adoption of multiple merit functions. In single-objective approaches, the optimal design solutions were not transparent in terms of the extent of satisfaction of other constraints on performance metrics. Therefore, researchers used multiple merit functions to provide the decision maker with a set of equivalent design solutions so that a selection could be made based on the specific requirements of the project (e.g. Li *et al.*, 1999; Liu *et al.*, 2006).

With the increase in the popularity of performance-based seismic design (PBSD) approaches towards the end of the 1990's, structural optimization tools were tailored to accommodate the new design concept. The multi-objective nature of PBSD naturally suits formulations that consider multiple merit functions, and several research works were published to formulate optimization frameworks from a PBSD standpoint with single (e.g. Ganzerli *et al.*, 2000; Fragiadakis and Papadrakakis, 2008; Sung and Su, 2009) or multiple objective functions (e.g. Liu, 2005; Lagaros and Papadrakakis, 2007; Ohsaki *et al.*, 2007).

Studies incorporating a fully probabilistic approach to performance evaluation into structural optimization are quite few. A non-exhaustive overview of some of the most notable studies is given in the following in a chronological order.

Beck *et al.* (1999) developed a reliability-based optimization method that considers uncertainties in modelling and loading for PBSD of steel structures. A hybrid optimization algorithm that combines GA and the quasi-Newton method was implemented. Performance criteria were selected as the lifetime drift risk, code-based maximum interstorey drift and beam and column stresses. The ground motion was characterized by a probabilistic response spectrum, and different hazard levels were considered. The methodology was applied to a three-storey structure. Section sizes were selected as design variables, and both continuous and discrete representations were considered. Linear elastic dynamic finite element analysis was used for performance assessment of the structure.

Ganzerli *et al.* (2000) studied the optimal PBSD of RC structures. Their purpose was the minimization of structural cost taking into account performance constraints (on plastic rotations of beams and columns) as well as behavioural constraints. Uncertainty associated with earthquake excitation and determination of the fundamental period of structure was taken into account. Static pushover analysis was used to determine the structural response.

Wen and Kang (2001a) developed an analytical formulation (optimal design is carried out with a gradient-based method) to evaluate the LCC of structures under multiple hazards. The methodology was then applied to a 9-storey steel building to find the minimum LCC under earthquake, wind and both hazards (Wen and Kang, 2001b). In this study, the simplified method based on an equivalent single-degree-of-freedom (SDOF) system developed by Collins *et al.* (1996) was used for structural assessment. The uncertainty in structural capacity was taken into account through a correction factor.

Liu *et al.* (2004) approached optimal PBSD of steel moment-resisting frames using GA. Three merit functions were defined: initial material costs, lifetime seismic damage costs, and

the number of different steel section types. Maximum interstorey drift was used for the performance assessment of the frames through static pushover analysis. Code provisions were taken into account in design. Different sources of uncertainty in estimating seismic demand and capacity were incorporated into the analysis by using SAC/FEMA guidelines, see (Cornell *et al.*, 2002) and §2.2.2. The results were presented as Pareto-fronts for competing merit functions. Final designs obtained from the optimization algorithm were assessed using inelastic time history analysis.

In a similar study, Liu (2005) formulated an optimal design framework for steel structures based on PBSO. The considered objectives were the material usage, initial construction expenses, degree of design complexity, seismic structural performance and lifetime seismic damage cost. Design variables were section types for frames' members. The designs were also checked for compliance with existing code provisions. A lumped plasticity model was used for structural modelling. Both static pushover and inelastic dynamic analysis were used, the latter when structural response parameters were directly taken as objective functions. In the case of pushover analysis, aleatory and epistemic uncertainties were taken into account following the SAC/FEMA guidelines (Cornell *et al.*, 2002), while in inelastic dynamic analysis the aleatory uncertainty is directly accounted for by considering a set of ground motions. The latter approach is also adopted in Liu *et al.* (2005); however, life-time seismic damage cost was not considered as an objective of the problem.

Lagaros, Fragiadakis and Papadrakakis explored a range of optimal design methods, mostly but not exclusively applied to steel MRF structures, based on the heuristic method of evolutionary algorithms (EA).

In Fragiadakis *et al.* (2006a) a single merit function on cost is used, subject to constraints on interstorey drift, determined with both inelastic static and dynamic analysis, the latter with ten ground motion records for each hazard level. Mean drift was taken as the performance measure. Discrete steel member sections were selected as design variables. Uncertainty associated with structural modelling was also taken into account in the form of an additional constraint.

In Fragiadakis *et al.* (2006b) initial construction and life-cycle costs were considered as merit functions. Probabilistic formulations were adopted for calculating the LCC. Deterministic constraints were based on the provisions of European design codes, in terms of limits on maximum interstorey drift. The latter was evaluated by means of static pushover analysis on a fiber-based finite element.

Lagaros *et al.* (2006) evaluated modal, elastic and inelastic time history analysis, taking the European seismic design code as a basis and with reference to steel structures, in an optimization framework. A fiber-based finite element modeling approach was adopted. Either ten natural or five artificial records were used to represent the hazard. Material weight was selected as the design objective. It was observed that lighter structures could be obtained when inelastic time history analysis (instead of elastic time history or modal analysis) and natural records were used instead of artificial spectrum-compatible records.

Lagaros and Papadrakakis (2007) evaluated the European seismic design code vs. a PBSO approach for 3D RC structures, in the framework of multi-objective optimization. The selected objective functions were the initial construction cost and the 10/50 maximum interstorey drift. Cross-sectional dimensions and the longitudinal and transverse reinforcement were the design variables. Three hazard levels were considered in the study, and the linear and nonlinear static procedures were used for design based on the European code and PBSO, respectively. It was concluded that there was considerable difference between the results obtained from the European code and PBSO, and at parity of initial cost, design solutions based on the former were more vulnerable to future earthquakes.

Rojas *et al.* (2007) used GA for optimal design of steel structures taking into account both structural and non-structural components. The merit function was selected as the initial cost,

and constraints were set in terms of probability of structural and non-structural performance levels. In particular, FEMA 350 (FEMA, 2000) and HAZUS (FEMA, 2003) procedures, for structural and non-structural damage respectively, were adopted to evaluate damage and to account for various sources of uncertainty. Two hazard levels were represented with two sets of seven records, inelastic time history analysis was conducted, and the median of the maximum response quantities (interstorey drift and floor accelerations) was used to evaluate the performance of designs.

Alimoradi *et al.* (2007; Foley *et al.*, 2007) studied the optimal design of steel frames with fully and partially restrained connections using GA. Uncertainty associated with structural capacity and demand was treated based on the formulation in FEMA 350 (2000). Seven ground motion records were used to represent each of the two considered hazard levels. A lumped plasticity model, with special connection models, was used for inelastic time history analysis. The methodology was applied to a portal and a three-storey four-bay frame. Interstorey drift and column axial compression force were selected as the performance metrics. For the portal frame, the objectives were selected as the median drift for IO, the median drift for CP, and the total weight of the structure; and for the multistorey frame the objectives were the minimization of member volume and minimization of the difference between the confidence levels (probability) in meeting a performance objective obtained from the global interstorey drift and the column axial compression force.

Finally, Fragiadakis and Papadrakakis (2008) studied the optimal design of RC structures. Both deterministic and probabilistic approaches were evaluated, and the latter was found to provide more economical solutions as well as more flexibility to the designer. In the probabilistic approach only the aleatory uncertainty was taken into account leaving out the epistemic uncertainties. The total cost of the structure was taken as the objective function, and compliance with European design codes was applied as a condition. EA was used to solve the optimization problem. Three hazard levels were considered. To reduce the computational time, fibre-based section discretization was used only at the member ends, and inelastic dynamic analysis was performed only if non-seismic checks performed through a linear elastic analysis were met.

3.2.4 Illustrative example

In this section optimization of the two-storey two-bay RC frame, shown in Fig. 3-4, is performed as an example for optimization-based probabilistic seismic design.

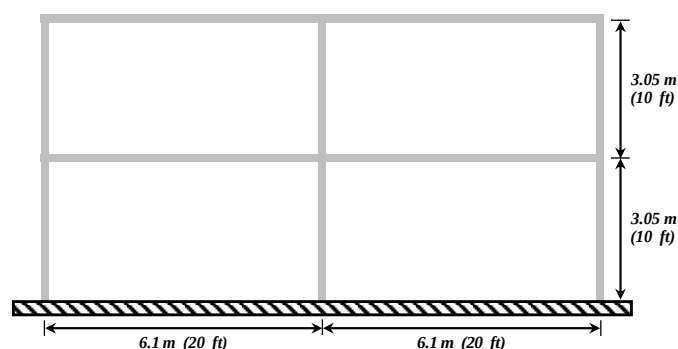


Fig. 3-4: RC frame used in application example

The design variables are selected as the column and beam longitudinal reinforcement ratios and section dimensions as provided in Table 3-1. First, practical upper and lower bounds are selected for each design variable. Then the design variables are discretized within these limits to convert the problem to a combinatorial one.

Three objectives are defined: the initial cost, LCC and seismic performance (in terms of maximum interstorey drift for a 2475 years return period intensity). The optimal solutions (in the Pareto-optimal sense) are obtained using the TS algorithm described in Section 3.2.3.

Table 3-1: Ranges and discretization of the design variables

	Minimum	Maximum	Increment
Column Reinforcement Ratio (%)	1.0	3.0	0.5
Beam Reinforcement Ratio (%)	1.0	3.0	0.5
Width of Exterior Columns (mm)	304.8	508	50.8
Width of Interior Columns (mm)	355.6	558.8	50.8
Depth of Columns (mm)	304.8	457.2	50.8
Depth of Beams (mm)	406.4	558.8	50.8
Width of Beams (mm)	304.8	406.4	50.8

The initial cost C_0 is estimated according to 2011 Building Construction Cost Data (RS Means, 2011) to include cost of steel, concrete, concrete formwork as well as the associated labour costs.

The LCC is a random quantity due to various sources of uncertainty including the ground motion variability, modeling error and unknown material properties, though not all LCC formulations take into account all these different sources, e.g. Liu (2003) and Fragiadakis *et al.* (2006b). The expected LCC of a structure, incorporating both aleatory uncertainty due to ground motion variability and epistemic uncertainty due to modeling error, is:

$$E[C_{LC}] = C_0 + \int_0^L E[C_{SD}] \left(\frac{1}{1+\lambda} \right)^t dt = C_0 + \alpha L E[C_{SD}] \quad (3-2)$$

where L is the service life of the structure and λ is the annual discount rate. Assuming that structural capacity does not degrade over time and that the structure is restored to its original condition after each earthquake occurrence, the annual expected seismic damage cost, $E[C_{SD}]$, is governed by a Poisson process (implicit in hazard modeling), hence does not depend on time and the above integral can be solved as shown. On the right hand side, α is the discount factor equal to $[1-\exp(-qL)]/qL$, where $q=\ln(1+\lambda)$.

The annual expected seismic damage cost $E[C_{SD}]$ is:

$$E[C_{SD}] = \sum_{i=1}^N C_i P_i \quad (3-3)$$

where N is the total number of damage-states considered, P_i is the total probability that the structure will be in the i^{th} damage state throughout its lifetime, and C_i is the corresponding cost (usually defined as a fraction of the initial cost of the structure). Four damage states are used: no damage, IO-LS (a state of light damage between the Immediate Occupancy and the Life Safety limit states), LS-CP (a state of severe damage between the Life safety and the Collapse prevention limit states) and total collapse. For the example problem here, the cost of repair, C_i , for IO-LS, LS-CP and total collapse are assumed to be 30, 70 and 100 percent, respectively, of the initial cost of the structure.

The probability of each damage state P_i is given by:

$$P_i = P(\Delta_D > \Delta_{C,i}) - P(\Delta_D > \Delta_{C,i+1}) \quad (3-4)$$

where Δ_D is the earthquake demand and $\Delta_{C,i}$ is the structural capacity, in terms of maximum interstorey drift ratio, defining the i^{th} limit-state. The probability of demand being greater than capacity is:

$$P(\Delta_D > \Delta_{C,i}) = \int_0^{\infty} P(\Delta_D > \Delta_{C,i} | IM = im) \left| \frac{dv(IM)}{dIM} \right| dIM \quad (3-5)$$

whereas explained before the first term inside the integral is the conditional probability of demand being greater than the capacity given the ground motion intensity, IM , and the second term is the absolute value of the slope of the hazard curve $v(IM)$. The hazard curve used for this example, in which PGA is selected as IM , is shown in Fig. 3-5 (solid blue line) together with its approximation (dashed red line):

$$v(IM) = c_7 \cdot e^{c_8 \cdot IM} + c_9 \cdot e^{c_{10} \cdot IM} \quad (3-6)$$

where c_7 through c_{10} are constants to be determined from curve fitting.

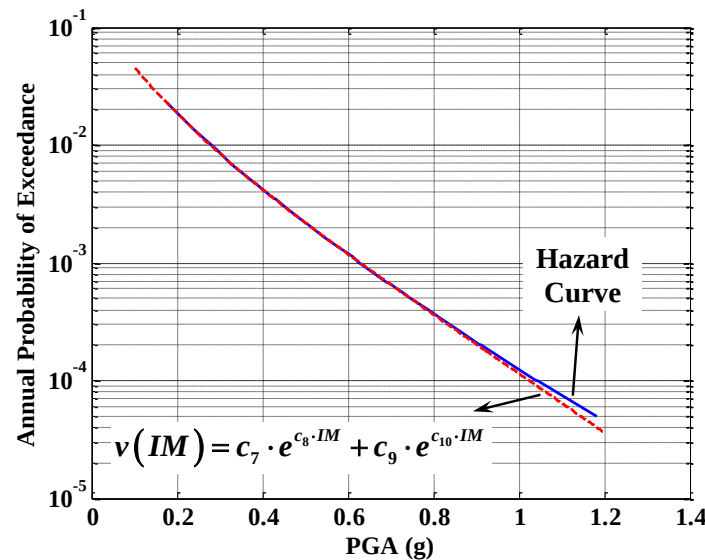


Fig. 3-5: The hazard curve for selected example problem

The conditional probability of demand being greater than the capacity (or fragility) is:

$$P(\Delta_D > \Delta_{C,i} | IM = im) = \int_0^{\infty} P(\Delta_D > \delta | IM = im) f_{C,i}(\delta) d\delta \quad (3-7)$$

where δ is the variable of integration and $f_{C,i}$ is the probability density function for structural capacity for the i^{th} damage state. This formulation assumes that demand and capacity are independent of each other.

Structural capacity is assumed to follow a lognormal distribution with logarithmic mean and the standard deviation $\Delta_{C,i}$ and β_C , respectively. The uncertainty in capacity represented with β_C accounts for factors such as modelling error and variation in material properties.

For the example problem presented here, the threshold values associated with each limit state are assumed to be invariant to changes in design variables. The specific values are adopted from FEMA 273 (1997) as 1, 2 and 4 percent interstorey drift, respectively, for IO, LS and CP. β_C is assumed to be constant for all damage states and taken as 0.35. A more detailed investigation of capacity uncertainty is available in Wen *et al.* (2004) and Kwon and Elnashai (2006). Although not used here, a preferred way of obtaining limit state threshold

values is through pushover analysis. An example pushover curve is shown Fig. 3-6(a) alongside the limit state threshold values 1.0, 2.0 and 4.0 percent interstorey drift for the IO, LS, and CP limit states, respectively. With an assumed value 0.35 for logarithmic standard deviation of β_C , Fig. 3-6(b) shows the corresponding lognormal probability density functions.

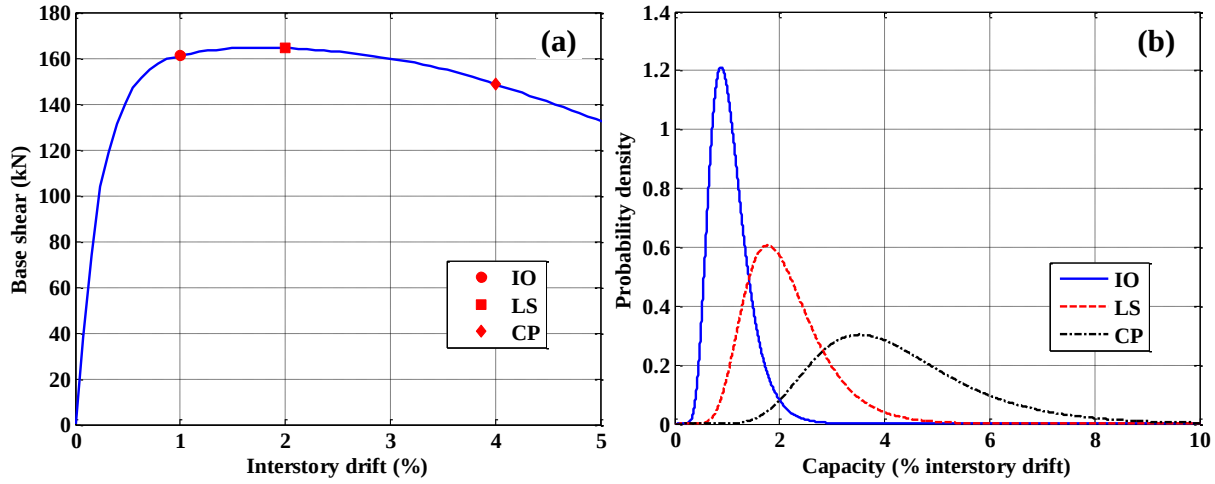


Fig. 3-6: (a) A typical pushover curve and the limit state points that delineate the performance levels, (b) illustration of lognormal probability distributions for the three structural limit states

The earthquake demand, given the intensity level, is also assumed to follow a lognormal distribution, and the probability of demand exceeding a certain value, δ , is given by:

$$P(\Delta_D > \delta \mid IM = im) = 1 - \Phi \left[\frac{\ln(\delta) - \lambda_{D|IM=im}}{\beta_D} \right] \quad (3-8)$$

where $\Phi[\cdot]$ is the standard normal cumulative distribution, $\lambda_{D|IM=im}$ is the mean of the natural logarithm of the earthquake demand as a function of the ground motion intensity, and β_D is the standard deviation of the corresponding normal distribution of the earthquake demand. Although β_D is dependent on ground motion intensity, in most studies it is taken as constant. The median, μ_D (λ_D in Eqn. (3-8) is equal to $\ln(\mu_D)$) and the logarithmic standard deviation, β_D , of earthquake demand as continuous functions of the ground motion intensity could be described using (Aslani and Miranda, 2005):

$$\mu_D(IM) = c_1 c_2^{IM} \cdot IM^{c_3} \quad (3-10)$$

$$\beta_D(IM) = c_4 + c_5 \cdot IM + c_6 \cdot IM^2 \quad (3-11)$$

where the constants c_1 through c_3 and c_4 through c_6 are determined by curve fitting to the data points of mean and logarithmic standard deviation, respectively, of earthquake demand evaluated using inelastic dynamic analysis, as shown in Fig. 3-7.

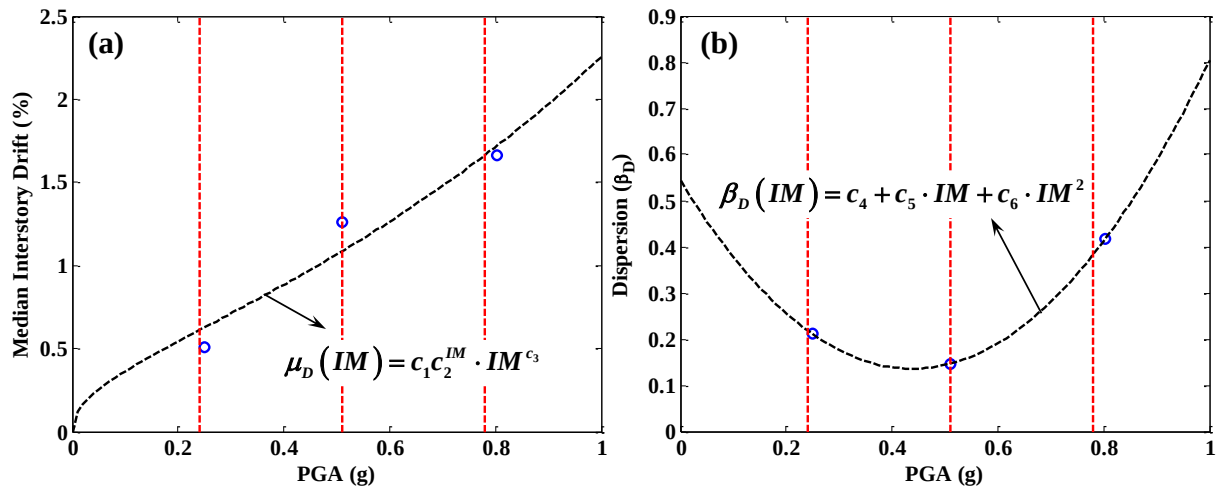


Fig. 3-7: Curve fitting to obtain the (a) mean and (b) logarithmic standard deviation of earthquake demand in continuous form

To obtain the earthquake demand, in terms of maximum interstorey drift ratio, the structural frame presented in Fig. 3-4 is modelled in the fibre-based finite element analysis software ZEUS-NL (Elnashai *et al.*, 2010), and inelastic time history analysis is carried out. At least three different earthquake intensities (here represented in terms of 75, 475 and 2475 years return periods) are needed in order to evaluate the constants that define the median of earthquake demand as a function of the intensity measure according to Eqn. (3-10). For each return period (intensity level) only one earthquake record that is compatible with the corresponding Uniform Hazard Spectrum (UHS) developed for the site are used so as to reduce the computational cost. The values are employed to evaluate μ_D and β_D as a function of the intensity measure as shown for an example case in Fig. 3-7.

With the above-described formulation, each term in Eqn. (3-5) is represented as an analytical function of ground motion intensity, IM . Thus, using numerical integration, the desired probabilities of Eqn. (3-4) are calculated. As mentioned above, the cost of repair for the IO, LS, and CP limit states, C_i , in Eqn. (3-3) are taken as a fraction of the initial cost of the structure. Finally, the expected value of the LCC is evaluated using Eqn. (3-2).

The results of the optimization runs are shown in Fig. 3-8 (a). Each dot in the plot represents a combination of the design variables that are evaluated by the TS algorithm. The solid line with circle markers indicate the Pareto-front for the two competing objective functions, i.e. initial cost and seismic performance under the 2475 years return period earthquake. The repair cost is calculated according the LCC formulation presented above and the optimal solutions are plotted in Fig. 3-8 (b) [LCC which is simply the repair cost plus the initial cost could also be shown in Fig. 3-8 (b)].

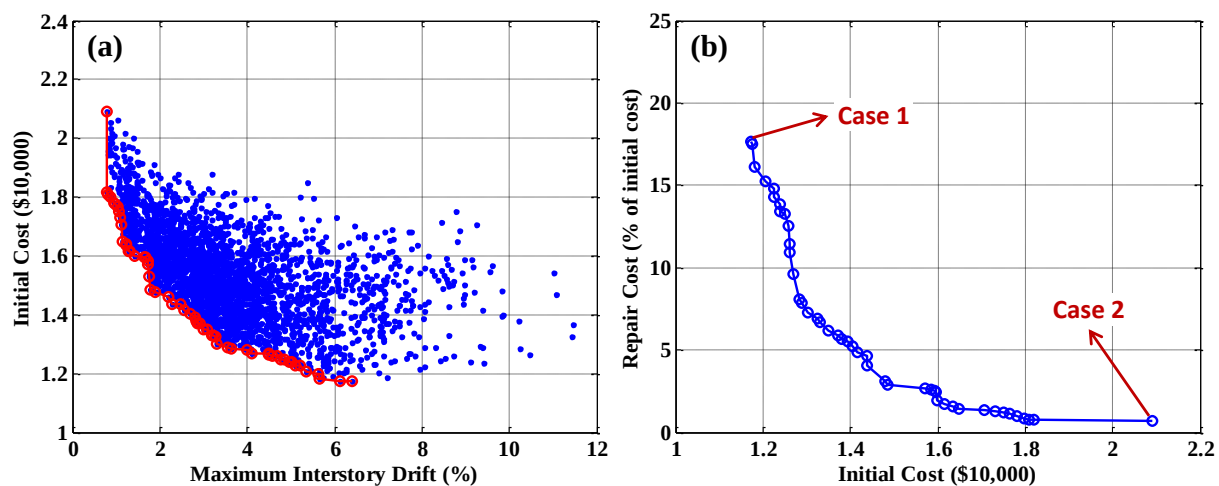


Fig. 3-8: (a) Initial cost vs. maximum interstorey under the 2475 years return period earthquake, (b) repair cost vs. initial cost (Pareto-front)

Two cases (the lowest and highest repair cost options) are identified in the equivalently optimal set of solutions as shown in Fig. 3-8 (b). The values of the design variables corresponding to these two cases are provided in Table 3-2. The representation of equivalently optimal solutions using Pareto-optimality is very useful for decision makers. It provides the decision maker with flexibility to choose among a set of equivalently optimal solutions depending on the requirements of the project. Furthermore, the extent to which the desired structural performance would be satisfied by a selected alternative can be easily observed.

Table 3-2 Values of the design variables for the two repair cost options

	Case 1	Case 2
Column Reinforcement Ratio (%)	1.5	3.0
Beam Reinforcement Ratio (%)	1.0	3.0
Width of Exterior Columns (mm)	304.8	508
Width of Interior Columns (mm)	355.6	558.8
Depth of Columns (mm)	304.8	457.2
Depth of Beams (mm)	406.4	558.8
Width of Beams (mm)	304.8	406.4

3.3 Non-optimization-based methods

3.3.1 Introduction

To the knowledge of the authors only two, non-optimization-based approaches for performance-based seismic design are available in the literature: Krawinkler *et al.* (2006) and Franchin and Pinto (2012).

The first design procedure cannot be considered as a fully probabilistic design procedure in the sense that the design satisfies performance targets in terms of risk. It actually is more in line with first-generation PBSD procedures, and iteratively enforces satisfaction of two performance targets in terms of cost, associated with 50/50 and 2/50 hazard levels (50% and

2% probability of exceedance in 50 years), respectively. The procedure makes use of median incremental dynamic analysis (IDA) curves (Vamvatsikos and Cornell 2002) to relate the hazard levels with the corresponding demand parameters, as well as of average loss curves, for both structural and nonstructural damage, to relate response with damage/cost.

The design variables are the fundamental period T_1 and the base shear ratio γ (ratio of base shear to the weight of the structure). The procedure requires a prior production of “design-aids” in the form of alternative median IDA curves for different values of the design variables.

The second proposal, described in some detail in the following, is fully probabilistic in that it employs constraints formulated explicitly in terms of MAF of exceedance of chosen performance-levels/limit-states.

The method, which is an approximate one, rests on the validity of two basic results of earthquake engineering: the closed-form expression for the MAF of exceedance of a limit-state from (Cornell *et al.*, 2002)(§2.2.2) and the so-called (empirical) “equal-displacement” rule (Veletsos and Newmark, 1960). Limits of validity of the above results are clearly recognized and are shared by the proposal.

With respect to the optimization approaches described in the previous section, the method differs in that it produces a solution that is feasible, i.e. that complies with the constraints, but not necessarily optimal. Extension to include an objective function related e.g. to minimum cost, is possible within the same framework. The extended method would retain its computational advantage, which is mainly due to the use of gradients (allowing a more efficient continuous optimization in place of discrete methods – e.g. genetic algorithms), and on the use of an elastic proxy for the nonlinear structure.

The next section illustrates the method, whose approximation is then explored in the following one with reference to an RC multi-storey frame structure.

3.3.2 Performance-based seismic design with analytical gradients

The method in (Franchin and Pinto, 2012) iteratively modifies a design solution until it satisfies multiple probabilistic constraints, i.e. constraints on the MAFs of multiple performance-levels (e.g. light damage, collapse, etc.), employing the analytical gradients with respect to the design variables of the closed-form MAFs. This is possible by virtue of the assumed validity of the “equal-displacement” rule, which allows the iteration process to be carried out on a (cracked) elastic model of the structure whose deformed shape, as obtained from multi-modal response spectrum analysis, is taken as a proxy for the true inelastic shape. This assumption of elasticity allows explicit analytical evaluation of the gradients of the MAFs, a fact that increases manifold the computational effectiveness of the procedure. Flexural reinforcement is designed only when the iteration process on the cross-section dimensions has ended. Shear reinforcement is capacity-designed as the last step.

3.3.2.1 Gradients

The gradients of the “SAC/FEMA” closed-form expression of the MAF of exceedance λ_{LS} of a structural limit-state:

$$\lambda_{LS} = k_0 \left(\frac{\hat{C}}{a} \right)^{\frac{k}{b}} \exp \left[\frac{1}{2} \frac{k^2}{b^2} (\beta_D^2 + \beta_C^2) \right] \quad (3-12)$$

with symbols introduced in §2.2.2, can be expressed as follows by the chain rule of differentiation:

$$\nabla_{\mathbf{d}_1} \lambda = \frac{\partial \lambda}{\partial k_0} \frac{\partial k_0}{\partial \mathbf{d}_1} + \frac{\partial \lambda}{\partial k} \frac{\partial k}{\partial \mathbf{d}_1} + \frac{\partial \lambda}{\partial a} \frac{\partial a}{\partial \mathbf{d}_1} + \frac{\partial \lambda}{\partial b} \frac{\partial b}{\partial \mathbf{d}_1} \quad (3-13)$$

where $\mathbf{d}_1$ is the sub-vector of “independent” parameters, i.e. the design variables, within the complete vector $\mathbf{d}$ of the structural dimensions (including also a sub-vector $\mathbf{d}_2$, dependent on $\mathbf{d}_1$ through appropriate rules expressing symmetries, regularity, etc.), and the derivatives of the MAF with respect to the hazard and demand parameters k_0 , k , a and b are:

$$\frac{\partial \lambda}{\partial k_0} = \frac{\lambda}{k_0} \quad (3-14a)$$

$$\frac{\partial \lambda}{\partial k} = \frac{\lambda}{b} \left[\frac{k}{b} (\beta_D^2 + \beta_C^2) - \ln \left(\frac{\hat{C}}{a} \right) \right] \quad (4-14b)$$

$$\frac{\partial \lambda}{\partial a} = \lambda \frac{k}{ab} \quad (4-14c)$$

$$\frac{\partial \lambda}{\partial b} = \lambda \frac{k}{b^2} \left[\ln \left(\frac{\hat{C}}{a} \right) - \frac{k}{b} (\beta_D^2 + \beta_C^2) \right] \quad (4-14d)$$

Eq.(3-13) does not contain terms with the derivatives of λ with respect to the dispersions β_D and β_C since the dependence of the latter on the design through response is generally minor and thus they are assumed to remain constant throughout iteration.

The derivatives of the hazard parameters $\partial k_0 / \partial \mathbf{d}_1$ and $\partial k / \partial \mathbf{d}_1$ can be obtained by the chain-rule, differentiating first with respect to the fundamental period of the structure (assuming the chosen IM is the spectral acceleration $S_a(T_1)$), which in turn depends on the design variables $\mathbf{d}_1$. The derivatives of the response parameters $\partial a / \partial \mathbf{d}_1$ and $\partial b / \partial \mathbf{d}_1$ can also be obtained by the chain-rule, differentiating first with respect to the nodal displacements, which in turn depend on the modal contributions to response, which are a function of the design variables $\mathbf{d}_1$. In both cases the method takes advantage of the availability of analytical expressions for the derivatives of modal frequencies and shapes of an elastic system with respect to its mass and stiffness terms (Lin *et al.* 1996). The reader is referred to (Franchin and Pinto, 2012) for the detailed derivation of the gradients.

3.3.2.2 Iterative search for a feasible solution

As anticipated performance constraints are expressed in terms of the MAF of limit-state violations for a number of limit-states of interest, e.g. a serviceability limit-state, such as light damage (LD), and a safety-related one, such as collapse prevention (CP). For example (the limits on the frequencies being arbitrary):

$$\lambda_{LD} \leq \lambda_{LD}^* = 1/100 \text{ years} \quad (3-15a)$$

$$\lambda_{CP} \leq \lambda_{CP}^* = 1/2500 \text{ years} \quad (3-15b)$$

The governing constraint at each iteration is defined as that having the largest value of the normalized MAF $\tilde{\lambda} = \lambda / \lambda^* - 1$. At the end of the process only one of the constraints is satisfied in equality, while the remaining ones are satisfied with more or less wide margins.

For simple cases with few design variables the search for the design solution can be performed with a steepest-descent (Newton) algorithm, however, in larger size applications this method is not acceptably reliable/accurate. The search for a feasible design solution, i.e. the problem of finding a zero for the $\tilde{\lambda} = \tilde{\lambda}(\mathbf{d})$ function, is carried out by means of a quasi-Newton method, transforming it into the problem of finding a minimum for $\tilde{\lambda}^2$, where the gradient $\nabla_{\mathbf{d}} \tilde{\lambda}^2 = \mathbf{0}$. In practice, since the feasible design must also satisfy a number of other practical constraints related, e.g. to construction, the problem is cast in the form of a constrained optimization:

$$\begin{cases} \min_{\mathbf{d}} s = \tilde{\lambda}^2 \\ \text{subject to } \mathbf{c} \leq 0 \end{cases} \quad (3-16)$$

where the vector $\mathbf{c}$ collects the n constraints $c_i(\mathbf{d})$ which are formulated to take upon positive values whenever the corresponding constraint is violated. Typical constraints employed in practice are of the form: a) $\gamma d_j \leq d_{j+1} \leq d_j$ regulating column tapering (with $\gamma < 1$ and column members ordering increasing upward), b) $d_j \leq d_{j,\max}$ limiting from above the cross-section dimension, or c) $d_j \geq d_{j,\min}$ limiting to a minimum (slenderness, axial load, etc.) the cross-section dimension. These constraints can all collectively be put in the form $\mathbf{c}(\mathbf{d}) = \mathbf{A}\mathbf{d} + \mathbf{b} \leq 0$. The problem is then solved e.g. with the well-known Broyden–Fletcher–Goldfarb–Shanno (BFGS) algorithm (Luenberger and Ye, 2008).

3.3.2.3 Design of reinforcement

Design of longitudinal reinforcement is carried out for a “seismic” combination of gravity loads and a seismic action characterized by a given average return period. This latter is chosen to limit structural damage (yielding) for frequent earthquakes, therefore design of longitudinal reinforcement is carried out for a seismic action with an average return period related to the λ_{LD}^* limit on the light damage performance-level. Since the frequency of exceedance of the response according to the Cornell’s formula is the product of the MAF of the seismic action inducing median demand equal to a median capacity $\lambda_{IM \hat{D}=\hat{C}}$, times an exponential amplification factor commonly between 1.2 and 2.2 (depending essentially on the hazard slope k , for usual values of β_D and β_C), one can conclude that the order of magnitude of the return period to be used for reinforcement design is in the order of $1.5/\lambda_{LD}^*$ (say, 150 years, according to Eq. (3-16)).

For what concerns the use of capacity design procedures, these are not necessary for the relative flexural strength of beams and columns, while they are clearly so for determining the shear strength of members and joints.

3.3.3 Illustrative example

3.3.3.1 Design

In this application the method is illustrated and validation of the obtained design is carried out by means of inelastic time-history analysis within the finite-element code OpenSEES (McKenna and Fenves, 2001).

The test of the method is carried out on the fifteen storeys RC plane frame shown in Fig. 2-37. Actually, the dimensions and properties of the frame in the figure are the result of the probabilistic design carried out with the present method.

Seven design variables are considered: three variables for the in-plane dimension of the three orders of external columns (each order corresponding to five floors, as shown in Fig. 2-37), three variables for the internal ones, and the seventh variable for beam height, constant for all floors. The out-of-plane dimensions for all members is kept constant and equal to 0.50, 0.40 and 0.30 m for external columns, internal columns and beams, respectively, as for the previous example.

Two constraints are imposed on the design, namely:

$$\lambda_{LD} = \lambda(\theta_{\max} \leq 0.004) \leq \lambda_{LD}^* = 1/100 \text{ years} \quad (3-17a)$$

$$\lambda_{CP} = \lambda(\theta_{\max} \leq 0.015) \leq \lambda_{CP}^* = 1/1950 \text{ years} \quad (3-17b)$$

The variables are constrained between a minimum and a maximum value, as shown in Table 3-3, which reports also the initial and final values. Further, column dimensions have been constrained with the additional eight constraints that prevent excessive or “inverse” tapering:

$$\begin{cases} t_{i+1,ext} \leq t_{i,ext} \\ t_{i+1,ext} \geq 0.85t_{i,ext} \\ t_{i+1,int} \leq t_{i,int} \\ t_{i+1,int} \geq 0.85t_{i,int} \end{cases} \quad i = 1,2 \quad (3-18)$$

The value of the demand dispersion term is set equal to $\beta_D = 0.30$, see e.g. (Dolšek and Fajfar, 2004). Capacity terms are set to $\beta_C = 0.30$, see e.g. (Panagiotakos and Fardis, 2001), and 0.0, for the CP and LD performance levels, respectively. Table 3-4 reports the evolution with the iterations of the fundamental period T_1 , the hazard coefficients k_0 and k , the slope a of the demand-intensity relation, as well as the non-normalized and normalized MAF for both limit-states. In this example the governing constraint is the collapse prevention one. Actually, the light damage limit state is already satisfied for the initial design. Table 3-5 reports the modal periods and participating mass ratios for the initial and final iterations. The frame exhibits a moderate second-mode contribution to response.

Table 3-3: Design variables, all dimensions in meters.

Var.	Min.	Max.	Initial	Final
$t_{1,ext}$	0.70	1.20	0.70	0.73
$t_{2,ext}$	0.50	1.00	0.50	0.73
$t_{3,ext}$	0.30	0.80	0.50	0.63
$t_{1,int}$	0.70	1.40	0.70	0.76
$t_{2,int}$	0.50	1.20	0.50	0.73
$t_{3,int}$	0.30	1.00	0.50	0.62
t_{beam}	0.55	0.65	0.55	0.68

Table 3-4: Iterations

Iter	T_1 (s)	k_0	k	a	λ_{LD}	$\tilde{\lambda}_{LD}$	λ_{CP}	$\tilde{\lambda}_{CP}$
1	3.526	0.001	1.467	0.0146	0.0046	-0.5347	0.0007	0.3309
2	2.938	0.001	1.532	0.0097	0.0043	-0.5687	0.0006	0.1372
3	2.636	0.001	1.611	0.0076	0.0039	-0.6016	0.0005	-0.0211
4	2.703	0.001	1.591	0.008	0.004	-0.5961	0.0005	0.0103
5	2.680	0.0012	1.598	0.0079	0.004	-0.5979	0.0005	-0.0005

Table 3-5: Modal properties for the initial and final iteration.

Mode	Initial		Final	
	T (s)	PMR (%)	T (s)	PMR (%)
1	3.527	76%	2.680	78%
2	1.174	11%	0.891	11%
3	0.683	4%	0.506	4%
4	0.467	2%	0.346	2%

3.3.3.2 Validation

In order to validate the final design, the final iteration structure is subjected to nonlinear time-history analysis for a suite of 35 ground motion records. The motions are spectrum-compatible artificial records generated in groups of 7 to match five uniform-hazard spectra of increasing intensity (mean return period ranging from 60 years to 2000 years), in order to span a sufficiently large range of spectral accelerations.

The predictive power of the elastic deformed shape as a proxy of the inelastic one is first checked by comparing the interstorey drift profiles as obtained from SRSS of modal responses for the five target spectra versus the average profiles from each of the five groups of artificial records matching those spectra.

A good prediction of $\theta_{\max}$ is a pre-requisite for the closeness of the risk λ to the target one λ^* , but a good prediction of the whole profile obviously increases the confidence in the designed structure. From the figure it is apparent how the maxima match quite satisfactorily, while the profiles show some discrepancy. The elastic profiles consistently overestimate the inelastic one at the lower floors: it is believed that the differences are mostly explained by the uniform stiffness reduction factor adopted to account for cracking, which, for a better approximation, might be made member-dependent and function of axial load ratio and response level.

Finally, it can be observed how the average $\theta_{\max}$ value for the records with $1/\lambda_{CP}^* \cong 2000\text{years}$ intensity is lower than the 1.5% limit. This is expected since the procedure does not enforce a constraint on the average demand, but, rather, on the probability of exceedance of the demand above a limit, which accounts also for the dispersion in this limit (β_C) and in the demand itself (β_D).

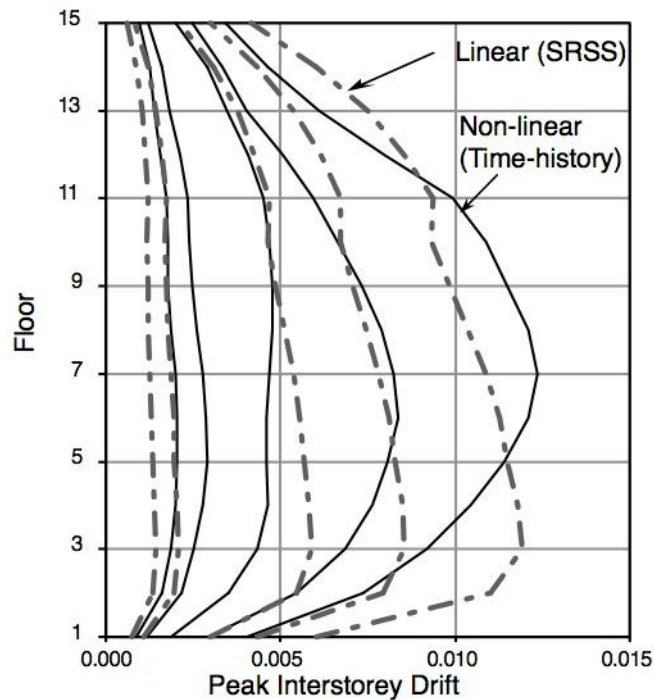


Fig. 3-9: Peak interstorey drift profiles for the five return periods as obtained from linear (SRSS of modal responses, red) and nonlinear (average over 7 records each, black) analyses

Finally, the designed structure has been checked through IM-based methods (§2.2) and found to have a limit-state MAF close to target one.

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4 Appendix

4.1 Excerpts from MATLAB script for PEER PBEE calculations

A MATLAB script has been developed for the application of PEER PBEE methodology. Excerpts from this script, brief explanation of different parts of the script, and results obtained for a one-bay one-storey frame example are presented below.

a) Hazard analysis

The POE of intensity measure, particularly, spectral acceleration (S_a) is computed using the OpenSHA application, *Hazard spectrum application* from <http://www.opensha.org/apps>. This application provides the POE of a certain value of S_a as a function of the period (uniform hazard spectrum) for given coordinates and site type. The hazard analysis part of the script reads the file provided by the application and interpolates the POE values to obtain the POE of S_a at the fundamental period of the considered structure, as shown in Fig. 4-1. The script then scales all the selected ground motions so that S_a at the fundamental period is equal to the smallest considered intensity value. Subsequently, the scale values for the other intensity values are obtained by linear amplification.

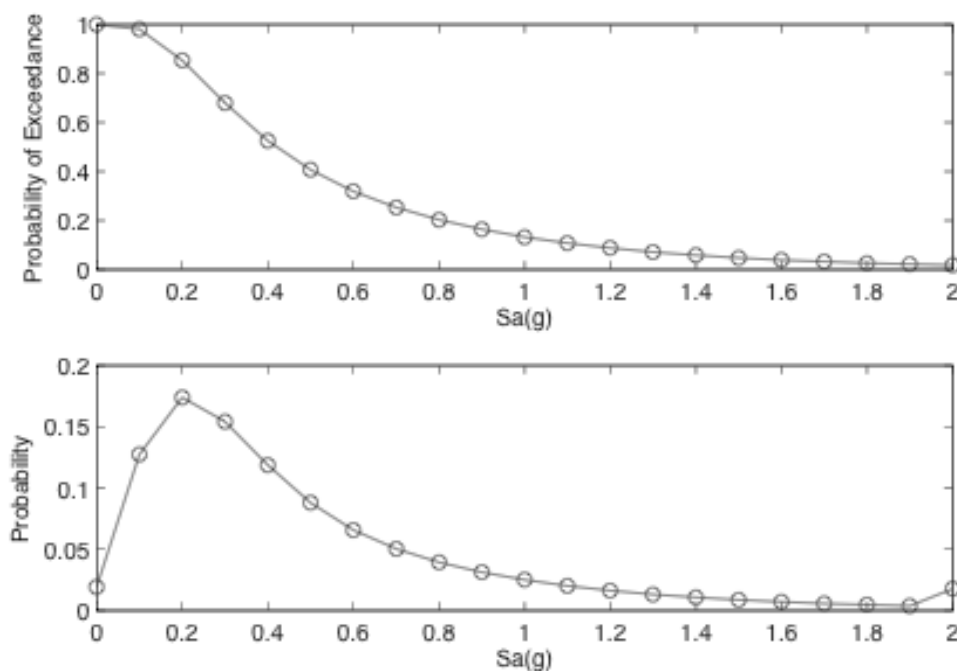


Fig. 4-1: Probability and probability of exceedance of the pseudo-acceleration S_a .

An excerpt from the hazard analysis is shown below.

```
% Hazard Curves
% Read the data file from Hazard Spectrum Application
% and Print the data in a matrix

% 1st column : period
% other columns : Probability of Exceedance of the pseudo-acceleration
% NB1 : 1st line contains the value of the pseudo acceleration (ex:1.1g)
% NB2 : 2nd line are only zeros

Tampon = zeros(21,2); % provisory memory used

wf=0; % counts the nb of data sets

pfin = fopen('Hazard Analysis Rough.txt');

if pfin > 0
    while ~feof(pfin)
        Ligne = fgetl(pfin);
        if ~isempty(strfind(Ligne,'Number of Data Sets:'))
            % get the nb of Data Sets from the first lines of the .txt
            disp(Ligne)
            get = textscan(Ligne,'%s %s %s %s %n',1) ;
            DataSets = get{1} ;
            Hazard = zeros(23,DataSets+1) ;
        end

        if ~isempty(strfind(Ligne,'Map Type = Prob@IML; IML ='))
            wf= wf+1;
            get = textscan(Ligne,'%s %s %s %s %s %s %n',1) ;
            Hazard(1,wf+1) = get{1} ;
        end

        if strcmp(Ligne,'X, Y Data:')
            for i=1:21
                Ligne = fgetl(pfin);
                Tampon(i,:) = strread(Ligne);
            end
        end

        if wf==1
            Hazard(3:23,1:2) = Tampon ;
        else
            Hazard(3:23,wf+1) = Tampon(:,2) ;
        end

    end
end
fclose(pfin);

% Print the matrix
% 'dispo' conducts the disposition

dispo='';
for i=1:DataSets+1
    dispo=strcat(dispo,'% 1.5e');
end
dispo = strcat(dispo,'\n');

% File Hazard_Curves.txt contains the matrix :
% 1st line : Sa (in g)
% 2nd line : zeros
% other line : probability of Sa as a function of the period
% 1st column : Period (in s)

fid = fopen('Intermediary/Hazard_Curves.txt','w') ;
fprintf(fid,dispo,Hazard);
fclose(fid);
```

```
%% Probabilities of Sa

% Find the probabilities of Sa from Hazard_Curves.txt corresponding to the
% period of the model

T1 = 0.45818 ; % period of the first mode of the model (in s)

line = 2 ; % start at the second line
while (T1 >= Hazard(line+1,1))
    line = line+1 ;
end

% Interpolation
Tampon = zeros(DataSets,1) ;
for i=1:DataSets
    Tampon(i,1) = Hazard(line,i+1) + (Hazard(line+1,i+1)- ...
    Hazard(line,i+1))/(Hazard(line+1,1)-Hazard(line,1))*(T1-...
    Hazard(line,1)) ;
end

% Print in Prob_Sa.txt
fid = fopen('Intermediary/Prob_Sa.txt','w') ;
fprintf(fid,'%1.10f\n',Tampon);
fclose(fid) ;
```

b) Structural analysis

The structural analysis part of the script runs OpenSees tcl file which executes all the nonlinear history simulations. Subsequently, the required engineering demand parameters (EDP, interstorey drift in this case) are post-processed from the output files and the parameters of a suitable statistical distribution, e.g. logarithmic distribution, are calculated for each level of the intensity measure. Probability and POE of EDP are obtained from the assumed statistical distribution (Fig. 4-2). In addition, based on introduced collapse criteria, the number of collapse cases and the probability of collapse for each level of the intensity measure are also calculated (Fig. 4-3).

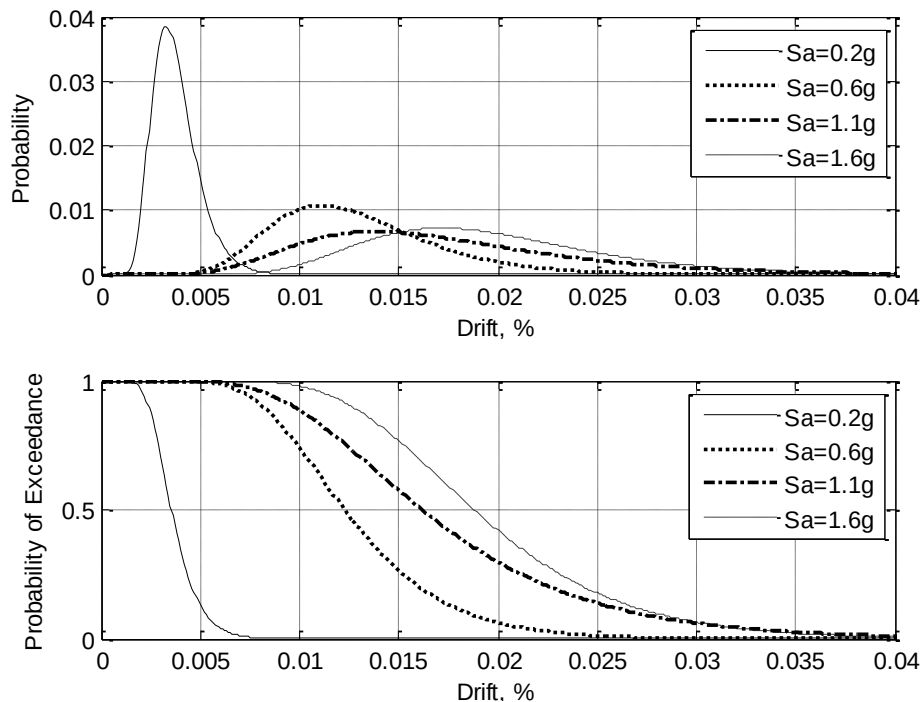


Fig. 4-2: Probability and probability of exceedance of the drift ratio for different levels of the intensity measure

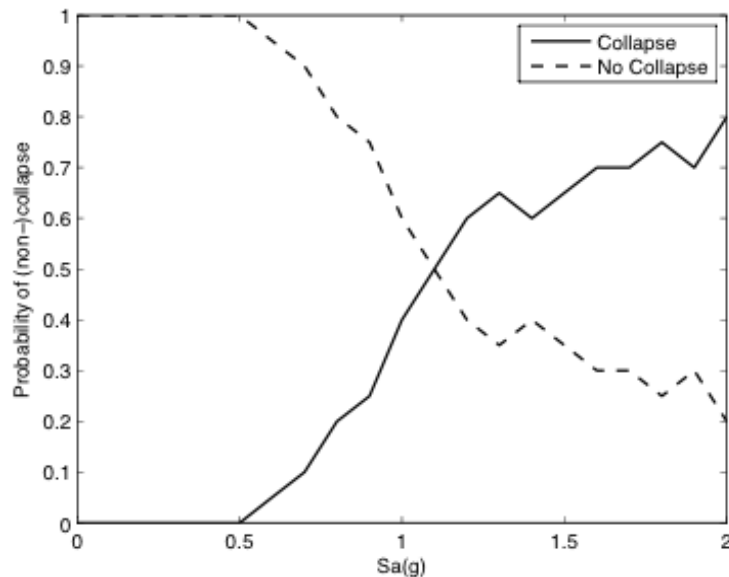


Fig. 4-3: Probability of collapse and no-collapse as a function of intensity measure

An excerpt from the structural analysis is shown below.

```
%% Structural Analysis

% Run OpenSees
!./openSees Complet1.tcl

% Read the file with the maximum values
fid = fopen('Intermediary/Max_Drift.txt','r');
Drift = textscan(fid,'%f %f %f %f %f %f %f %f %f %f %f %f %f %f %f %f %f %f %f %f',20);
fclose(fid);

collapse = zeros(1,SF) ; % # of collapses for each scale factor
MeanD = zeros(1,SF) ; % mean of drifts
VarD = zeros(1,SF) ; % Variance of drifts

% compute the mean of each column (ie each scale factor)
% for the cases without collapses
for i=1:SF % for each Scale Factor
    for j=1:GM % take all the ground motions
        if (Drift{i}(j) > 0) % No collapse
            MeanD(1,i) = MeanD(1,i) + Drift{i}(j) ;
        else % Drift=-1 if collapse
            collapse(1,i) = collapse(1,i) + 1 ;
        end
    end
end

for i=1:SF
    MeanD(1,i) = MeanD(1,i)/(20-collapse(1,i)) ;
end

% compute the variance of each column (ie each scale factor)
for i=1:SF
    for j=1:GM
        if (Drift{i}(j) > 0)
            VarD(1,i) = VarD(1,i) + (Drift{i}(j)-MeanD(1,i))^2 ;
        end
    end
end

for i=1:SF
    VarD(1,i) = VarD(1,i)/(20-collapse(1,i)) ;
end
```

```
% compute the standard deviation of each column
DevD = sqrt(VarD) ;

muD = zeros(1,20) ; % parameters for lognormal distribution
sigD = zeros(1,20) ;

for i=1:SF          % for each scale factor
    muD(1,i) = log(MeanD(1,i)) - 0.5*log(1+VarD(1,i)/(MeanD(1,i))^2);
    sigD(1,i) = sqrt(log(1+VarD(1,i)/(MeanD(1,i))^2)) ;
end
```

c) Damage analysis

The script calculates the fragility functions using the input median and standard deviation values of EDP. The median values are obtained as a result of pushover analysis. Standard deviation values can be obtained from pushover analyses conducted by varying the material properties, which has not been conducted for the presented example. Fragility curves are shown in Fig. 4-4.

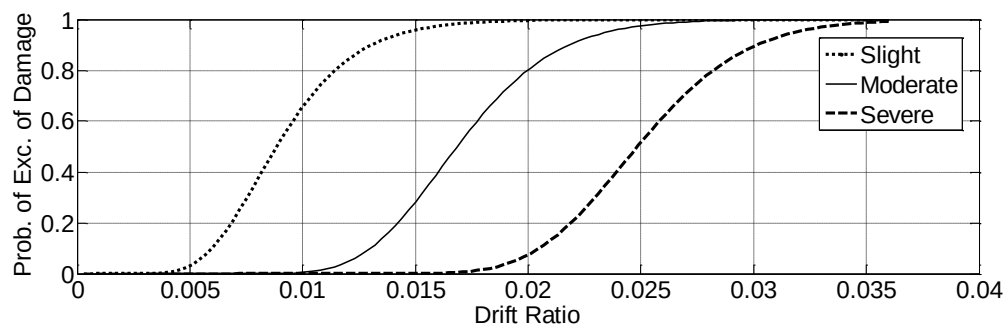


Fig. 4-4: Fragility curves

An excerpt from the script for the damage analysis is given below.

```
%% Damage Analysis

% The medians come from the pushover analysis
% Medians (Med) and logarithmic standard deviation (Var) are used
MedDam(1)=0.009;
VarDam(1)=0.3;
MedDam(2)=0.017;
VarDam(2)=0.2;
MedDam(3)=0.025;
VarDam(3)=0.15;

numDR=size(Dr,2);
for i=1:3
    muDam(i) = log(MedDam(i)) ;
    sigDam(i) = VarDam(i) ;

    for j=1:numDR
        fdamts(j)=0;
        if j==1
            prDam(i,j)=(1/(Dr(j)*sigDam(i)*sqrt(2*pi)))*...
                exp(-(log(Dr(j))-muDam(i))^2/(2*sigDam(i)^2))*Dr(j);
            PrDam(i,j)=prDam(i,j);
        else
            prDam(i,j)=(1/(Dr(j)*sigDam(i)*sqrt(2*pi)))*exp(-...
                (log(Dr(j))-muDam(i))^2/(2*sigDam(i)^2))*(Dr(j)-Dr(j-1));
            PrDam(i,j)=PrDam(i,j-1)+prDam(i,j);
        end
    end
end
end
```

d) Loss analysis

Since no realistic figures are available for the frame studied in this example, the loss analysis has been scaled to unity where unity represents the median loss in case of collapse. Median values for slight, moderate and severe damages have been subsequently chosen equal to 0.2, 0.5 and 0.75, respectively. Loss curves are plotted in Fig. 4-5.

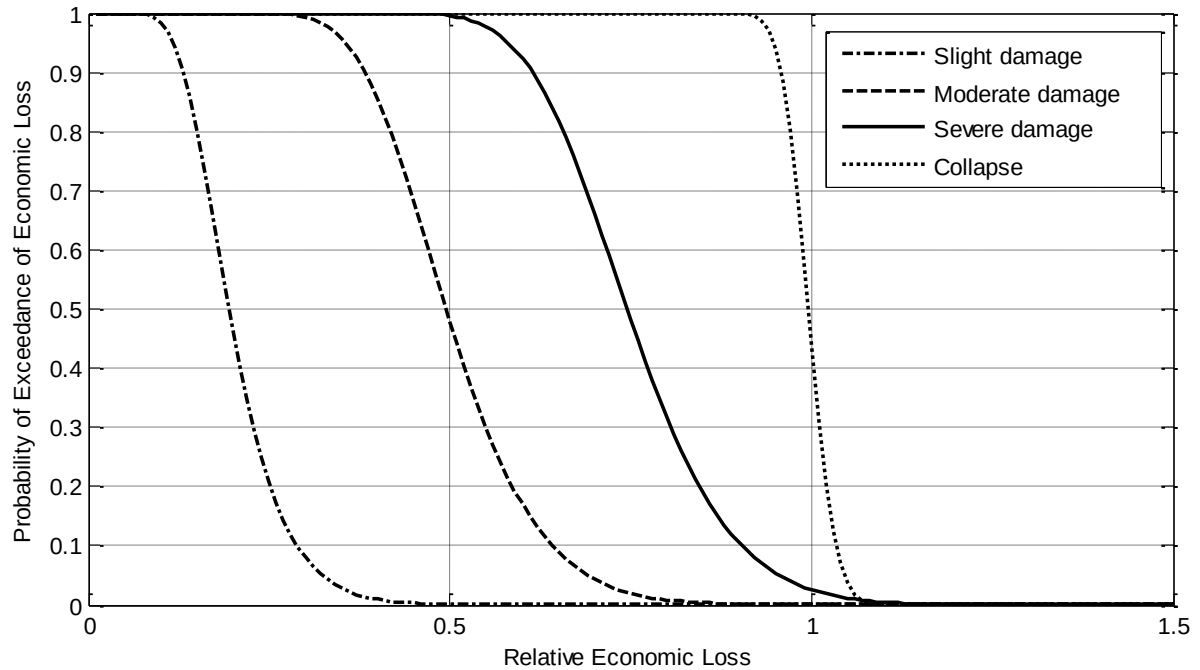


Fig. 4-5: Loss curves for different damage measure levels

An excerpt from the loss analysis is presented below.

```
%% Loss Analysis

% Lognormal distribution is assumed for the loss given a damage state

% For structural damageable group, three median and three COV values are for the assumed
three damage states,
% 1) slight damage, 2) moderate damage, 3) severe damage
MedLoss(1)=0.2;
VarLoss(1)=0.3;
MedLoss(2)=0.5;
VarLoss(2)=0.2;
MedLoss(3)=0.75;
VarLoss(3)=0.15;
% Loss for Damages

for i=1:3
    muLoss(i)=log(MedLoss(i));
    sigLoss(i)=VarLoss(i);
    j1=0;
    for j=0.01:0.01:1.5
        j1=j1+1;
        Lossvec(j1)=j;
        if j1==1
            prLoss(i,j1)=(1/(j*sigLoss(i)*sqrt(2*pi)))*...
                exp(-(log(j)-muLoss(i))^2/(2*sigLoss(i)^2))*Lossvec(j1);
            PrLoss1(i,j1)=prLoss(i,j1);
            PrLoss(i,j1)=1-PrLoss1(i,j1);
        else
            prLoss(i,j1)=(1/(j*sigLoss(i)*sqrt(2*pi)))*...
                exp(-(log(j)-muLoss(i))^2/(2*sigLoss(i)^2))*...

```

```

        (Lossvec(j1)-Lossvec(j1-1));
        PrLoss1(i,j1)=PrLoss1(i,j1-1)+prLoss(i,j1);
        PrLoss(i,j1)=1-PrLoss1(i,j1);
    end
end
end

% Loss for collapse

MedLosC=1;
VarLosC=0.030;

muLosC=log(MedLosC);
sigLosC=VarLosC;
j1=0;
for j=0.01:0.01:1.5
    j1=j1+1;
    Lossvec(j1)=j;
    if j1==1
        prLosC(j1)=(1/(j*sigLosC*sqrt(2*pi)))*...
            exp(-(log(j)-muLosC)^2/(2*sigLosC^2))*Lossvec(j1);
        PrLosC1(j1)=prLosC(j1);
        PrLosC(j1)=1-PrLosC1(j1);
    else
        prLosC(j1)=(1/(j*sigLosC*sqrt(2*pi)))*...
            exp(-(log(j)-muLosC)^2/(2*sigLosC^2))*...
            (Lossvec(j1)-Lossvec(j1-1));
        PrLosC1(j1)=PrLosC1(j1-1)+prLosC(j1);
        PrLosC(j1)=1-PrLosC1(j1);
    end
end
end

```

e) Combination of all analyses

Probabilities obtained in the four analyses discussed above are combined together following Equation 3-5 to obtain the final loss curve, i.e. POE of the economic loss is plotted in Fig. 4-6.

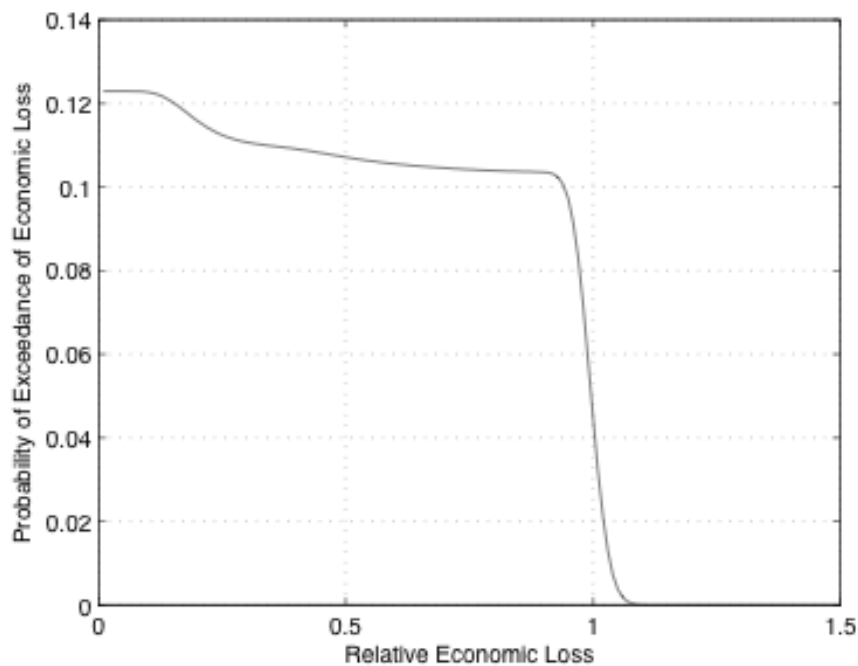


Fig. 4-6: Final loss curve

An excerpt from the combination of all analyses is shown below.

```
%% Combination of the Probabilities
% following Equation 5

%-----
% Notations used

% PrLoss(k,j) is Prob. of Exc. of DV value j for given Damage level k
% prDam(k,i) is the prob. of Damage Level k for given Drift value i
% prDr(i,m) is the prob. of Drift value i for the Intensity Measure m
%-----

% Equation 5a
Pr_Loss1=zeros(nLoss,np);
for j=1:nLoss      % for all the points of the DV curve
    for i=1:np      % for all the points of the EDP curve
        for k=1:3    % Sum over the Damage levels
            Pr_Loss1(j,i)=Pr_Loss1(j,i) + PrLoss(k,j)*prDam(k,i);
        end
    end
end

%Equation 5b
Pr_Loss2=zeros(nLoss,SF);
for j=1:nLoss      % for all the points of the DV curve
    for m=1:SF      % for all the points of the IM curve
        for i=1:np    % Sum over the EDP
            Pr_Loss2(j,m)=Pr_Loss2(j,m) + Pr_Loss1(j,i)*prDr(i,m);
        end
    end
end

% Compute the probabilities of Collapse and non-Collapse
for m=1:SF
    prC(m)=collapse(m)/20;
    prNC(m)=1-prC(m);
end

%Equation 5d
Pr_Loss3=zeros(nLoss,SF);
for j=1:nLoss      % for all the points of the DV curve
    for m=1:SF      % for all the points of the IM curve
        Pr_Loss3(j,m)=Pr_Loss2(j,m)*prNC(m) + PrLosC(j)*prC(m);
    end
end

%Equation 5e
Pr_Lossf=zeros(nLoss,1);
for j=1:nLoss      % For all the points of the DV curve
    for m=1:SF      % Sum over the IM
        Pr_Lossf(j)=Pr_Lossf(j) + Pr_Loss3(j,m)*prob_Sa(m);
    end
end
```

4.2 Excerpts from MATLAB script for unconditional simulation calculations

A MATLAB code has been developed for the unconditional probabilistic approach presented in §2.3. The code is written according to the object-oriented paradigm. Some familiarity with this way of programming allows easier reading, but it is not necessary to understand the main operations carried out within each function/method. Excerpts from this code and a brief explanation of different parts are presented below.

The probabilistic analysis according to the Monte Carlo simulation scheme starts with a loop over the total number of simulations (stored in the variable `dataAn{3,2}`) in which the variables `Z`, `M` and `E` are sampled from their distributions using appropriate methods. Once `E` is known `R` is evaluated with `Source2SiteDistance`.

```
for j = 1:dataAn{3,2}
    % calling method to sample Sourcei from discrete distribution.
    seismic.sampleSourceFromDiscreteDistribution(j);
    % calling method to sample (M,E)|Sourcei.
    seismic.event.sampleEventParameters(j);
    % compute source to site distance
    dist = seismic.event.Source2SiteDistance(flipdim(siteLocation,2),j);
    seismic.event.states(j,1).source2siteDistance = dist(3); % Rjb distance
end
```

The source is sampled with the method (function) `sampleSourceFromDiscreteDistribution` of the object `seismic`. A random number between 0 and 1 is sampled and compared with the discrete cumulative distribution function (CDF) of `Z` to find the corresponding source number. Source parameters are then assigned for the current event from the sampled source (like faulting mechanism).

```
function sampleSourceFromDiscreteDistribution(obj,kEvent)
tmpEvent = obj.event;
CDF = cumsum(obj.sourceDiscreteDistribution);
numSource = find(CDF>=rand,1,'first');
tmpEvent.states(kEvent,1).source = ...
    [numSource obj.sourceType(numSource) obj.sourceMech(numSource)]; % source #, source type,
    source mechanism
tmpEvent.states(kEvent,1).F = obj.sourceMech(numSource);
end
```

Other macro-seismic data for the current event are sampled from their distributions with parameters depending on the current source. The method/function is `sampleEventParameters`. Two options are given, depending on the type of source: 1) area source 2) finite fault source. In the former case the rupture is assumed to be located at a single point, in the latter the rupture has an extension proportional to the sampled magnitude (this method calls a further method for the rupture simulation, `FaultRuptureCASimulator`, which is not reported and is not used in the illustrative example).

```
function sampleEventParameters(obj,kEvent)
tmpSeis = obj.parent.source;
numSource = obj.states(kEvent,1).source(1);
beta = tmpSeis(numSource).beta;
m1 = tmpSeis(numSource).lowerM;
mu = tmpSeis(numSource).upperM;
v = (1-rand(1)*(1-exp(-beta*(mu-m1))))*exp(-beta*m1);
obj.states(kEvent,1).magnitude = -log(v)/beta;
switch obj.states(kEvent,1).source(2)
    case 2
        % Fault Source - Implement Cellular Automata based simulation
        [rupture, hypocentre, ztor, rupA] = FaultRuptureCASimulator(tmpSeis(numSource).mesh,...
            tmpSeis(numSource).faultClosePoint,tmpSeis(numSource).faultIDX,...
            obj.states(kEvent,1).magnitude,tmpSeis(numSource).mech,...
            tmpSeis(numSource).numericalArea,tmpSeis(numSource).meshRes);

        obj.states(kEvent,1).rupture = rupture; % [long lat depth]
        obj.states(kEvent,1).hypo = hypocentre;
        obj.states(kEvent,1).ztor = ztor;
        obj.states(kEvent,1).RupArea = rupA;
    case 1
        % Point Source - Simple Point Simulation
        d1 = tmpSeis(numSource).meshRes/6371.01;
        d1 = d1*(180/pi);
        % Sample epicentre point randomly from mesh
        nelem = size(tmpSeis(numSource).mesh,1);
        loc = ceil(nelem*rand);
```

```

    epicentre = tmpSeis(numSource).mesh(loc,:); % [long lat]
    % Shift epicentre randomly around sample point
    shift1 = dl*rand(1,2)-dl;
    hypocentre = epicentre+shift1;
    obj.states(kEvent,1).epi = epicentre; % [long lat]
    obj.states(kEvent,1).hypo = [hypocentre 10]; % [long lat depth]
    obj.states(kEvent,1).rupture = obj.states(kEvent,1).hypo;
    obj.states(kEvent,1).ztor = 10;
    obj.states(kEvent,1).RupArea = dl^2;
end
end

```

After generating all events, the input values of the faulting style F , the magnitude M and the distance R for the synthetic ground motion model by Rezaeian and der Kiureghian are known and the corresponding model (function `Rez(F, M, R, Vs30,0,dt)`) can be called to generate the ground motions at the site. Notice that the generation involves a nonlinear set of equations that is solved numerically and on extremely rare occasions (combinations of input parameters) does not yield a solution. In these cases no motion is sampled and the variable `pos` serves to purpose of recording them and removing them from the set of motions in order to avoid problems with the following structural analysis.

```

pos = [];
for iEvent = 1:dataAn{3,2}
    disp(['sample # ',num2str(iEvent),' out of ',num2str(dataAn{3,2})])
    F = mech(iEvent);
    M = magn(iEvent);
    R = dist(iEvent);
    RezADK(iEvent,1) = Rez(F, M, R, Vs30,0,dt);
    if ~isempty(RezADK(iEvent,1).flag)
        pos = [pos;iEvent];
    end
end
% delete aborted events
RezADK(pos) = [];

```

The following is the code that implements the Rezaeian and Der Kiureghian model. It starts with the `classdef` keyword which indicates that `Rez` is a class (actually a sub-class of the class `Signal`). A motion or a set of motions will be an object from this class (created with the method/function that has the same name of the class, which is called the constructor method), in the jargon of object-oriented programming. Two syntaxes are possible, one to sample motions specifying directly the inner model parameters (syntax 1), the second to sample motions starting from macro-seismic data (syntax 2).

```

%Earthquake Engng Struct. Dyn. 2010; 39:1155-1180
%Sanaz Rezaeian and Armen Der Kiureghian
%
%SYNTAX 1
%Rez(Ia, D5_95, tmid, wmid, w_, xsif, tmax, dt)
%
% Ia      Arias intensity of the signal
% D5_95   significant duration t95-t5, (s)
% tmid    reference instant t45, (s)
% wmid    circular frequency at the reference time instant t45, (rad/s)
% w_      first derivative of the filter circular frequency, at t45, (rad/s/s)
% xsif    filter damping ratio, (-)
% tmax    total duration, (s)
% dt      time step, (s)
%
%SYNTAX 2
%Rez(F, M, R, V, tmax, dt)
%
% F      F = 0 strike-slip fault, F =1 reverse fault
% M      moment magnitude
% R      Joiner Boore distance from the fault (Rjb), (km)
% V      vs30 average shear wave velocity in the upper 30 m, (m/s)
% tmax   total duration, (s) (set to 0 for automatic evaluation)
% dt     time step, (s)
%

```

```

classdef Rez < Signal
properties
    Ia, D5_95, tmid, wmid, w_, xsif, F, M, R, V
    alpha %parameters of the envelope function (alpha1, alpha2, alpha3)
    q %envelope function, eq.2
    I %Arias intensity as a function of time (tq)
    tq %time vector for q and I
    tt %time instants t5, t45 and t95 (arias = 5%, 45%, 95% Ia_max)
    flag % sampling is correct(empty) or incorrect (0)
end
properties(Constant)
    np = 200 %# of pts in q
    wc = 2*pi*0.1; %corner frequency, eq. 6
    q_limit = 0.10; %value of q where conventionally the record ends (to get tmax)
end
methods
    function obj=Rez(varargin)
        obj@Segnale();
        %-----
        if nargin>6 %syntax 1
            obj.Ia = varargin{1}; obj.D5_95 = varargin{2}; obj.tmid = varargin{3};
            obj.wmid = varargin{4}; obj.w_ = varargin{5}; obj.xsif = varargin{6};
            tmax = varargin{7};
            dt = varargin{8};
        else %syntax 2
            obj.F = varargin{1}; obj.M = varargin{2};
            obj.R = varargin{3}; obj.V = varargin{4};
            [obj.Ia, obj.D5_95, obj.tmid, obj.wmid, obj.w_, obj.xsif] = ...
                Rez.Parameters(obj.F, obj.M, obj.R, obj.V, 1);
            obj.Ia = obj.Ia * (2*9.81/pi)^2; %nb <<< conversione da articolo
            obj.wmid = obj.wmid * 2*pi; %nb <<< conversione da articolo
            obj.w_ = obj.w_ * 2*pi; %nb <<< conversione da articolo
            tmax = varargin{5};
            dt = varargin{6};
        end
        %-----
        %evaluates parameters of envelope q (from Ia, D5_95, tmid)
        obj.tq = linspace(0,obj.D5_95*3,obj.np);
        options = optimset('TolX',1e-4, 'TolFun',1e-3, 'Display','off');
        [x, F, exitflag] = fsolve(@obj.qsystem, [3 1], options);
        if exitflag~=1, warning('convergenza shape parameters'); end
        %-----
        %if tmax is not specified is evaluated from q
        if tmax==0, tmax = obj.tq( find(obj.q>=obj.q_limit*max(obj.q),1,'last') ); end
        %-----
        %filtering of stationary white noise
        t = 0:dt:tmax;
        a = zeros(size(t));
        s2 = zeros(size(t));
        for i=1:length(t)
            h = obj.hIRF(t, t(i));
            a = a + h * randn;
            s2 = s2 + h.^2;
        end
        s2(1) = 1;
        a = a ./ sqrt(s2);
        %-----
        %modulation through envelope function
        alpha = a .* interp1([obj.tq 1000], [obj.q 1e-6], t);
        %-----
        %base line correction through a high-pass filter
        [u, udot, u2dot] = Signal.newmark(t, alpha, 0, 0, 1, obj.wc^2, 1.0, 0.25, 0.5);
        %-----
        try
            obj.setSignal(t, u2dot);
        catch ME
            if strcmp(ME.identifier,'MATLAB:interp1:NaNInX')
                disp('...signal generation aborted')
                obj.flag = 0;
            end
        end
    end
end

function plot_IRF(obj, tau)
    plot(obj.t, obj.hIRF(obj.t, tau), 'r-'); grid on
end

function plot_q(obj)
    subplot(2,1,1); plot(obj.tq, obj.q, 'b-', obj.tt, interp1(obj.tq, obj.q, obj.tt), 'ro');
    xlabel('t - s'); ylabel('modulating function');
end

```

```

title(sprintf('D 5_, 9_5=%5.2f t_m i_d=%5.2f\nt_5=%5.2f t_4_5=%5.2f t_9_5=%5.2f\n
a_1=%5.3f a_2=%5.3f a_3=%5.3f', [obj.D5_95, obj.tmid, obj.tt, obj.alpha]));
subplot(2,1,2); plot(obj.tq, obj.I, 'b-', obj.tt, interp1(obj.tq, obj.I, obj.tt), 'ro');
title(sprintf('I_a=%8.4f I_a*=%8.4f', [obj.Ia, obj.I(end)]));
xlabel('t - s'); ylabel('arias intensity');
end

end

methods(Hidden)
function h = hIRF(obj, t, tau)
%unit impulse response function for the time-varying filter, eq. 3
dt = t(2)-t(1);
h = zeros(size(t));
wf = obj.wmid+obj.w*(tau-obj.tmid);
xx = sqrt(1-obj.xsif^2);
i = round(tau/dt)+1;
wf_tt = wf * ( t(i:end)-tau );
h(i:end) = wf/xx * exp(-obj.xsif*wf_tt) .* sin(wf_tt*xx);
end

function F = qsystem(obj, x)
%systems of nonlinear equations in the parameters of the envelope q, eq. 10
if x(1)<=1.0, x(1) = 1.00001; end
if x(2)<=0.0, x(2) = 0.00001; end

obj.alpha(2) = x(1);
obj.alpha(3) = x(2);
tmp = 2*9.81/pi; %da assumere per far tornare il calcolo con Ia prescritta !
tmp = 1; %<< cfr. fattore di conversione da articolo
obj.alpha(1) = sqrt( obj.Ia*tmp*(2*obj.alpha(3))^(2*obj.alpha(2)-
1)/gamma(2*obj.alpha(2)-1));

obj.q = obj.alpha(1)*obj.tq.^ (obj.alpha(2)-1) .*exp(-obj.alpha(3)*obj.tq);
dt = obj.tq(2)-obj.tq(1);
obj.I = cumtrapz(obj.q.^2)*dt*(pi/(2*9.81))+obj.tq/1e6; %arias intensity
obj.tt = interp1(obj.I,obj.tq,[0.05 0.45 0.95]*obj.I(end)); %t5, t45, t95

F = [obj.D5_95-(obj.tt(3)-obj.tt(1)) obj.tmid-obj.tt(2)]; %to be minimized by MATLAB
solver
end

end

methods(Static)
function [Ia, D5_95, tmid, wmid, w_, xsif] = Parameters(F, M, R, V, switchValues)
% switchValues = 0 -> mean values
% switchValues = 1 -> mean values + variability

np = length(F);
if nargin<5 || isempty(switchValues), switchValues=1; end

% beta1 beta2 beta3 beta4 beta5 tau sigma
data = [-1.844 -0.071 2.944 -1.356 -0.265 0.274 0.594;... %Ia
-6.195 -0.703 6.792 0.219 -0.523 0.457 0.569;... %D5_95
-5.011 -0.345 4.638 0.348 -0.185 0.511 0.414;... %tmid
2.253 -0.081 -1.810 -0.211 0.012 0.692 0.723;... %wmid
-2.489 0.044 2.408 0.065 -0.081 0.129 0.953;... %w_
-0.258 -0.477 0.905 -0.289 0.316 0.682 0.760]; %xsif

rho = [ 1.0 -0.36 0.01 -0.15 0.13 -0.01; ...
-0.36 1.0 0.67 -0.13 -0.16 -0.20; ...
0.01 0.67 1.0 -0.28 -0.20 -0.22; ...
-0.15 -0.13 -0.28 1.0 -0.20 0.28; ...
0.13 -0.16 -0.20 -0.20 1.0 -0.01; ...
-0.01 -0.20 -0.22 0.28 -0.01 1.0];

D = diag( sqrt(data(:,6).^2 + data(:,7).^2) );

nu = [data( 1,1:5) * [ones(1,np); F; M/7; log(R/25); log(V/750)] ; ...
data(2:6,1:5) * [ones(1,np); F; M/7; R/25 ; V/750 ] ];

if switchValues==0
%nothing
elseif switchValues==1
nu = nu + D*chol(rho)'*randn(6,np);
end

tmp = cdf('norm', nu, 0, 1);

md = 0.0468; vr = (0.164)^2; Ia = icdf('lognormal', tmp(1,:), log(md^2/sqrt(vr+md^2)),
sqrt(log(vr/md^2+1)));
md = 5.87; vr = (3.11)^2; wmid = icdf('gamma', tmp(4,:), md^2/vr, vr/md);

D5_95 = Rez.invbeta( tmp(2,:), 17.3, 9.31, 5, 45);
tmid = Rez.invbeta( tmp(3,:), 12.4, 7.44, 0.5, 40);

```

```

xsisf = Rez.invbeta( tmp(6,:), 0.213, 0.143, 0.02, 1);

%inverse cdf of the two-sided exponential, eq. 15
i = tmp(5,:) > 0.7163949205;
w_ = 0.1477104874 * log(0.000001317203012 + 1.395876289 * tmp(5,:));
w_(i) = -0.0524790747527685 * log(3.525846007 - 3.525773196 * tmp(5,i));
end

function Y = invbeta(x, md, dv, lbound, ubound)
%inverse cdf of beta with bounds different from 0 and 1
md_ = (md-lbound)/(ubound-lbound);
vr_ = (dv/(ubound-lbound))^2;
tmp = md * (1-md_) / vr_-1;
alfa = md_*tmp;
beta = (1-md_)*tmp;
Y = icdf('beta', x, alfa, beta) * (ubound-lbound) + lbound;
end
end
end

```

Once the motion time-series have been generated the corresponding hazard curve at any vibration period can be obtained by simple post processing of the spectral amplitudes (which are computed automatically through a function/method of the container class `Signal`). If the spectral ordinates at the period of interest, say T_1 , are stored in `SaT1mcs`, the following code evaluates the corresponding MAF with the built-in MATLAB function `ecdf`:

```

% HAZARD CURVE FROM MCS
[CDF,Sa] = ecdf(SaT1mcs);
CCDF = 1-CDF;
MAF_Sa_MCS = lambda0 * CCDF;

```

If a script to run FE analysis with the sampled motions and record the corresponding $\theta_{\max}$ values is used, the evaluation of the structural MAF is carried out in an analogous manner.

Probabilistic analysis with the Importance Sampling Simulation is more involved. In this case the loop over the total number of runs starts after the function `sampleMagnitudeIS` has produced a corresponding number of magnitudes (stratified sampling). The active source in each event is then sampled conditionally on the current M value (see Fig. 2-38, left).

```

% calling method to sample M.
seismic.event.sampleMagnitudeIS(dataAn{6,2});
for j = 1:length(seismic.event.states)
    % calling method to compute source and epicentre.
    seismic.event.computeSourceIS(j);
    % compute source to site distance
    dist = seismic.event.Source2SiteDistance(flipdim(siteLocation,2),j);
    seismic.event.states(j,1).source2siteDistance = dist(3); % Rjb distance
end

```

The following function, a method from the `event` class, implements the stratified sampling procedure described in §2.3.2.3.

```

function sampleMagnitudeIS(obj,totMaps)
% iteratively change the M discretization so to have all the weights precisely equal to 1.
global mapTot CDFm point
mapTot = totMaps;
tmpSeis = obj.parent;
tmpSource = obj.parent.source;
% STEP 1: compute magnitude CDF for all sources
Mdiscr = linspace(tmpSeis.MvectorIS(1),tmpSeis.MvectorIS(end),1e6);
fun = zeros(tmpSeis.sourceNumber,length(Mdiscr));
for iSource = 1:tmpSeis.sourceNumber
    alpha(iSource) = tmpSource(iSource).alpha;
    beta(iSource) = tmpSource(iSource).beta;
    upM(iSource) = tmpSource(iSource).upperM;
    lowM(iSource) = tmpSource(iSource).lowerM;
    Madm = find(Mdiscr>=lowM(iSource) & Mdiscr<=upM(iSource));

```

```

    fun(iSource,Madm) = alpha(iSource)*(1-((exp(-beta(iSource)*Mdiscr(Madm))-exp(-
beta(iSource)*upM(iSource))))...
    / (exp(-beta(iSource)*lowM(iSource))-exp(-beta(iSource)*upM(iSource))));
    if ~isempty(find((Mdiscr>upM(iSource))))
        fun(iSource,Mdiscr>upM(iSource)) = alpha(iSource);
    end
end
if size(fun,1) > 1
    magnCDF = sum(fun) / sum(tmpSeis.sourceAlpha);
else
    magnCDF = fun / tmpSeis.sourceAlpha;
end
% STEP 2: refine MvectorIS so to have the M weights equal to 1
for inter = 1:length(tmpSeis.MvectorIS)-1
    interval = [tmpSeis.MvectorIS(inter) tmpSeis.MvectorIS(inter+1)];
    indices = find(Mdiscr>=interval(1) & Mdiscr<=interval(2));
    CDFinter = magnCDF(indices);
    mapsPerMagnitude0(inter,1) = round((max(CDFinter)-min(CDFinter)) * totMaps);
end
Mmin = tmpSeis.MvectorIS(1);
Mmax = tmpSeis.MvectorIS(end);
options = optimset('fsolve');
options.TolFun = 1e-8;
options.TolX = 1e-8;
options.LargeScale = 'off';
options.NonlEqnAlgorithm = 'gn';
options.LineSearchType = 'cubicpoly';
CDFm(1) = 0;
Mvector(1) = Mmin;
for point = 2:length(tmpSeis.MvectorIS)-1
    CDFiplus1_0 = interp1(Mdiscr,magnCDF,tmpSeis.MvectorIS(point));
    Ni_0 = mapsPerMagnitude0(point-1);
    X0 = [CDFiplus1_0;Ni_0];

    [sol,~,exitflag] = fsolve(@discrM,X0,options);
    if exitflag <= 0
        disp('Optimization of magnitude discretization stopped')
        break
    end
    CDFm(point) = sol(1);
    mapsPerMagnitude(point-1,1) = sol(2);
    Mvector(point) = interp1(magnCDF,Mdiscr,CDFm(point));
    if isnan(Mvector(point))
        break
    end
end
if exitflag <= 0 || isnan(Mvector(point))
    for inter = 1:length(tmpSeis.MvectorIS)-1
        interval = [tmpSeis.MvectorIS(inter) tmpSeis.MvectorIS(inter+1)];
        indices = find(Mdiscr>=interval(1) & Mdiscr<=interval(2));
        CDFinter = magnCDF(indices);
        mapsPerMagnitude(inter,1) = round((max(CDFinter)-min(CDFinter)) * totMaps);
    end
else
    CDFm(length(tmpSeis.MvectorIS)) = 1;
    Mvector(length(tmpSeis.MvectorIS)) = Mmax;
    mapsPerMagnitude(length(tmpSeis.MvectorIS)-1,1) = mapTot - sum(mapsPerMagnitude);
    tmpSeis.MvectorIS = Mvector;
end
% STEP 3: sample M for each interval in MvectorIS
numStates = 0;
for inter = 1:length(tmpSeis.MvectorIS)-1
    if mapsPerMagnitude(inter) == 0
        continue
    else
        interval = [tmpSeis.MvectorIS(inter) tmpSeis.MvectorIS(inter+1)];
        indices = find(Mdiscr>=interval(1) & Mdiscr<=interval(2));
        Minter = Mdiscr(indices);
        CDFinter = magnCDF(indices);

        prob = min(CDFinter) + rand(1,mapsPerMagnitude(inter))*(max(CDFinter)-min(CDFinter));
        M(1,:) = interp1(CDFinter,Minter,prob);
        tmp = num2cell(M(:));
        [obj.states(numStates+1:numStates+mapsPerMagnitude(inter),1).magnitude] = tmp{:};
        clear M
        ISweight = (max(CDFinter)-min(CDFinter))/(mapsPerMagnitude(inter)/sum(mapsPerMagnitude));
        [obj.states(numStates+1:numStates+mapsPerMagnitude(inter),1).magnISweight] =
deal(ISweight);

```

```
[obj.states(numStates+1:numStates+mapsPerMagnitude(inter),1).totalISweight] =
deal(ISweight);

    numStates = numStates + mapsPerMagnitude(inter);
end
end
clear global
end
```

The following function samples source and epicenter conditional on the current event magnitude.

```
function computeSourceIS(obj,kEvent)
tmpSeis = obj.parent;
tmpSource = obj.parent.source;
% STEP 1: compute probabilities for all sources
M = obj.states(kEvent,1).magnitude;
for iSource = 1:tmpSeis.sourceNumber
    alpha(iSource) = tmpSource(iSource).alpha;
    beta(iSource) = tmpSource(iSource).beta;
    upM(iSource) = tmpSource(iSource).upperM;
    lowM(iSource) = tmpSource(iSource).lowerM;
    if ~isempty(find(M>=lowM(iSource) & M<=upM(iSource), 1))
        pdfun(iSource,kEvent) = beta(iSource)*exp(-beta(iSource)*M) /...
            (exp(-beta(iSource)*lowM(iSource))-exp(-beta(iSource)*upM(iSource)));
    else
        pdfun(iSource,kEvent) = 0;
    end
    prob(iSource) = alpha(iSource)*pdfun(iSource,kEvent);
end
prob = prob ./ sum(prob);
% STEP 2: sample source
CDF = cumsum(prob);
numSource = find(CDF>=rand,1,'first');
obj.states(kEvent,1).source = ...
    [numSource tmpSeis.sourceType(numSource) tmpSeis.sourceMech(numSource)]; % source #, source
type, source mechanism
obj.states(kEvent,1).F = tmpSeis.sourceMech(numSource);
% STEP 3: sample epicentre
switch obj.states(kEvent,1).source(2)
    case 2
        % Fault Source - Implement Cellular Automata based simulation
        [rupture, hypocentre, ztor, rupA] = FaultRuptureCASimulator(tmpSource(numSource).mesh,...
            tmpSource(numSource).faultClosePoint,tmpSource(numSource).faultIDX,...
            obj.states(kEvent,1).magnitude,tmpSource(numSource).mech,...
            tmpSource(numSource).numericalArea,tmpSource(numSource).meshRes);

        obj.states(kEvent,1).rupture = rupture; % [long lat depth]
        obj.states(kEvent,1).hypo = hypocentre;
        obj.states(kEvent,1).ztor = ztor;
        obj.states(kEvent,1).RupArea = rupA;
    case 1
        % Point Source - Simple Point Simulation
        dl = tmpSource(numSource).meshRes/6371.01;
        dl = dl*(180/pi);
        % Sample epicentre point randomly from mesh
        nelem = size(tmpSource(numSource).mesh,1);
        loc = ceil(nelem*rand);
        epicentre = tmpSource(numSource).mesh(loc,:); % [long lat]
        % Shift epicentre randomly around sample point
        shift1 = dl*rand(1,2)-dl;
        hypocentre = epicentre+shift1;
        obj.states(kEvent,1).epi = epicentre; % [long lat]
        obj.states(kEvent,1).hypo = [hypocentre 10]; % [long lat depth]
        obj.states(kEvent,1).rupture = [hypocentre 10];
        obj.states(kEvent,1).ztor = 10;
        obj.states(kEvent,1).RupArea = dl^2;
end
end
```

As for the MCS case, macro-seismic data for the events are used as an input to the ground motion model to sample the time-series at the site. Once these, are known spectral ordinates and structural response values obtained from FE analysis can be post processed to yield the corresponding MAFs. Evaluation of the hazard curves cannot use in the case the built-in function `ecdf` and uses instead the IS weights to compute the CDF.

```
% HAZARD CURVE FROM ISS
[SaISS,ord] = sort(SaTliss);
w = weights(ord);
prob = w/sum(w);
CDF = cumsum(prob);
CCDF = 1-CDF;
MAF_Sa_ISS = lambda0 * CCDF;
```

4.3 Excerpts from MATLAB script for TS algorithm calculations

Here, the relevant parts of the Matlab script that is used to obtain the optimal properties of the RC frame example are provided.

```
% Obtain the Pareto-optimal solutions for a given earthquake intensity

clear; clc; close all

% Load the variables that includes all the combinations of design variables
% and the corresponding initial cost
load cost

% Initialize the story height
H=3048;

% Initialize the output requests from the finite element analysis
outrequests={'ND:n1:x';...
            'ND:n7:x';...
            'ND:n13:x';...
            'ND:n24:x';...
            'ND:n30:x';...
            'ND:n36:x';...
            'ND:n47:x';...
            'ND:n53:x';...
            'ND:n59:x';...
            };

% Initialize other variables of the finite element analysis
timeend=13; dt=0.0025; data_freq_dynamic=4;
filename='2S2B_optim_frame.dat'; % input file name for ZEUS NL
EQinputfile='2475y_EQ.txt';
conc_prop=[35 2.7 0.0035 1
           35 2.7 0.0035 1.1];
steel_prop=[210000 0.001 250 17];

% Sort the combinations based on the initial cost
combsRC=[combs zeros(size(combs,1),1)];
[CRCS,IX]=sort(CRCS(:,11),'ascend');
combsS=combsRC(IX,:);
store=[CRCS combsS];

% Set the parameters of the TS algorithm
maxnumiter=375;
allindices=(1:size(store,1))';
numneigh=8;
```

```

var=1:8;
combs=store(:,2:end);

for ii=1:numneigh;
    varind=setdiff(var,ii);
    allvar{ii}=combs(:,varind);
end

% Initialize a pool of 8 processor for parallel processing in TS algorithm
matlabpool open 8

% Initialize the memory structures and TS variables
count=1;
seed=store(1,2:end); seedin=1;
taboolist=[1]; seedlist=[1]; Paretolist=[]; results=[];
% Start TS
while count<=maxnumiter;
    % Create the neighbouring points around the current seed point
    for ii=1:numneigh;
        fixind=mod(ii,numneigh); if fixind==0; fixind=numneigh; end
        varind=setdiff(var,fixind);
        seedvar=seed(varind);
        tf=ismember(allvar{fixind},seedvar,'rows');
        indchoices=find(tf==1);
        validind=setdiff(indchoices,taboolist);
        if isempty(validind)==1;
            validind=setdiff(allindices,taboolist);
        end
        rndnum1=unidrnd(size(validind,1));
        nindices(ii,1)=validind(rndnum1);
        taboolist=[taboolist; validind(rndnum1)];
    end
    % Perform the inelastic dynamic analysis for the neighbouring points
    parfor jj=1:size(nindices,1); % Use parallel processing
        dirname=['dir',num2str(jj)];
        mkdir(dirname);
        copyfile('Afunction.m',[dirname,'\']);
        copyfile(EQinputfile,[dirname,'\']);
        copyfile(filename,[dirname,'\']);
        copyfile('Reader.exe',[dirname,'\']);
        copyfile('Solver.exe',[dirname,'\']);
        copyfile('RunZeus.bat',[dirname,'\']);
        copyfile('ZEUSPostM.m',[dirname,'\']);
        cd(dirname);
        dec_var=combs(nindices(jj),:);
        Afunction(filename,conc_prop,ecc_prop,steel_prop,dec_var,...
            EQinputfile,timeend,dt,data_freq_dynamic);
        dos('runzeus.bat');
        resultsfile='result.num';
        [output]=ZEUSPostM(resultsfile,outrequests);
        cd('..');
        rmdir(dirname,'s');
        time=output.time;
        intdrifts=[output.NDout{2,1}-output.NDout{1,1} output.NDout{5,1}-...
            output.NDout{4,1} output.NDout{8,1}-output.NDout{7,1} ...
            output.NDout{3,1}-output.NDout{2,1} output.NDout{6,1}- ...
            output.NDout{5,1} output.NDout{9,1}-output.NDout{8,1}]/H*100;
        maxid=max(abs(intdrifts));
        maxid=[maxid(1:3); maxid(4:end)];
    end
    count=count+1;
end

```

```

        maxid=max(max(maxid));
        if time(end)~=timeend;
            resultstemp(jj,1)=1000;
        else
            resultstemp(jj,1)=maxid;
        end
    end

% Compile the results
results=[results; [nindices resultstemp]];
ofv=[nindices resultstemp store(nindices,:)];
Paretolisttemp=[Paretolist; ofv];

% Find the Pareto-front for the results obtained up to current step
candidates1=PFplot_taboo(ofv,2,3,10,Inf);
candidates2=PFplot_taboo(Paretolisttemp,2,3,10,Inf);
[candidates,ia,ib]=intersect(candidates1(:,2),candidates2(:,2));
candidates=candidates1(ia,:);

% Find the seed combination for the next iteration
if isempty(candidates)==1;
    possibleseeds=setdiff(Paretolist(:,1),seedlist);
    if isempty(possibleseeds)==1;
        validind=setdiff(allindices,taboolist);
        rndnum1=unidrnd(size(validind,1));
        seed=combs(validind(rndnum1),:);
        seedlist=[seedlist; validind(rndnum1)];
    else
        rndnum1=unidrnd(size(possibleseeds,1));
        seed=store(possibleseeds(rndnum1),2:end);
        seedlist=[seedlist; possibleseeds(rndnum1)];
    end
else
    [B,IX]=sort(candidates(:,3));
    candidates=candidates(IX,:);
    seed=candidates(1,4:end);
    seedlist=[seedlist; candidates(1,1)];
end
Paretolist=candidates2;
[B,IX]=sort(Paretolist(:,3));
Paretolist=Paretolist(IX,:);
count=count+1;

end

% Visualize the results
figure; plot(results(:,2),CRCS(results(:,1)),'.b','linewidth',2); grid;
hold on; plot(Paretolist(:,2),Paretolist(:,3),'--om','linewidth',2);
xlabel('Drift (%)'); ylabel('Initial Cost ($)');

figure; plot(Paretolist(:,2),Paretolist(:,3),'--or','linewidth',2); grid;
legend('Taboo Search');
xlabel('Drift (%)'); ylabel('Initial Cost ($)');

```

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